

Review exercise 2

1 The equation of the line is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 8}{6 - 8} = \frac{x + 2}{4 + 2}$$

$$\frac{y - 8}{-2} = \frac{x + 2}{6}$$

$$3y - 24 = -x - 2$$

$$x + 3y - 22 = 0$$

2 $y - (-4) = \frac{1}{3}(x - 9)$

$$y + 4 = \frac{1}{3}(x - 9)$$

$$3y + 12 = x - 9$$

$$x - 3y - 21 = 0$$

$$a = 1, b = -3, c = -21$$

3 Using points A and B:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 3}{5 - 3} = \frac{x - 0}{k - 0}$$

$$\frac{y - 3}{2} = \frac{x}{k}$$

$$ky - 3k = 2x$$

Substituting point C into the equation:

$$k(2k) - 3k = 2(10)$$

$$2k^2 - 3k - 20 = 0$$

$$(2k + 5)(k - 4) = 0$$

$$k = -\frac{5}{2} \text{ or } k = 4$$

4 a Using points (160, 72) and (180, 81):

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{81 - 72}{180 - 160}$$

$$= \frac{9}{20}$$

$$= 0.45$$

b $l = kh$, where k is the gradient.

$$\text{So } l = 0.45h$$

c The model may not be valid for young people/children who are still growing.

5 a The gradient of l_1 is 3.

So the gradient of l_2 is $-\frac{1}{3}$.

The equation of line l_2 is:

$$y - 2 = -\frac{1}{3}(x - 6)$$

$$y - 2 = -\frac{1}{3}x + 2$$

$$y = -\frac{1}{3}x + 4$$

b $y = 3x - 6$

$$y = -\frac{1}{3}x + 4$$

$$3x - 6 = -\frac{1}{3}x + 4$$

$$3x + \frac{1}{3}x = 4 + 6$$

$$\frac{10}{3}x = 10$$

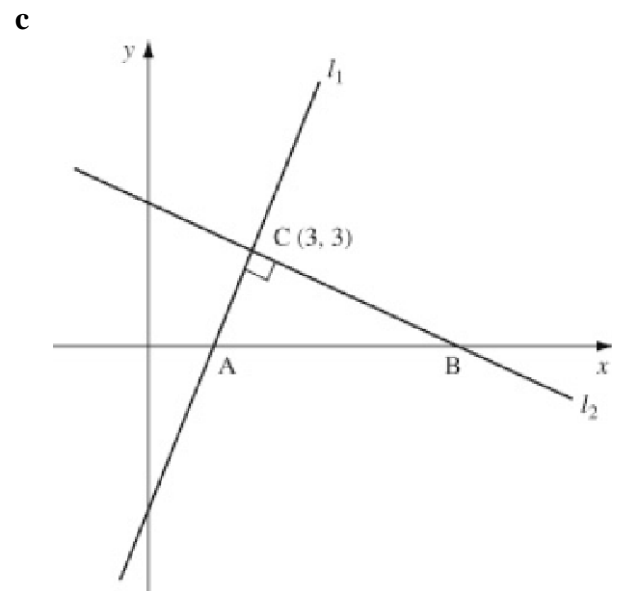
$$x = 3$$

$$y = (3 \times 3) - 6 = 3$$

$$x = 3$$

$$y = 3 \times 3 - 6 = 3$$

The point C is (3, 3).



Where l_1 meets the x -axis, $y = 0$:

$$0 = 3x - 6$$

$$3x = 6$$

$$x = 2$$

The point A is (2, 0).

Where l_2 meets the x -axis, $y = 0$:

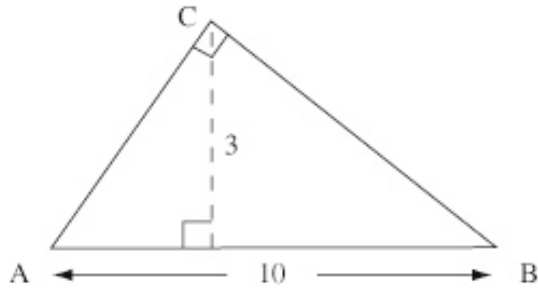
$$0 = -\frac{1}{3}x + 4$$

$$\frac{1}{3}x = 4$$

$$x = 12$$

The point B is (12, 0).

