Poisson distributions 2G

1 a i $P(X=4)=\binom{100}{4}\times(0.05)^4\times(0.95)^{96}=0.1781$ (4 d.p.)

ii Using a calculator:

$P(X\leqslant 2)=0.1183$ (4 d.p.)

b Use the approximation $X\sim \text{Po}(100\times 0.05)$, i.e. $X\sim \text{Po}(5)$

i $P(X=4)=\dfrac{e^{-5}5^4}{4!}=0.1755$ (4 d.p.)

ii Using the tables on page 191 of the textbook:

$P(X\leqslant 2)=0.1247$

2 a i $P(X=5)=\binom{150}{5}\times(0.04)^5\times(0.96)^{145}=0.1628$ (4 d.p.)

ii Using a calculator:

$P(X\leqslant 3)=0.1458$ (4 d.p.)

b Use the approximation $X\sim \text{Po}(150\times 0.04)$, i.e. $X\sim \text{Po}(6)$

i $P(X=5)=\dfrac{e^{-6}6^5}{5!}=0.1606$ (4 d.p.)

ii Using the tables in the textbook:

$P(X\leqslant 3)=0.1512$

3 a Let $X=200-Y$ so that $X\sim B(200,0.02)$

i $P(Y=197)=P(X=3)=\binom{200}{3}\times(0.02)^3\times(0.98)^{197}=0.1963$ (4 d.p.)

ii Using a calculator:

$P(Y\geqslant 198)=P(X\leqslant 2)=0.2351$ (4 d.p.)

b Use the approximation $X\sim \text{Po}(200\times 0.02)$, i.e. $X\sim \text{Po}(4)$

i $P(X=3)=\dfrac{e^{-4}4^3}{3!}=0.1954$ (4 d.p.)

ii Using the tables in the textbook:

$P(X\leqslant 2)=0.2381$

4 Let X be the number of pupils with a birthday on 1 April, assuming a uniform distribution of birthdays across 365 days of the year.

Then $X\sim B\left(800,\dfrac{1}{365}\right)$

a $P(X=4)=\binom{800}{4}\times\left(\dfrac{1}{365}\right)^4\times\left(\dfrac{364}{365}\right)^{796}=0.1075$ (4 d.p.)

b Use the approximation $X\sim \text{Po}\left(800\times\dfrac{1}{365}\right)$, i.e. $X\sim \text{Po}\left(\dfrac{160}{73}\right)$

$P(X=4)=\dfrac{e^{-\frac{160}{73}}\left(\frac{160}{73}\right)^4}{4!}=0.1074$ (4 d.p.)

4 c The two answers are very similar, which shows that the Poisson approximation is a good approximation in this case. This is to be expected as the Poisson approximation is extremely accurate for this very low value p and high n.

5 Let X be the number of defective items in a batch of 100. Then $X \sim \mathrm{B}(100, 0.03)$ and this can be approximated by $X \sim \mathrm{Po}(100 \times 0.03)$, i.e. $X \sim \mathrm{Po}(3)$

a Using the tables in the textbook:

$\mathrm{P}(X \leqslant 3) = 0.6472$

b $\mathrm{P}(X = 2) = \dfrac{\mathrm{e}^{-3}3^2}{2!} = 0.2240$ (4 d.p.)

6 Let X be the number of patients with the condition in a sample of 180. Then $X \sim \mathrm{B}(180, 0.02)$ and this can be approximated by $X \sim \mathrm{Po}(180 \times 0.02)$, i.e. $X \sim \mathrm{Po}(3.6)$

a $\mathrm{P}(X = 1) = \dfrac{\mathrm{e}^{-3.6}3.6^1}{1!} = 0.0984$ (4 d.p.)

b As $\lambda = 3.6$ use a calculator to find the required value:

$$\mathrm{P}(X \geqslant 2) = 1 - \mathrm{P}(X \leqslant 1)$$
$$= 1 - 0.1257 = 0.8743 \text{ (4 d.p.)}$$

7 a Let X be the number of people who catch the virus in a sample of 20. Then $X \sim \mathrm{B}\left(20, \dfrac{1}{120}\right)$

$$\mathrm{P}(X = 1) = \binom{20}{1}\left(\frac{1}{120}\right)^1\left(\frac{119}{120}\right)^{19} = 0.1422 \text{ (4 d.p.)}$$

b Let Y be the number of people who catch the virus in a sample of 900.

