Hypothesis testing 4B

1 a $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 5.5 \quad \text{H}_1 : \lambda < 5.5$

Assume H_0, so that $X \sim \text{Po}(5.5)$

Significance level 5%, so require $\text{P}(X \leqslant c) < 0.05$

From the tables

$\text{P}(X \leqslant 1) = 0.0266$ and $\text{P}(X \leqslant 2) = 0.0884$

$\text{P}(X \leqslant 1) < 0.05$ and $\text{P}(X \leqslant 2) > 0.05$ so the critical value is 1

Hence the critical region is $X \leqslant 1$

Actual significance level $= \text{P}(X \leqslant 1) = 0.0266$

b $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 8 \quad \text{H}_1 : \lambda > 8$

Assume H_0, so that $X \sim \text{Po}(8)$

Significance level 1%, so require $\text{P}(X \geqslant c) < 0.01$

From the tables

$\text{P}(X \geqslant 15) = 1 - \text{P}(X \leqslant 14) = 1 - 0.9827 = 0.0173$

and $\text{P}(X \geqslant 16) = 1 - \text{P}(X \leqslant 15) = 1 - 0.9918 = 0.0082$

$\text{P}(X \geqslant 15) > 0.01$ and $\text{P}(X \geqslant 16) < 0.01$ so the critical value is 16

Hence the critical region is $X \geqslant 16$

Actual significance level $= \text{P}(X \geqslant 16) = 0.0082$

c $X \sim \text{Po}(\lambda)$

$\text{H}_0 : \lambda = 4 \quad \text{H}_1 : \lambda > 4$

Assume H_0, so that $X \sim \text{Po}(4)$

Significance level 5%, so require $\text{P}(X \geqslant c) < 0.05$

From the tables

$\text{P}(X \geqslant 8) = 1 - \text{P}(X \leqslant 7) = 1 - 0.9489 = 0.0511$

and $\text{P}(X \geqslant 9) = 1 - \text{P}(X \leqslant 8) = 1 - 0.9786 = 0.0214$

$\text{P}(X \geqslant 8) > 0.05$ and $\text{P}(X \geqslant 9) < 0.05$ so the critical value is 9

Hence the critical region is $X \geqslant 9$

Actual significance level $= \text{P}(X \geqslant 9) = 0.0214$

2 Let the random variable X denote the number of fish caught by the fisherman in a two-hour period.

$\text{H}_0 : \lambda = 2 \times 5 = 10 \quad \text{H}_1 : \lambda > 10$

Assume H_0, so that $X \sim \text{Po}(10)$

Significance level 5%, so require $\text{P}(X \geqslant c) < 0.05$

From the tables

$\text{P}(X \geqslant 15) = 1 - \text{P}(X \leqslant 14) = 1 - 0.9165 = 0.0835$

and $\text{P}(X \geqslant 16) = 1 - \text{P}(X \leqslant 15) = 1 - 0.9513 = 0.0487$

$\text{P}(X \geqslant 15) > 0.05$ and $\text{P}(X \geqslant 16) < 0.05$ so the critical value is 16

Hence the critical region is $X \geqslant 16$

3 Let the random variable X denote the number of sales made by Hans in a 10-day period.

$H_0: \lambda = 10 \times 0.8 = 8 \quad H_1: \lambda > 8$

Assume H_0, so that $X \sim \text{Po}(8)$

Significance level 5%, so require $P(X \geqslant c) < 0.05$

From the tables

$P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9362 = 0.0638$

and $P(X \geqslant 14) = 1 - P(X \leqslant 13) = 1 - 0.9658 = 0.0342$

$P(X \geqslant 13) > 0.05$ and $P(X \geqslant 14) < 0.05$ so the critical value is 14

Hence the critical region is $X \geqslant 14$

4 Let the random variable X denote the number of defects found in $25\,\text{m}^2$ of cloth.

$H_0: \lambda = 5 \times 1.3 = 6.5 \quad H_1: \lambda < 6.5$

Assume H_0, so that $X \sim \text{Po}(6.5)$

Significance level 5%, so require $P(X \leqslant c) < 0.05$

From the tables

$P(X \leqslant 2) = 0.0430$ and $P(X \leqslant 3) = 0.1118$

$P(X \leqslant 2) < 0.05$ and $P(X \leqslant 3) > 0.05$ so the critical value is 2