5 c



$$AB = 12 - 2 = 10$$

The perpendicular height, using AB as the base is 3.

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 10 \times 3 \\ &= 15 \end{aligned}$$

6 Substituting $y = 2x$ into $5y + x - 33 = 0$:

$$5(2x) + x - 33 = 0$$

$$11x - 33 = 0$$

$$x = 3$$

$$y = 2 \times 3 = 6$$

The point P is $(3, 6)$.

$$\begin{aligned} \text{Distance from origin} &= \sqrt{3^2 + 6^2} \\ &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

7 Gradient of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-4 - 8}{7 - 5}$$

$$= -6$$

Gradient of the perpendicular bisector is $\frac{1}{6}$

$$\begin{aligned} \text{Midpoint of line} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{5 + 7}{2}, \frac{8 - 4}{2} \right) \\ &= (6, 2) \end{aligned}$$

Equation of the perpendicular bisector is:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{6}(x - 6)$$

$$y = \frac{1}{6}x + 1$$

This line crosses the x -axis at $y = 0$:

$$\frac{1}{6}x + 1 = 0$$

$$x = -6$$

The point Q is $(-6, 0)$.

8 Equation of circle with centre $(-3, 8)$

and radius r :

$$(x + 3)^2 + (y - 8)^2 = r^2$$

$$r = \text{distance from } (-3, 8) \text{ to } (0, 9)$$

$$r^2 = (0 + 3)^2 + (9 - 8)^2 = 9 + 1 = 10$$

The equation for C is:

$$(x + 3)^2 + (y - 8)^2 = 10$$

9 a Rearranging:

$$x^2 - 6x + y^2 + 2y = 10$$

Completing the square:

$$(x - 3)^2 - 9 + (y + 1)^2 - 1 = 10$$

$$(x - 3)^2 + (y + 1)^2 = 20$$

$$a = 3, b = -1, r = \sqrt{20}$$

b The circle has centre $(3, -1)$ and radius $\sqrt{20}$.

10 a Rearranging $3x + y = 14$:

$$y = 14 - 3x$$

Solving simultaneously using substitution:

$$(x - 2)^2 + (14 - 3x - 3)^2 = 5$$

$$(x - 2)^2 + (-3x + 11)^2 = 5$$

$$x^2 - 4x + 4 + 9x^2 - 66x + 121 - 5 = 0$$

$$10x^2 - 70x + 120 = 0$$

$$x^2 - 7x + 12 = 0$$

$$(x - 3)(x - 4) = 0$$

$$\text{So } x = 3 \text{ and } x = 4$$

$$x = 3: y = 14 - 3 \times 3 = 5$$

$$x = 4: y = 14 - 3 \times 4 = 2$$

Point A is $(3, 5)$ and point B is $(4, 2)$.

b Using Pythagoras' theorem:

$$\begin{aligned} \text{Length } AB &= \sqrt{(4 - 3)^2 + (2 - 5)^2} \\ &= \sqrt{10} \end{aligned}$$

11 The equation of the circle is $x^2 + y^2 = r^2$.

Solving simultaneously using substitution:

$$x^2 + (3x - 2)^2 = r^2$$

$$x^2 + 9x^2 - 12x + 4 - r^2 = 0$$

$$10x^2 - 12x + 4 - r^2 = 0$$

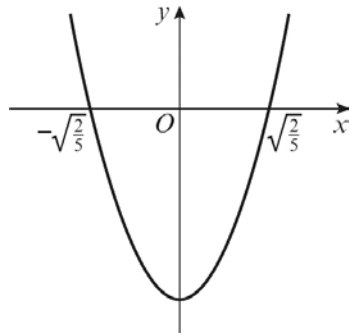
Using the discriminant for no solutions:

$$b^2 - 4ac < 0$$

$$(-12)^2 - 4(10)(4 - r^2) < 0$$

$$144 - 160 + 40r^2 < 0$$

11 $40r^2 - 16 < 0$
 When $40r^2 - 16 = 0$
 $8(5r^2 - 2) = 0$
 $r^2 = \frac{2}{5}$
 $r = \pm\sqrt{\frac{2}{5}}$



