

## Review Exercise 1

$$1 \text{ a } 8^{\frac{1}{3}}$$

$$\begin{aligned} \text{Use } a^{\frac{1}{m}} &= \sqrt[m]{a}, \text{ so } a^{\frac{1}{3}} = \sqrt[3]{a} \\ &= \sqrt[3]{8} \\ &= 2 \end{aligned}$$

$$1 \text{ b } 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} \left( \text{Use } a^{-m} = \frac{1}{a^m} \right)$$

First find  $8^{\frac{2}{3}}$   $a^{\frac{n}{m}} = \sqrt[m]{(a^n)}$  or  $(\sqrt[m]{a})^n$

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$$

$$8^{\frac{2}{3}} = 2^2 = 4$$

$$\begin{aligned} 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} \\ &= \frac{1}{4} \end{aligned}$$

$$2 \text{ a } 125^{\frac{4}{3}}$$

$$a^{\frac{n}{m}} = \sqrt[m]{(a^n)} \text{ or } (\sqrt[m]{a})^n$$

$$= (\sqrt[3]{125})^4$$

$$= 5^4$$

$$= 625$$

$$2 \text{ b } 24x^2 \div 18x^{\frac{4}{3}}$$

(Use  $a^m \div a^n = a^{m-n}$ )

$$= \frac{24x^2}{18x^{\frac{4}{3}}} = \frac{4x^2}{3x^{\frac{4}{3}}} \quad \text{cancelling by 6}$$

$$= \frac{4x^{\frac{2}{3}}}{3} \quad \text{because } 2 - \frac{4}{3} = \frac{2}{3}$$

$$3 \text{ a } \sqrt{80}$$

$$\begin{aligned} \text{Use } \sqrt{(bc)} &= \sqrt{b}\sqrt{c} \\ &= \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \quad (a=4) \end{aligned}$$

$$\begin{aligned} 3 \text{ b } (4-\sqrt{5})^2 &= (4-\sqrt{5})(4-\sqrt{5}) \\ &= 4(4-\sqrt{5}) - \sqrt{5}(4-\sqrt{5}) \\ &= 16 - 4\sqrt{5} - 4\sqrt{5} + 5 \\ &= 21 - 8\sqrt{5} \\ &(b=21 \text{ and } c=-8) \end{aligned}$$

$$\begin{aligned} 4 \text{ a } (4+\sqrt{3})(4-\sqrt{3}) \\ &= 4(4-\sqrt{3}) + \sqrt{3}(4-\sqrt{3}) \\ &= 16 - 4\sqrt{3} + 4\sqrt{3} - 3 \\ &= 13 \end{aligned}$$

$$\begin{aligned} 4 \text{ b } \frac{26}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} &= \frac{26(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})} \\ &= \frac{26(4-\sqrt{3})}{13} \\ &= 2(4-\sqrt{3}) \\ &= 8 - 2\sqrt{3} \\ &(a=8 \text{ and } b=-2) \end{aligned}$$

$$\begin{aligned} 5 \text{ a } \text{mean} &= \frac{1-\sqrt{k}+2+5\sqrt{k}+2\sqrt{k}}{3} \\ &= \frac{3+6\sqrt{k}}{3} \\ &= 1+2\sqrt{k} \end{aligned}$$

$$\begin{aligned} 5 \text{ b } \text{ range} &= 2 + 5\sqrt{k} - (1 - \sqrt{k}) \\ &= 1 + 6\sqrt{k} \end{aligned}$$

$$\begin{aligned} 6 \text{ a } y^{-1} &= \left(\frac{1}{25}x^4\right)^{-1} \\ &= \frac{1}{\frac{1}{25}x^4} \\ &= \frac{25}{x^4} \\ &= 25x^{-4} \end{aligned}$$

$$\begin{aligned} 6 \text{ b } 5y^{\frac{1}{2}} &= 5\left(\frac{1}{25}x^4\right)^{\frac{1}{2}} \\ &= 5\left(\frac{1}{5}x^2\right) \\ &= x^2 \end{aligned}$$

$$\begin{aligned} 7 \text{ Area} &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2}(2\sqrt{2})(3+\sqrt{2}+5+3\sqrt{2}) \\ &= \sqrt{2}(8+4\sqrt{2}) \\ &= 8\sqrt{2}+8 \end{aligned}$$