Then $Y \sim \mathrm{B}\left(900, \dfrac{1}{120}\right)$ and this can be approximated by $Y \sim \mathrm{Po}(7.5)$

Using the tables in the textbook:

$\mathrm{P}(Y \leqslant 6) = 0.3782$

8 a Let X be the number of defective articles in a sample of 10. Then $X \sim \mathrm{B}(10, 0.025)$

$$\mathrm{P}(X = 1) = \binom{10}{1} = (0.025)^1(0.975)^9 = 0.1991 \text{ (4 d.p.)}$$

b Let Y be the number of defective articles in a sample of 120. Then $Y \sim \mathrm{B}(120, 0.025)$ and this can be approximated by $Y \sim \mathrm{Po}(120 \times 0.025)$, i.e. $Y \sim \mathrm{Po}(3)$

Using the tables in the textbook:

$\mathrm{P}(Y \leqslant 3) = 0.6472$

9 a Let X be the number of chipped pots in a sample of 10. Then $X \sim \mathrm{B}(10, 0.05)$

b $\mathrm{P}(X = 3) = \dbinom{10}{3}(0.05)^3(0.95)^7 = 0.0105$ (4 d.p.)

9 c Let Y be the number of chipped pots in a sample of 140. Then $Y \sim B(140, 0.05)$ and this can be approximated by $Y \sim Po(140 \times 0.05)$, i.e. $Y \sim Po(7)$
Using the tables in the textbook:

$$P(6 \leqslant Y \leqslant 9) = P(Y \leqslant 9) - P(Y \leqslant 5)$$
$$= 0.8305 - 0.3007 = 0.5298$$

10 Let X be the number of tomato plants growing over 2 metres in a sample of 50. Then $X \sim B(50, 0.08)$ and this can be approximated by $X \sim Po(50 \times 0.08)$, i.e. $X \sim Po(4)$
Using the tables in the textbook:

$$P(5 \leqslant X \leqslant 8) = P(X \leqslant 8) - P(X \leqslant 4)$$
$$= 0.9786 - 0.6288 = 0.3498$$

11 a Let X be the number of damaged genes in an insect cell. Assuming genes are damaged independently, $X \sim B(1200, 0.005)$

b $E(X) = np = 1200 \times 0.005 = 6$
$Var(X) = np(1-p) = 1200 \times 0.005 \times 0.995 = 5.97$

c Use the approximation $X \sim Po(1200 \times 0.005)$, i.e. $X \sim Po(6)$
Using the tables in the textbook:
$P(X \leqslant 4) = 0.2851$

12 a Let X be the number of defective nails in a sample of 200. Then $X \sim B(200, 0.025)$ and this can be approximated by $X \sim Po(200 \times 0.025)$, i.e. $X \sim Po(5)$
Using the tables in the textbook:

$$P(X > 6) = 1 - P(X \leqslant 6)$$
$$= 1 - 0.7622 = 0.2378$$

b Let Y be the number of packets with more than 6 defective nails in a sample of 6 packets. Then $Y \sim B(6, 0.2378)$.
Using a calculator:

$$P(Y > 3) = 1 - P(X \leqslant 3)$$
$$= 1 - 0.9685 = 0.0315 \text{ (4 d.p.)}$$

13 a Let X be the number of defective components in a sample of 400. Then $X \sim B(400, 0.0125)$ and this can be approximated by $X \sim Po(400 \times 0.0125)$, i.e. $X \sim Po(5)$
Using the tables in the textbook:

$$P(X > 3) = 1 - P(X \leqslant 3)$$
$$= 1 - 0.2650 = 0.7350$$

b Let Y be the number of boxes containing more than 3 defective components in a sample of 5. Then $Y \sim B(5, 0.7350)$

$$P(Y = 3) = \binom{5}{3} \times (0.735)^3 \times (0.265)^2 = 0.2788 \text{ (4 d.p.)}$$

14 a Let X be the number of 1st class letters arriving next day in a sample of 180. Then $Y = 180 - X$ is the number of 1st class letters failing to arrive next day. Then $Y \sim \mathrm{B}(180, 0.05)$ and this can be approximated by $Y \sim \mathrm{Po}(180 \times 0.05)$, i.e. $Y \sim \mathrm{Po}(9)$

Using the tables in the textbook:

$\mathrm{P}(X > 173) = \mathrm{P}(Y \leqslant 6) = 0.2068$

b $\mathrm{P}(X < 168) = \mathrm{P}(Y > 12) = 1 - \mathrm{P}(Y \leqslant 12)$

$= 1 - 0.8758 = 0.1242$

15 a Let X be the number of broken eggs in a sample of 150. Then $X \sim \mathrm{B}(150, 0.01)$ and this can be approximated by $X \sim \mathrm{Po}(150 \times 0.01)$, i.e. $X \sim \mathrm{Po}(1.5)$

Using the tables in the textbook:

$\mathrm{P}(X > 4) = 1 - \mathrm{P}(X \leqslant 4)$

$= 1 - 0.9814 = 0.0186$

b Let Y be the number of consignments containing more than 4 broken eggs in a sample of 5.

Then $Y \sim \mathrm{B}(5, 0.0186)$

$$\mathrm{P}(Y = 1) = \binom{5}{1} \times (0.0186)^1 \times (0.9814)^4 = 0.0863 \text{ (4 d.p.)}$$