$-\sqrt{\frac{2}{5}} < r < \sqrt{\frac{2}{5}}$
 However, the radius cannot be negative.
 So $0 < r < \sqrt{\frac{2}{5}}$

12 a Equation of circle with centre (1, 5) and radius r :
 $(x - 1)^2 + (y - 5)^2 = r^2$
 $r = \text{distance from } (1, 5) \text{ to } (4, -2)$
 $r^2 = (4 - 1)^2 + (-2 - 5)^2$
 $= 9 + 49$
 $= 58$

The equation for C is:
 $(x - 1)^2 + (y - 5)^2 = 58$

b Gradient of the radius of the circle at P
 $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 5}{4 - 1} = -\frac{7}{3}$
 Gradient of the tangent $= \frac{3}{7}$

Equation of the tangent at P :
 $y - y_1 = m(x - x_1)$
 $y + 2 = \frac{3}{7}(x - 4)$
 $3x - 7y - 26 = 0$

13 a $AB^2 = (6 - 2)^2 + (5 - 1)^2$
 $= 4^2 + 4^2 = 32$
 $BC^2 = (8 - 6)^2 + (3 - 5)^2$
 $= 2^2 + 2^2 = 8$

13 a $AC^2 = (8 - 2)^2 + (3 - 1)^2$
 $= 6^2 + 2^2 = 40$

Using Pythagoras' theorem:
 $AB^2 + BC^2 = 32 + 8 = 40 = AC^2$
 Therefore, $\angle ABC$ is 90° .

- b** As triangle ABC is a right-angled triangle, AC is a diameter of the circle.
- c** AC is a diameter of the circle, so the midpoint of AC is the centre.

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
 $= \left(\frac{2 + 8}{2}, \frac{1 + 3}{2} \right)$
 $= (5, 2)$

Radius $= \frac{1}{2} \times AC$
 $= \frac{1}{2} \times \sqrt{40}$
 $= \frac{1}{2} \times 2\sqrt{10}$
 $= \sqrt{10}$

The equation of the circle is:
 $(x - 5)^2 + (y - 2)^2 = 10$

14 $\frac{2x^2 + 20x + 42}{224x + 4x^2 - 4x^3} = \frac{x^2 + 10x + 21}{112x + 2x^2 - 2x^3}$
 $= \frac{(x + 3)(x + 7)}{-2x(x^2 - x - 56)}$
 $= \frac{(x + 3)(x + 7)}{-2x(x + 7)(x - 8)}$
 $= \frac{(x + 3)}{-2x(x - 8)}$
 $a = 3, b = -2, c = -8$

15 a Using the factor theorem:
 $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 7\left(\frac{1}{2}\right)^2 - 17\left(\frac{1}{2}\right) + 10$
 $= \frac{1}{4} - \frac{7}{4} - \frac{17}{2} + 10$
 $= 0$

So $(2x - 1)$ is a factor of $2x^3 - 7x^2 - 17x + 10$.

- 19 a** $p = 5: 5^2 = 25 = 24 + 1$
 $p = 7: 7^2 = 49 = 2(24) + 1$
 $p = 11: 11^2 = 121 = 5(24) + 1$
 $p = 13: 13^2 = 169 = 7(24) + 1$
 $p = 17: 17^2 = 289 = 12(24) + 1$
 $p = 19: 19^2 = 361 = 15(24) + 1$
- b** $3(24) + 1 = 73$ and 73 is not a square number.