The area of the trapezium is  $8+8\sqrt{2}$  cm<sup>2</sup>.

$$\begin{aligned} 8 \quad \frac{p+q}{p-q} &= \frac{(3-2\sqrt{2})+(2-\sqrt{2})}{(3-2\sqrt{2})-(2-\sqrt{2})} \\ &= \frac{5-3\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{(5-3\sqrt{2})}{(1-\sqrt{2})} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} \\ &= \frac{5+5\sqrt{2}-3\sqrt{2}-6}{1+\sqrt{2}-\sqrt{2}-2} \\ &= \frac{-1+2\sqrt{2}}{-1} \\ &= 1-2\sqrt{2} \quad (m=1, n=-2) \end{aligned}$$

$$9 \text{ a } x^2 - 10x + 16 = (x-8)(x-2)$$

$$\begin{aligned} \text{b Let } x &= 8^y \\ 8^{2y} - 10(8^y) + 16 &= (8^y - 8)(8^y - 2) = 0 \\ \text{So } 8^y &= 8 \text{ or } 8^y = 2 \\ y &= 1 \text{ or } y = \frac{1}{3} \end{aligned}$$

$$10 \text{ a } x^2 - 8x = (x-4)^2 - 16$$

Complete the square for  $x^2 - 8x - 29$

$$\begin{aligned} x^2 - 8x - 29 &= (x-4)^2 - 16 - 29 \\ &= (x-4)^2 - 45 \\ (a &= -4 \text{ and } b = -45) \end{aligned}$$

$$\begin{aligned} \text{b } x^2 - 8x - 29 &= 0 \\ (x-4)^2 - 45 &= 0 \end{aligned}$$

Use the result from part a:

$$(x-4)^2 = 45$$

Take the square root of both sides:

$$x-4 = \pm\sqrt{45}$$

$$x = 4 \pm \sqrt{45}$$

$$\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$

$$\text{since } \sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

$$\text{Roots are } 4 \pm 3\sqrt{5}$$

$$(c = 4 \text{ and } d = \pm 3)$$

$$11 \quad f(a) = a(a-2) \text{ and } g(a) = a+5$$

$$a(a-2) = a+5$$

$$a^2 - 2a - a - 5 = 0$$

$$a^2 - 3a - 5 = 0$$

Using the quadratic formula:

$$a = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{29}}{2}$$

$$= 4.19 \text{ or } -1.19$$

As  $a > 0$ ,  $a = 4.19$  (3 s.f.)

**12 a** The height of the athlete's shoulder above the ground is 1.7 m

**b**  $1.7 + 10t - 5t^2 = 0$

Using the quadratic formula when  $a = -5$ ,  $b = 10$  and  $c = 1.7$

$$t = \frac{-10 \pm \sqrt{(10)^2 - 4(-5)(1.7)}}{2(-5)}$$

$$= \frac{-10 \pm \sqrt{134}}{-10}$$

$= -0.16$  or  $2.16$

As  $t > 0$ ,  $t = 2.16$  s (3 s.f.)

**c**  $1.7 + 10t - 5t^2 = 1.7 - 5(t^2 - 2t)$

$$= 1.7 - 5((t - 1)^2 - 1)$$

$$= 1.7 - 5(t - 1)^2 + 5$$

$$= 6.7 - 5(t - 1)^2$$

$A = 6.7$ ,  $B = 5$  and  $C = 1$

**d** Maximum when  $(t - 1) = 0$ ,  $t = 1$  s and maximum height = 6.7 m

**13 a**  $f(x) = x^2 - 6x + 18$

$$x^2 - 6x = (x - 3)^2 - 9$$

Complete the square for  $x^2 - 6x + 18$

$$x^2 - 6x + 18 = (x - 3)^2 - 9 + 18$$

$$= (x - 3)^2 + 9$$

$a = 3$  and  $b = 9$

**b**  $y = x^2 - 6x + 18$

$$y = (x - 3)^2 + 9$$

$$(x - 3)^2 \geq 0$$

Squaring a number cannot give a negative result.

The minimum value of  $(x - 3)^2$  is 0, when  $x = 3$ .

So the minimum value of  $y$  is  $0 + 9 = 9$ , when  $x = 3$ .

$Q$  is the point (3, 9).