Hence the critical region is $X \leqslant 2$

5 Let the random variable X denote the number of accidents at the crossroads in a 12-month period.

$H_0: \lambda = 12 \times 0.6 = 6 \quad H_1: \lambda < 6$

Assume H_0, so that $X \sim \text{Po}(6)$

Significance level 5%, so require $P(X \leqslant c) < 0.05$

From the tables

$P(X \leqslant 1) = 0.0174$ and $P(X \leqslant 2) = 0.0620$

$P(X \leqslant 1) < 0.05$ and $P(X \leqslant 2) > 0.05$ so the critical value is 1

Hence the critical region is $X \leqslant 1$

6 Let the random variable X denote the number of sales of the computer game in a 20-day period.

$H_0: \lambda = 20 \times 0.35 = 7 \quad H_1: \lambda > 7$

Assume H_0, so that $X \sim \text{Po}(7)$

Significance level 5%, so require $P(X \geqslant c) < 0.05$

From the tables

$P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.9467 = 0.0533$

and $P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9730 = 0.0270$

$P(X \geqslant 12) > 0.05$ and $P(X \geqslant 13) < 0.05$ so the critical value is 13

Hence the critical region is $X \geqslant 13$

7 a $H_0 : \lambda = 4 \quad H_1 : \lambda \neq 4$

Assume H_0, so that $X \sim \text{Po}(4)$

Significance level 5%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1)$ to be as close as possible to 2.5%

From the tables

$P(X = 0) = 0.0183$ and $P(X \leqslant 1) = 0.0916$

0.0183 is closer to 0.025, so $c_1 = 0$ and the lower critical region is $X = 0$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2)$ to be as close as possible to 2.5%

From the tables

$P(X \geqslant 8) = 1 - P(X \leqslant 7) = 1 - 0.9489 = 0.0511$

and $P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9786 = 0.0214$

0.0214 is closer to 0.025, so $c_2 = 9$ and the upper critical region is $X \geqslant 9$

Critical region is $X = 0$ or $X \geqslant 9$

Actual significance level $= P(X = 0) + P(X \geqslant 9) = 0.0183 + 0.0214 = 0.0397$

b $H_0 : \lambda = 8 \quad H_1 : \lambda \neq 8$

Assume H_0, so that $X \sim \text{Po}(8)$

Significance level 5%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1)$ to be as close as possible to 2.5%

From the tables

$P(X \leqslant 2) = 0.0138$ and $P(X \leqslant 3) = 0.0424$

0.0138 is closer to 0.025, so $c_1 = 2$ and the lower critical region is $X \leqslant 2$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2)$ to be as close as possible to 2.5%

From the tables

$P(X \geqslant 14) = 1 - P(X \leqslant 13) = 1 - 0.9658 = 0.0342$

and $P(X \geqslant 15) = 1 - P(X \leqslant 14) = 1 - 0.9827 = 0.0173$

0.0173 is closer to 0.025, so $c_2 = 15$ and the upper critical region is $X \geqslant 15$

Critical region is $X \leqslant 2$ or $X \geqslant 15$

Actual significance level $= P(X \leqslant 2) + P(X \geqslant 15) = 0.0138 + 0.0173 = 0.0311$

7 c $H_0: \lambda = 9.5 \quad H_1: \lambda \neq 9.5$

Assume H_0, so that $X \sim \text{Po}(9.5)$

Significance level 5%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1)$ to be as close as possible to 2.5%

From the tables

$P(X \leqslant 3) = 0.0149$ and $P(X \leqslant 4) = 0.0403$

0.0149 is closer to 0.025, so $c_1 = 3$ and the lower critical region is $X \leqslant 3$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2)$ to be as close as possible to 2.5%

From the tables

$P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.9665 = 0.0335$

and $P(X \geqslant 17) = 1 - P(X \leqslant 16) = 1 - 0.9823 = 0.0177$

0.0177 is closer to 0.025, so $c_2 = 17$ and the upper critical region is $X \geqslant 17$

Critical region is $X \leqslant 3$ or $X \geqslant 17$

Actual significance level $= P(X \leqslant 3) + P(X \geqslant 17) = 0.0149 + 0.0177 = 0.0326$

8 a Let the random variable X denote the number of incoming calls made in a 30-minute interval.