- 20 a** Rearranging:
 $x^2 - 10x + y^2 - 8y = -32$
 Completing the square:
 $(x - 5)^2 - 25 + (y - 4)^2 - 16 = -32$
 $(x - 5)^2 + (y - 4)^2 = 9$
 $(x - 5)^2 + (y - 4)^2 = 3^2$
 $a = 5, b = 4, r = 3$
- b** Centre of circle C is $(5, 4)$.
 Centre of circle D is $(0, 0)$.
 Using Pythagoras' theorem:
 Distance = $\sqrt{(5-0)^2 + (4-0)^2} = \sqrt{41}$
- c** Radius of circle $C = 3$
 Radius of circle $D = 3$
 Distance between the centres = $\sqrt{41}$
 $3 + 3 < \sqrt{41}$
 Therefore, the circles C and D do not touch.

- 21 a** $(1 - 2x)^{10}$
 $= 1^{10} + \binom{10}{1} 1^9 (-2x) + \binom{10}{2} 1^8 (-2x)^2$
 $+ \binom{10}{3} 1^7 (-2x)^3 + \dots$
 $= 1 + 10(-2x) + \frac{10(9)}{2} (-2x)^2$
 $+ \frac{10(9)(8)}{6} (-2x)^3 + \dots$
 $= 1 - 20x + 180x^2 - 960x^3 + \dots$
- b** $(0.98)^{10}$
 $= (1 - 2(0.01))^{10}$
 $= 1 - 20(0.01) + 180(0.01)^2 - 960(0.01)^3$
 $+ \dots$
 $= 0.817$ (3 d.p.)

- 22** $(1 + 2x)^5$
 $= 1^5 + \binom{5}{1} 1^4 (2x) + \binom{5}{2} 1^3 (2x)^2 + \dots$
 $= 1 + 5(2x) + \frac{5(4)}{2} (2x)^2 + \dots$
 $= 1 + 10x + 40x^2 + \dots$
 $(2 - x)(1 + 2x)^5$
 $= (2 - x)(1 + 10x + 40x^2 + \dots)$
 $= 2 + 20x + 80x^2 + \dots - x - 10x^2 + \dots$
 $= 2 + 19x + 70x^2 + \dots$
 $\approx 2 + 19x + 70x^2$
 $a = 2, b = 19, c = 70$

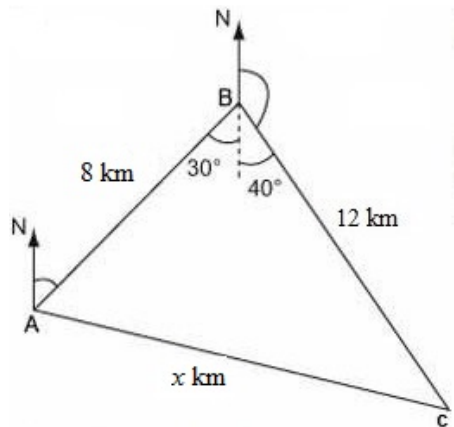
- 23** $(2 - 4x)^q$
 x term = $\binom{q}{q-1} 2^{q-1} (-4x)^1$
 $= q \times 2^{q-1} \times -4x$
 $= -4 \times 2^{q-1} qx$
 $-4 \times 2^{q-1} q = -32q$
 $2^{q-1} = 8$
 $q - 1 = 3$
 $q = 4$