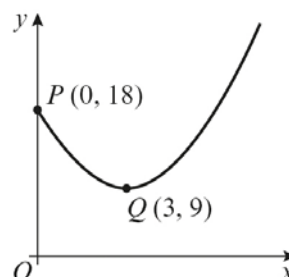
The curve crosses the  $y$ -axis where  $x = 0$ .

**13 b** For  $x = 0$ ,  $y = 18$

$P$  is the point (0, 18).

The graph of  $y = x^2 - 6x + 18$  is a  $\cup$  shape.

For  $y = ax^2 + bx + c$ , if  $a > 0$ , the shape is  $\cup$ .



Use the information about  $P$  and  $Q$  to sketch the curve  $x \geq 0$ , so the part where  $x < 0$  is not needed.

**c**  $y = (x - 3)^2 + 9$

Put  $y = 41$  into the equation of  $C$ .

$$41 = (x - 3)^2 + 9$$

Subtract 9 from both sides.

$$32 = (x - 3)^2$$

$$(x - 3)^2 = 32$$

Take the square root of both sides.

$$x - 3 = \pm\sqrt{32}$$

$$x = 3 \pm \sqrt{32}$$

$$\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$$

using  $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$

$$x = 3 \pm 4\sqrt{2}$$

$x$ -coordinate of  $R$  is  $3 + 4\sqrt{2}$

The other value is  $3 - 4\sqrt{2}$  which is less than 0, so is not needed.

**14 a** Using the discriminant

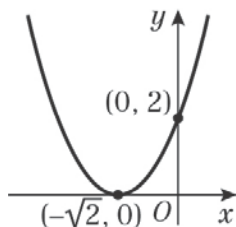
$b^2 - 4ac = 0$  for equal roots

$$(2\sqrt{2})^2 - 4(1)(k) = 0$$

$$8 - 4k = 0$$

$$k = 2$$

**14 b**  $y = x^2 + 2\sqrt{2}x + 2$   
 $= (x + \sqrt{2})^2$   
 When  $y = 0$ ,  $(x + \sqrt{2})^2 = 0$   
 $x = -\sqrt{2}$   
 When  $x = 0$ ,  $y = 2$



**15 a**  $g(x) = x^9 - 7x^6 - 8x^3$   
 $= x^3(x^6 - 7x^3 - 8)$   
 To factorise  $x^6 - 7x^3 - 8$ , let  $y = x^3$   
 $y^2 - 7y - 8 = (y + 1)(y - 8)$   
 So  $g(x) = x^3(x^3 + 1)(x^3 - 8)$   
 $a = 1, b = -8$

**b**  $g(x) = x^3(x^3 + 1)(x^3 - 8) = 0$   
 $x^3 = 0, x^3 = -1$  or  $x^3 = 8$   
 $x = -1, x = 0$  or  $x = 2$

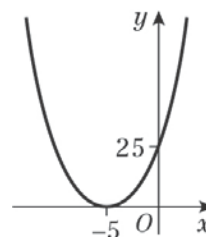
**16 a**  $x^2 + 10x + 36$   
 $x^2 + 10x = (x + 5)^2 - 25$   
 Complete the square for  $x^2 + 10x + 36$   
 $x^2 + 10x + 36 = (x + 5)^2 - 25 + 36$   
 $= (x + 5)^2 + 11$   
 $a = 5$  and  $b = 11$

**b**  $x^2 + 10x + 36 = 0$   
 $(x + 5)^2 + 11 = 0$   
 'Hence' implied in part **a** must be used  
 $(x + 5)^2 = -11$   
 A real number squared cannot be negative. There are no real roots.

**c**  $x^2 + 10x + k = 0$   
 $a = 1, b = 10, c = k$

**16 c** For equal roots,  $b^2 = 4ac$   
 $10^2 = 4 \times 1 \times k$   
 $4k = 100$   
 $k = 25$

**d** The graph of  $x^2 + 10x + 25$  is a  $\cup$  shape.  
 For  $y = ax^2 + bx + c$ , if  $a > 0$ , the shape is  $\cup$ .  
 $x = 0$ :  $y = 0 + 0 + 25 = 25$   
 Meets  $y$ -axis at  $(0, 25)$ .  
 $y = 0$ :  $x^2 + 10x + 25 = 0$   
 $(x + 5)(x + 5) = 0$   
 $x = -5$   
 Meets  $x$ -axis at  $(-5, 0)$ .