$H_0: \lambda = 30 \times 0.25 = 7.5 \quad H_1: \lambda \neq 7.5$

Assume H_0, so that $X \sim \text{Po}(7.5)$

Significance level 5%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1)$ to be as close as possible to 2.5%

From the tables

$P(X \leqslant 2) = 0.0203$ and $P(X \leqslant 3) = 0.0591$

0.0203 is closer to 0.025, so $c_1 = 2$ and the lower critical region is $X \leqslant 2$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2)$ to be as close as possible to 2.5%

From the tables

$P(X \geqslant 13) = 1 - P(X \leqslant 12) = 1 - 0.9573 = 0.0427$

and $P(X \geqslant 14) = 1 - P(X \leqslant 13) = 1 - 0.9784 = 0.0216$

0.0216 is closer to 0.025, so $c_2 = 14$ and the upper critical region is $X \geqslant 14$

Critical region is $X \leqslant 2$ or $X \geqslant 14$

b Actual significance level $= P(X \leqslant 2) + P(X \geqslant 14) = 0.0203 + 0.0216 = 0.0419$

c $X = 11$ is not in the critical region, so there is insufficient evidence to reject H_0 and to conclude that the rate of incoming calls has changed.

9 a Let the random variable X denote the number of defects found in a 35 m length of material.

$H_0: \lambda = 35 \times \frac{1}{7} = 5 \quad H_1: \lambda \neq 5$

Assume H_0, so that $X \sim \text{Po}(5)$

Significance level 10%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1) < 0.05$

From the tables

$P(X \leqslant 1) = 0.0404$ and $P(X \leqslant 2) = 0.1247$

$P(X \leqslant 1) < 0.05$ and $P(X \leqslant 2) > 0.05$ so $c_1 = 1$ and the lower critical region is $X \leqslant 1$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2) < 0.05$

From the tables

$P(X \geqslant 9) = 1 - P(X \leqslant 8) = 1 - 0.9319 = 0.0681$

and $P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9682 = 0.0318$

$P(X \geqslant 9) > 0.05$ and $P(X \geqslant 10) < 0.05$ so $c_2 = 10$ and the upper critical region is $X \geqslant 10$

Critical region is $X \leqslant 1$ or $X \geqslant 10$

b Actual significance level $= P(X \leqslant 1) + P(X \geqslant 10) = 0.0404 + 0.0318 = 0.0722$

10 a A Poisson distribution would be suitable if emails arrive independently and at random, and at a constant average rate.

b Let the random variable X denote the number of emails received in a 15-minute period.

$H_0: \lambda = 3 \times 3 = 9 \quad H_1: \lambda \neq 9$

Assume H_0, so that $X \sim \text{Po}(9)$

Significance level 5%

If $X = c_1$ is the upper boundary of the lower critical region, require $P(X \leqslant c_1)$ to be as close as possible to 2.5%

From the tables

$P(X \leqslant 3) = 0.0212$ and $P(X \leqslant 4) = 0.0550$

0.0212 is closer to 0.025, so $c_1 = 3$ and the lower critical region is $X \leqslant 3$

If $X = c_2$ is the lower boundary of the upper critical region, require $P(X \geqslant c_2)$ to be as close as possible to 2.5%

From the tables

$P(X \geqslant 15) = 1 - P(X \leqslant 14) = 1 - 0.9585 = 0.0415$

and $P(X \geqslant 16) = 1 - P(X \leqslant 15) = 1 - 0.9780 = 0.0220$

0.0220 is closer to 0.025, so $c_2 = 16$ and the upper critical region is $X \geqslant 16$

Critical region is $X \leqslant 3$ or $X \geqslant 16$

c Actual significance level $= P(X \leqslant 3) + P(X \geqslant 16) = 0.0212 + 0.0220 = 0.0432$

d $X = 13$ is not in the critical region, so there is insufficient evidence to reject H_0 and to conclude that the mean rate is different to 9 every 15 minutes (or 3 every 5 minutes as claimed by the company).

11 a From the cumulative probability tables,

$P(X \leqslant 2) = 0.0296$ for $\lambda = 7$, and $P(X \leqslant 2) = 0.0138$ for $\lambda = 8$

This means $c \geqslant 8$

Considering the upper critical region

$P(X \geqslant 15) = 1 - P(X \leqslant 14) = 1 - 0.9585 = 0.0415$ for $\lambda = 9$

and $P(X \geqslant 15) = 1 - P(X \leqslant 14) = 1 - 0.9827 = 0.0173$ for $\lambda = 8$

This means $c \leqslant 8$

The only positive interger satisfying both conditions is $c = 8$

b $P(X \leqslant 2) + P(X \geqslant 15) = 0.0138 + 0.0173 = 0.0311$