- 24** Using the sine rule:
 $\frac{b}{\sin B} = \frac{c}{\sin C}$
 $\frac{b}{\sin 45^\circ} = \frac{\sqrt{5}}{\sin 30^\circ}$
 $b = \frac{\sqrt{5} \sin 45^\circ}{\sin 30^\circ}$
 $b = \frac{\sqrt{5} \times \frac{\sqrt{2}}{2}}{\frac{1}{2}}$
 $b = \sqrt{10}$
 $AC = \sqrt{10}$ cm
- 25 a** Using the cosine rule:
 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
 $\cos 60^\circ = \frac{(2x-3)^2 + 5^2 - (x+1)^2}{2(2x-3)(5)}$
 $\frac{1}{2} = \frac{4x^2 - 12x + 9 + 25 - (x^2 + 2x + 1)}{10(2x-3)}$
 $5(2x-3) = 3x^2 - 14x + 33$
 $3x^2 - 24x + 48 = 0$
 $x^2 - 8x + 16 = 0$

25 b $x^2 - 8x + 16 = 0$
 $(x - 4)^2 = 0$
 $x = 4$

c Area = $ac \sin B$
 $a = 2 \times 4 - 3 = 5$
 $c = 5$
 Area = $\frac{1}{2} \times 5 \times 5 \times \sin 60^\circ$
 $= 10.8253\dots$
 $= 10.8 \text{ cm}^2$ (3 s.f.)

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a Using the cosine rule
 $x^2 = 8^2 + 12^2 - 2 \times 8 \times 12 \times \cos 70^\circ$
 $= 142.332\dots$
 $x = 11.93 \text{ km}$
 The distance of ship C from ship A is 11.93 km.

b Using the sine rule:
 $\frac{\sin 70^\circ}{11.93} = \frac{\sin A}{12}$
 $\sin A = 0.94520\dots$
 $A = 70.9^\circ$
 The bearing of ship C from ship A is 100.9° .

27 a If triangle ABC is isosceles, then two of the sides are equal.
 $AB = \sqrt{(6+2)^2 + (10-4)^2} = \sqrt{100} = 10$
 $BC = \sqrt{(16-6)^2 + (10-10)^2} = \sqrt{100} = 10$
 $AC = \sqrt{(16+2)^2 + (10-4)^2} = \sqrt{360} = 6\sqrt{10}$
 $AB = BC$
 Therefore ABC is an isosceles triangle.

27 b Using the cosine rule:
 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
 $= \frac{10^2 + 10^2 - (\sqrt{360})^2}{2(10)(10)}$
 $= \frac{100 + 100 - 360}{200}$
 $= -\frac{4}{5}$
 $B = 143.13010\dots$
 Angle ABC = 143.1° (1 d.p.)

28 Using the sine rule in triangle ABD:
 $\frac{\sin \angle BDA}{4.3} = \frac{\sin 40^\circ}{3.5}$
 $\sin \angle BDA = \frac{4.3 \sin 40^\circ}{3.5}$
 $= 0.78971\dots$
 $\angle BDA = 52.16^\circ$

Using the angle sum of a triangle:
 $\angle ABD = 180^\circ - (52.16^\circ + 40^\circ)$
 $= 87.84^\circ$

Using the sine rule in triangle ABD:
 $\frac{AD}{\sin 87.84} = \frac{3.5}{\sin 40^\circ}$
 $AD = 5.44 \text{ cm}$

$AC = AD + DC$
 $= 5.44 + 8.6$
 $= 14.04 \text{ cm}$

Area of triangle ABC
 $= \frac{1}{2} \times 4.3 \times 14.04 \times \sin 40^\circ$
 $= 19.4 \text{ cm}^2$

29 a $(x - 5)^2 + (y - 2)^2 = 5^2$
 $(x - 5)^2 + (y - 2)^2 = 25$
 b Substituting $x = 8$ and $y = k$ into the equation of the circle:
 $(8 - 5)^2 + (k - 2)^2 = 25$
 $9 + k^2 - 4k + 4 - 25 = 0$
 $k^2 - 4k - 12 = 0$
 $(k + 2)(k - 6) = 0$
 $k = -2$ or $k = 6$
 k is positive, therefore $k = 6$.