The graph meets the  $x$ -axis at just one point, so it 'touches' the  $x$ -axis.

**17 a**  $x^2 + 2x + 3$   
 $x^2 + 2x = (x + 1)^2 - 1$   
 Complete the square for  $x^2 + 2x + 3$   
 $x^2 + 2x + 3 = (x + 1)^2 - 1 + 3$   
 $= (x + 1)^2 + 2$   
 $a = 1$  and  $b = 2$

**b** The graph of  $y = x^2 + 2x + 3$  is a  $\cup$  shape.  
 For  $y = ax^2 + bx + c$ , if  $a > 0$ , the shape is  $\cup$ .  
 $x = 0$ :  $y = 0 + 0 + 3$   
 Put  $x = 0$  to find the intersection with the  $y$ -axis:  
 Meets  $y$ -axis at  $(0, 3)$ .

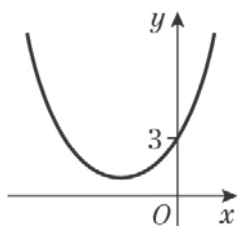
**17 b** Put  $y = 0$  to find the intersection with the  $x$ -axis:

$$y = 0 : x^2 + 2x + 3 = 0$$

$$(x + 1)^2 + 2 = 0$$

$$(x + 1)^2 = -2$$

A real number squared cannot be negative, therefore, no real roots, so not intersection with the  $x$ -axis.



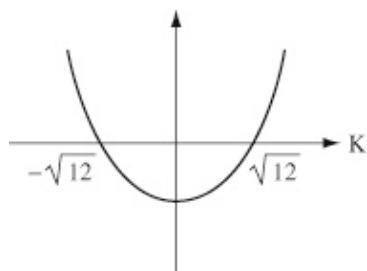
**c**  $x^2 + 2x + 3$   
 $a = 1, b = 2, c = 3$   
 $b^2 - 4ac = 2^2 - 4 \times 1 \times 3$   
 $= -8$

Since the discriminant is negative, the equation has no real roots, so the graph does not cross the  $x$ -axis.

**d**  $x^2 + kx + 3 = 0$   
 $a = 1, b = k, c = 3$   
 For no real roots,  $b^2 < 4ac$   
 $k^2 < 12$   
 $k^2 - 12 < 0$

$$(k + \sqrt{12})(k - \sqrt{12}) < 0$$

This is a quadratic inequality with critical values  $-\sqrt{12}$  and  $\sqrt{12}$ .



Critical values:

$$k = -\sqrt{12}, k = \sqrt{12}$$

$$-\sqrt{12} < k < \sqrt{12}$$

**17 d** The surds can be simplified

$$\text{using } \sqrt{(ab)} = \sqrt{a}\sqrt{b}$$

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$(-2\sqrt{3} < k < 2\sqrt{3})$$

**18 a**  $2x^2 - x(x - 4) = 8$   
 $2x^2 - x^2 + 4x = 8$   
 $x^2 + 4x - 8 = 0$

**b**  $x^2 + 4x - 8 = 0$   
 $x^2 + 4x = (x + 2)^2 - 4$

$$(x + 2)^2 - 4 - 8 = 0$$

$$(x + 2)^2 = 12$$

$$x + 2 = \pm\sqrt{12}$$

$$x = -2 \pm \sqrt{12}$$

$$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

$$x = -2 \pm 2\sqrt{3}$$

$$a = -2 \text{ and } b = 2$$

Using  $y = x - 4$ :

