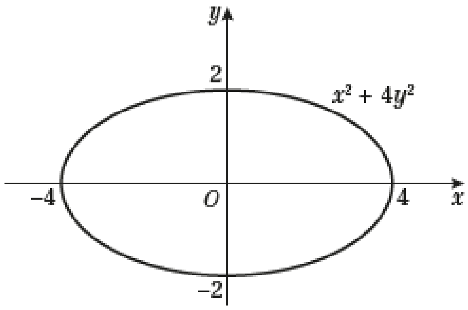
Conic sections 2 3A

1 i a $x^2+4y^2=16 \Rightarrow \dfrac{x^2}{16}+\dfrac{y^2}{4}=1$

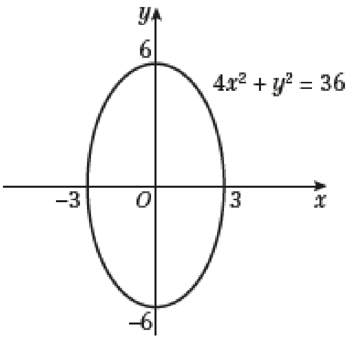
So $a=4$ and $b=2$



b Parametric equations
$x=4\cos\theta, y=2\sin\theta$

ii a $4x^2+y^2=36 \Rightarrow \dfrac{x^2}{9}+\dfrac{y^2}{36}=1$

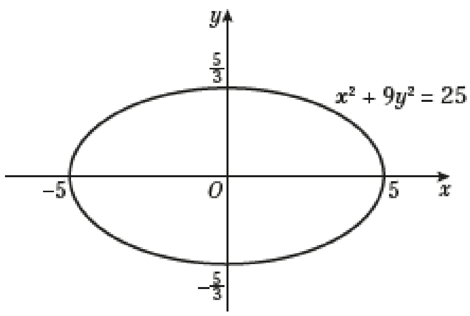
So $a=3$ and $b=6$



b Parametric equations
$x=3\cos\theta, y=6\sin\theta$

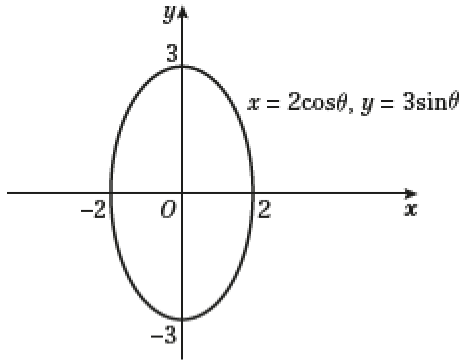
iii a $x^2+9y^2=25 \Rightarrow \dfrac{x^2}{25}+\dfrac{y^2}{\left(\frac{5}{3}\right)^2}=1$

So $a=5$ and $b=\frac{5}{3}$



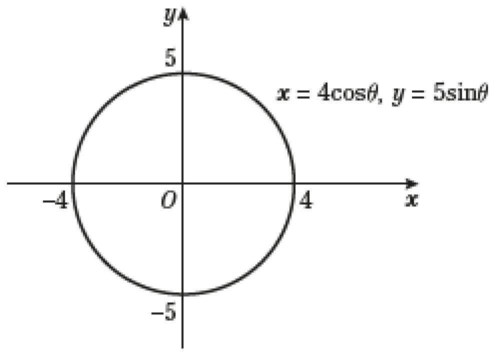
b Parametric equations
$x=5\cos\theta, y=\frac{5}{3}\sin\theta$

2 i a $x=2\cos\theta, y=3\sin\theta$
$-2\le x\le 2;\ -3\le y\le 3$



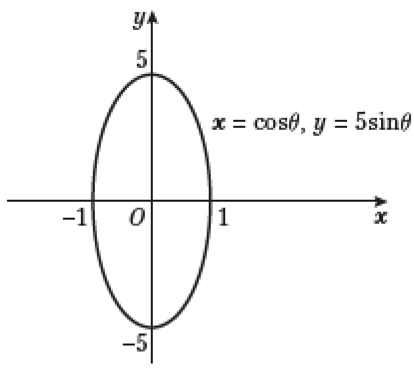
b $a=2$ and $b=3$, so the Cartesian equation is $\dfrac{x^2}{2^2}+\dfrac{y^2}{3^2}=1$

ii a $x=4\cos\theta, y=5\sin\theta$
$-4\le x\le 4;\ -5\le y\le 5$



b $a=4$ and $b=5$, so the Cartesian equation is $\dfrac{x^2}{4^2}+\dfrac{y^2}{5^2}=1$

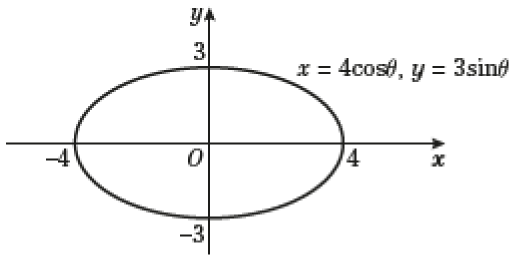
iii a $x=\cos\theta, y=5\sin\theta$
$-1\le x\le 1;\ -5\le y\le 5$



2 iii b $a = 1$ and $b = 5$, so the Cartesian equation is $x^2 + \frac{y^2}{5^2} = 1$

iv a $x = 4\cos\theta, y = 3\sin\theta$

$-4 \leq x \leq 4; -3 \leq y \leq 3$



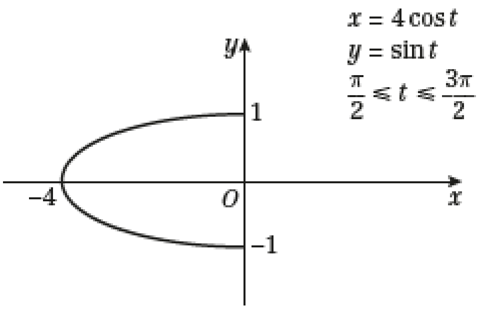
b $a = 4$ and $b = 3$, so the Cartesian equation is $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

3 a Clearly P has coordinates $(a\cos\theta, a\sin\theta)$ and Q has coordinates $(b\cos\theta, b\sin\theta)$. Then by definition $R = (b\cos\theta, a\sin\theta)$.

b Since $\cos^2\theta + \sin^2\theta = 1$, we have that the locus described by R as θ varies from 0 to 2π is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

This is the equation of an ellipse.

c



Challenge

The matrix of a rotation of 45° anticlockwise is

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

If we apply this to the general vector $\begin{pmatrix} a\cos\theta \\ b\cos\theta \end{pmatrix}$

we get $\begin{pmatrix} \frac{a}{\sqrt{2}}\cos\theta - \frac{b}{\sqrt{2}}\sin\theta \\ \frac{a}{\sqrt{2}}\cos\theta + \frac{b}{\sqrt{2}}\sin\theta \end{pmatrix}$

Then we can compute:

$$\frac{(x+y)^2}{2a^2} + \frac{(x-y)^2}{2b^2} = \frac{(a\sqrt{2}\cos\theta)^2}{2a^2} + \frac{(-b\sqrt{2}\sin\theta)^2}{2b^2}$$

$$= \frac{2a^2\cos^2\theta}{2a^2} + \frac{2b^2\sin^2\theta}{2b^2}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$