29 c $XY = \sqrt{(10-1)^2 + (2+1)^2} = \sqrt{90}$
 $YZ = \sqrt{(8-10)^2 + (6-2)^2} = \sqrt{20}$
 $XZ = \sqrt{(8-1)^2 + (6+1)^2} = \sqrt{98}$

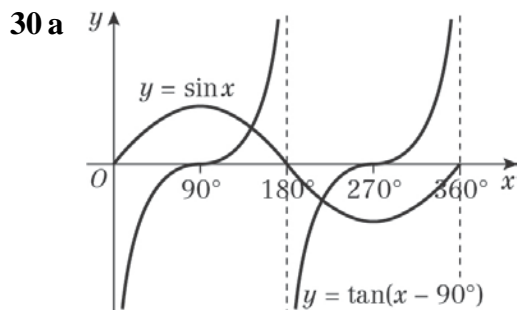
Using the cosine rule:

$$\begin{aligned} \cos Y &= \frac{x^2 + z^2 - y^2}{2xz} \\ &= \frac{20 + 90 - 98}{2\sqrt{1800}} \\ &= \frac{12}{60\sqrt{2}} \\ &= \frac{1}{5\sqrt{2}} \end{aligned}$$

Rationalising the denominator:

$$\cos Y = \frac{\sqrt{2}}{10}$$

So $\cos \angle XYZ = \frac{\sqrt{2}}{10}$



b There are two solutions in the interval $0 \leq x \leq 360^\circ$.

31 a The curve $y = \sin x$ crosses the x -axis at $(-360^\circ, 0)$, $(-180^\circ, 0)$, $(0^\circ, 0)$, $(180^\circ, 0)$ and $(360^\circ, 0)$.

$y = \sin(x + 45^\circ)$ is a translation of

$$\begin{pmatrix} -45^\circ \\ 0 \end{pmatrix}$$

so subtract 45° from the x -coordinates.

The curve crosses the x -axis at $(-405^\circ, 0)$, $(-225^\circ, 0)$, $(-45^\circ, 0)$, $(135^\circ, 0)$ and $(315^\circ, 0)$.

$(-405^\circ, 0)$ is not in the range, so $(-225^\circ, 0)$, $(-45^\circ, 0)$, $(135^\circ, 0)$ and $(315^\circ, 0)$

31 b The curve $y = \sin(x + 45^\circ)$ crosses the y -axis when $x = 0$.

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\left(0, \frac{\sqrt{2}}{2}\right)$$

32 Each of the four triangular faces is an equilateral triangle.

Area of one triangle

$$= \frac{1}{2}ac \sin B$$

$$= \frac{1}{2} \times s \times s \times \sin 60^\circ$$

$$= \frac{s^2}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}s^2}{4} \text{ cm}^2$$

Total area

= area of 4 triangles + area of square

$$= 4 \times \frac{\sqrt{3}s^2}{4} + s^2$$

$$= \sqrt{3}s^2 + s^2$$

$$= (\sqrt{3} + 1)s^2$$

The total surface area of the pyramid is

$$(\sqrt{3} + 1)s^2 \text{ cm}^2.$$

33 a $\sin \theta = \cos \theta$

$$\frac{\sin \theta}{\cos \theta} = 1$$

So $\tan \theta = 1$

b When $\tan \theta = 1$

$\theta = 45^\circ$ or 225°

So $\sin \theta = \cos \theta$ when $\theta = 45^\circ$ or 225°

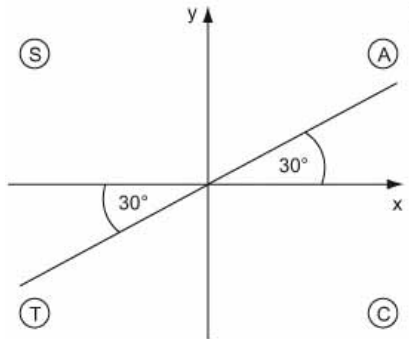
34 $3 \tan^2 x = 1$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