$$y = (-2 \pm 2\sqrt{3}) - 4$$

$$= -6 \pm 2\sqrt{3}$$

Solution:  $x = -2 \pm 2\sqrt{3}$

$$y = -6 \pm 2\sqrt{3}$$

**19 a**  $3(2x + 1) > 5 - 2x$

$$6x + 3 > 5 - 2x$$

$$6x + 2x + 3 > 5$$

$$8x > 2$$

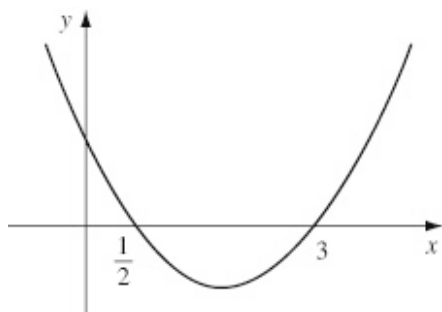
$$x > \frac{1}{4}$$

**b**  $2x^2 - 7x + 3 = 0$   
 $(2x - 1)(x - 3) = 0$   
 $(2x - 1) = 0$

$$(x - 3) = 0$$

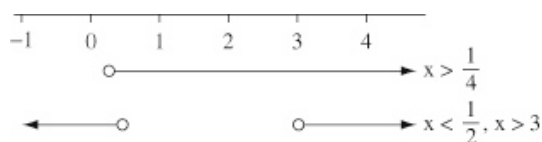
$$x = \frac{1}{2} \text{ or } x = 3$$

19 b



$2x^2 - 7x + 3 > 0$  where  
 $x < \frac{1}{2}$  or  $x > 3$

c



$\frac{1}{4} < x < \frac{1}{2}$  or  $x > 3$

20

$-2(x + 1) = x^2 - 5x + 2$   
 $-2x - 2 = x^2 - 5x + 2$   
 $x^2 - 3x + 4 = 0$   
 Using the discriminant  
 $b^2 - 4ac = (-3)^2 - 4(1)(4) = -7$   
 As  $b^2 - 4ac < 0$ , there are no real roots.  
 Hence there is no value of  $x$  for which  
 $p(x) = q(x)$ .

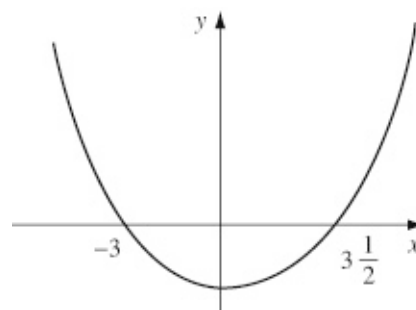
21 a

$y = 5 - 2x$   
 $2x^2 - 3x - (5 - 2x) = 16$   
 $2x^2 - 3x - 5 + 2x = 16$   
 $2x^2 - x - 21 = 0$   
 $(2x - 7)(x + 3) = 0$   
 $x = 3\frac{1}{2}, x = -3$   
 $x = 3\frac{1}{2}: y = 5 - 7 = -2$   
 $x = -3: y = 5 + 6 = 11$

Solution  $x = 3\frac{1}{2}, y = -2$   
 and  $x = -3, y = 11$

21 b

The equation in part a could be written as  $y = 5 - 2x$  and  $y = 2x^2 - 3x - 16$ .  
 Therefore, the solution to  $2x^2 - 3x - 16 = 5 - 2x$  are the same as for part a.  
 These are the critical values for  $2x^2 - 3x - 16 > 5 - 2x$ :  
 $x = 3\frac{1}{2}$  and  $x = -3$ .  
 $2x^2 - 3x - 16 > 5 - 2x$   
 $(2x^2 - 3x - 16 - 5 + 2x > 0)$   
 $2x^2 - x - 21 > 0$



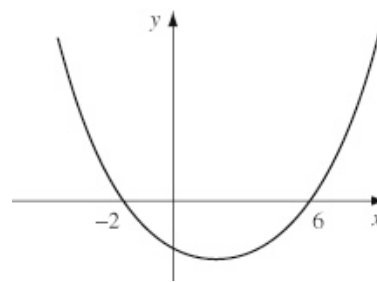
$x < -3$  or  $x > 3\frac{1}{2}$

22 a

$x^2 + kx + (k + 3) = 0$   
 $a = 1, b = k, c = k + 3$   
 $b^2 > 4ac$   
 $k^2 > 4(k + 3)$   
 $k^2 > 4k + 12$   
 $k^2 - 4k - 12 > 0$

b

$k^2 - 4k - 12 = 0$   
 $(k + 2)(k - 6) = 0$   
 $k = -2, k = 6$



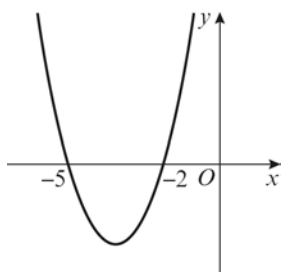
$k^2 - 4k - 12 > 0$  where  $k < -2$  or  $k > 6$

23  $\frac{6}{x+5} < 2$

Multiply both sides by  $(x + 5)^2$   
 $6(x + 5) < 2(x + 5)^2$   
 $6x + 30 < 2x^2 + 20x + 50$   
 $2x^2 + 14x + 20 > 0$

Solve the quadratic to find the critical values.

$2x^2 + 14x + 20 = 0$   
 $2(x^2 + 7x + 10) = 0$   
 $2(x + 5)(x + 2) = 0$   
 $x = -5$  or  $x = -2$



The solution is  $x < -5$  or  $x > -2$ .

24 a  $9 - x^2 = 0$   
 $(3 + x)(3 - x) = 0$   
 $x = -3$  or  $x = 3$   
 When  $x = 0$ ,  $y = 9$

To work out the points of intersection, solve the equations simultaneously.

$9 - x^2 = 14 - 6x$   
 $x^2 - 6x + 5 = 0$   
 $(x - 5)(x - 1) = 0$   
 $x = 1$  or  $x = 5$

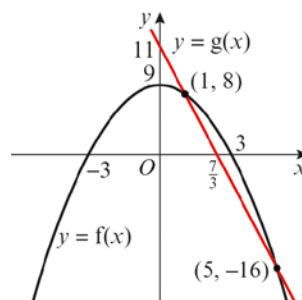
When  $x = 1$ ,  $y = 8$   
 When  $x = 5$ ,  $y = -16$

Let  $14 - 6x = 0$

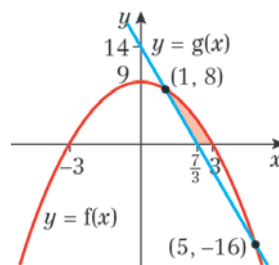
$x = \frac{14}{6} = \frac{7}{3}$

The line crosses the  $x$ -axis at  $(\frac{7}{3}, 0)$ .

24 a



b



25 a  $x^3 - 4x = x(x^2 - 4)$   
 $= x(x + 2)(x - 2)$

b Curve crosses the  $x$ -axis where  $y = 0$

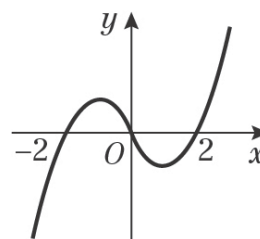
$x(x + 2)(x - 2) = 0$

$x = 0$ ,  $x = -2$ ,  $x = 2$

When  $x = 0$ ,  $y = 0$

When  $x \rightarrow \infty$ ,  $y \rightarrow \infty$

When  $x \rightarrow -\infty$ ,  $y \rightarrow -\infty$



Crosses the  $y$ -axis at  $(0, 0)$ .

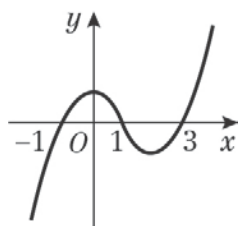
Crosses the  $x$ -axis at  $(-2, 0)$ ,  $(2, 0)$ .

c  $y = x^3 - 4x$

$y = (x - 1)^3 - 4(x - 1)$

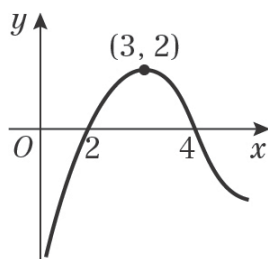
This is a translation of  $+1$  in the  $x$ -direction.

25 c



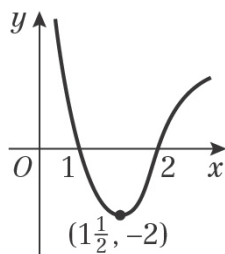
Crosses the  $x$ -axis at  $(-1, 0)$ ,  $(1, 0)$  and  $(3, 0)$ .

26 a



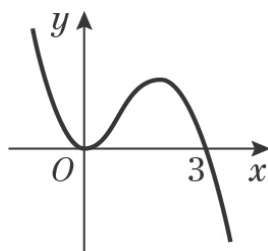
Crosses the  $x$ -axis at  $(2, 0)$ ,  $(4, 0)$ .  
Image of  $P$  is  $(3, 2)$ .