For $\tan x = \frac{1}{\sqrt{3}}$

$$x = 30^\circ$$

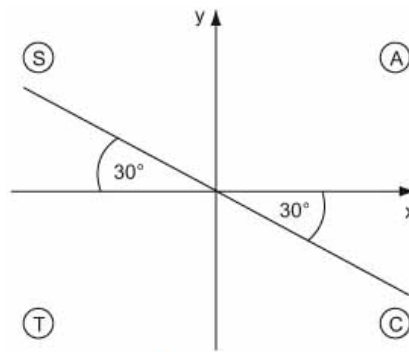
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So $x = 30^\circ$ or $x = 210^\circ$

$$\text{For } \tan x = -\frac{1}{\sqrt{3}}$$

$$x = 330^\circ \text{ (or } -30^\circ)$$



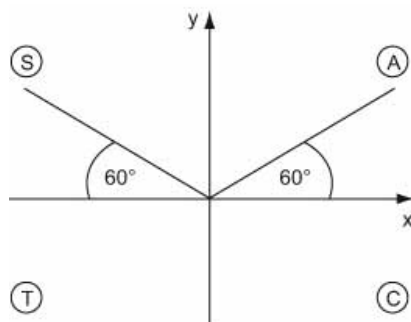
So $x = 330^\circ$ or $x = 150^\circ$

So $x = 30^\circ, 150^\circ, 210^\circ$ or 330°

35 $2 \sin(\theta - 30^\circ) = \sqrt{3}$

$$\sin(\theta - 30^\circ) = \frac{\sqrt{3}}{2}$$

$$\theta - 30^\circ = 60^\circ$$



So $\theta - 30^\circ = 60^\circ$ or $\theta - 30^\circ = 120^\circ$

When $\theta - 30^\circ = 60^\circ$

$$\begin{aligned} \theta &= 60^\circ + 30^\circ \\ &= 90^\circ \end{aligned}$$

When $\theta - 30^\circ = 120^\circ$

$$\begin{aligned} \theta &= 120^\circ + 30^\circ \\ &= 150^\circ \end{aligned}$$

So $\theta = 90^\circ$ or 150°

36 a

$$2 \cos^2 x = 4 - 5 \sin x$$

$$2(1 - \sin^2 x) = 4 - 5 \sin x$$

$$2 - 2 \sin^2 x = 4 - 5 \sin x$$

$$2 \sin^2 x - 5 \sin x + 2 = 0 \text{ (as required)}$$

b Let $\sin x = y$

$$2y^2 - 5y + 2 = 0$$

$$(2y - 1)(y - 2) = 0$$

$$\text{So } y = \frac{1}{2} \text{ or } y = 2$$

$$\text{When } \sin x = \frac{1}{2}, x = 30^\circ$$

$$\text{or } x = 180^\circ - 30^\circ = 150^\circ$$

$\sin x = 2$ is impossible.

$$x = 30^\circ \text{ or } 150^\circ$$

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$$2 \tan^2 x - 4 = 5 \tan x$$

$$2 \tan^2 x - 5 \tan x - 4 = 0$$

Using the quadratic formula:

$$\tan x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{57}}{4}$$

$$\text{When } \tan x = \frac{5 + \sqrt{57}}{4}, x = 72.3^\circ$$

$$\text{or } x = 72.3^\circ + 180^\circ = 252.3^\circ$$

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$$\text{When } \tan x = \frac{5 - \sqrt{57}}{4}, x = -32.5^\circ$$

$$\text{or } x = -32.5^\circ + 180^\circ = 147.5^\circ$$

$$\text{or } x = 147.5^\circ + 180^\circ = 327.5^\circ$$

$$x = 72.3^\circ, 147.5^\circ, 252.3^\circ \text{ or } 327.5^\circ$$

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$$5 \sin^2 x = 6(1 - \cos x)$$

$$5 \sin^2 x + 6 \cos x - 6 = 0$$

$$5(1 - \cos^2 x) + 6 \cos x - 6 = 0$$

$$5 - 5 \cos^2 x + 6 \cos x - 6 = 0$$

$$5 \cos^2 x - 6 \cos x + 1 = 0$$

$$(5 \cos x - 1)(\cos x - 1) = 0$$

$$\text{So } \cos x = \frac{1}{5} \text{ or } \cos x = 1$$

$$\text{When } \cos x = \frac{1}{5}, x = 78.5^\circ$$

$$\text{or } x = 360^\circ - 78.5^\circ = 281.5^\circ$$

$$\text{When } \cos x = 1, x = 0^\circ \text{ or } 360^\circ$$

$$x = 0^\circ, 78.5^\circ, 281.5^\circ \text{ or } 360^\circ$$

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$$\text{LHS} = \cos^2 x (\tan^2 x + 1)$$

$$= \cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1 \right)$$

$$= \sin^2 x + \cos^2 x$$

$$= \text{RHS}$$

Challenge

- 1 a** Finding points B and C using $y = 3x - 12$:

When $y = 0$, $x = 4$

When $x = 0$, $y = -12$

The point B is $(4, 0)$ and

the point C is $(0, -12)$.

Using Pythagoras' theorem to find the length of the square:

$$BC = \sqrt{(0-4)^2 + (-12-0)^2} = \sqrt{160}$$

$$\text{Area of square} = (\sqrt{160})^2 = 160$$

- b** The point A is $(-8, 4)$ and the point D is $(-12, -8)$.

$$\begin{aligned} \text{The gradient of line } AD &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 4}{-12 + 8} \\ &= \frac{-12}{-4} \\ &= 3 \end{aligned}$$

The equation of line AD is:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3(x + 8)$$

$$y = 3x + 28$$

When $y = 0$, $x = -\frac{28}{3}$

The point S is $(-\frac{28}{3}, 0)$.

- 2** Rearranging $x^2 + y^2 + 8x - 10y = 59$:

$$x^2 + 8x + y^2 - 10y = 59$$

Completing the square:

$$(x + 4)^2 - 16 + (y - 5)^2 - 25 = 59$$

$$(x + 4)^2 + (y - 5)^2 = 100$$

Both circles have the same centre at

$(-4, 5)$. The radius of one circle is 8 and

the other is 10, so

$(x + 4)^2 + (y - 5)^2 = 8^2$ lies completely inside $x^2 + y^2 + 8x - 10y = 59$.

$$\begin{aligned} \mathbf{3} \quad \text{LHS} &= \binom{n}{k} + \binom{n}{k+1} \\ &= \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} \\ &= \frac{n!(k+1)}{(k+1)!(n-k)!} + \frac{n!(n-k)}{(k+1)!(n-k)!} \\ &= \frac{n!((k+1) + (n-k))}{(k+1)!(n-k)!} \\ &= \frac{n!(n+1)}{(k+1)!(n-k)!} \\ &= \frac{(n+1)!}{(k+1)!(n-k)!} \\ &= \binom{n+1}{k+1} \\ &= \text{RHS} \end{aligned}$$

$$\begin{aligned} \mathbf{4} \quad &2 \sin^3 x - \sin x + 1 = \cos^2 x \\ &2 \sin^3 x - \sin x + 1 = 1 - \sin^2 x \\ &2 \sin^3 x + \sin^2 x - \sin x = 0 \\ &\sin x(2 \sin^2 x + \sin x - 1) = 0 \\ &\sin x(2 \sin x - 1)(\sin x + 1) = 0 \\ &\text{So } \sin x = 0, \sin x = \frac{1}{2} \text{ or } \sin x = -1 \\ &\text{When } \sin x = 0, x = 0^\circ, 180^\circ \text{ or } 360^\circ \\ &\text{When } \sin x = \frac{1}{2}, x = 30^\circ \\ &\text{or } x = 180^\circ - 30^\circ = 150^\circ \\ &\text{When } \sin x = -1, x = 270^\circ \\ &\text{So } x = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 270^\circ \text{ or } 360^\circ \end{aligned}$$