26 b



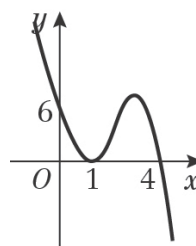
Crosses the  $x$ -axis at  $(1, 0)$ ,  $(2, 0)$ .  
Image of  $P$  is  $(1\frac{1}{2}, -2)$ .

27 a



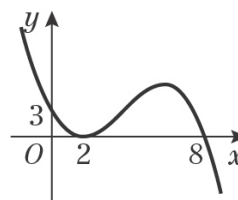
Meets the  $y$ -axis at  $(0, 0)$ .  
Meets the  $x$ -axis at  $(0, 0)$ ,  $(3, 0)$ .

27 b



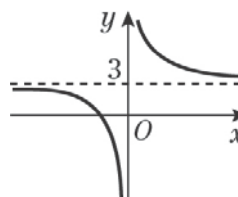
Meets the  $y$ -axis at  $(0, 6)$ .  
Meets the  $x$ -axis at  $(1, 0)$ ,  $(4, 0)$ .

c



Meets the  $y$ -axis at  $(0, 3)$ .  
Meets the  $x$ -axis at  $(2, 0)$ ,  $(8, 0)$ .

28 a



$y = 3$  is an asymptote.  
 $x = 0$  is an asymptote.

b The graph does not cross the  $y$ -axis  
(see sketch in part a).

Crosses the  $x$ -axis where  $y = 0$ :

$$\frac{1}{x} + 3 = 0$$

$$\frac{1}{x} = -3$$

$$x = -\frac{1}{3}, \left(-\frac{1}{3}, 0\right)$$



**29 a**  $(x^2 - 5x + 2)(x^2 - 5x + 4) = 0$   
 For  $x^2 - 5x + 2 = 0$

Using the quadratic formula:

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{17}}{2}$$

$$x = 4.56 \text{ or } x = 0.438$$

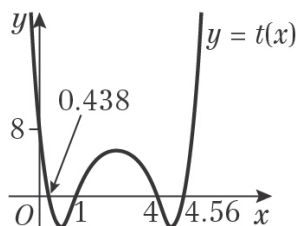
For  $x^2 - 5x + 4 = 0$

$$(x - 1)(x - 4) = 0$$

$$x = 1 \text{ or } x = 4$$

$$x = 0.438, 1, 4 \text{ or } 4.56$$

**b** When  $x = 0, y = 8$



**30 a**  $y = -f(x)$  is a reflection in the  $x$ -axis of  $y = f(x)$ , so  $P$  is transformed to  $(6, 8)$ .

**b**  $y = f(x - 3)$  is a translation 3 units to the right of  $y = f(x)$ , so  $P$  is transformed to  $(9, -8)$ .

**c**  $2y = f(x)$  is  $y = \frac{1}{2}f(x)$  which is a vertical stretch scale factor  $\frac{1}{2}$  of  $y = f(x)$ , so  $P$  is transformed to  $(6, -4)$ .

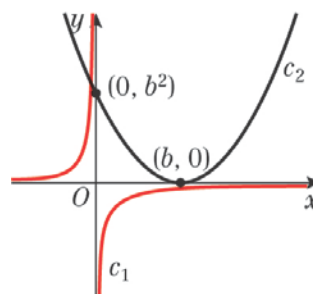
**31 a**  $y = -\frac{a}{x}$  is the curve  $y = \frac{k}{x}, k < 0$

$y = (x - b)^2$  is a translation,  $b$  units to the right of the curve  $y = x^2$

When  $x = 0, y = b^2$

When  $y = 0, x = b$

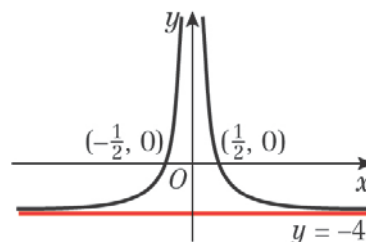
**31 a**



**b** The graphs intersect at 1 point, so have 1 point of intersection.

**32 a**  $y = \frac{1}{x^2} - 4$  is a translation  $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$  of

$$y = \frac{1}{x^2}$$



**b** When  $y = \frac{1}{(x+k)^2} - 4$  passes through the origin,  $x = 0$  and  $y = 0$ .

$$\text{So } \frac{1}{k^2} - 4 = 0$$

$$\frac{1}{k^2} = 4$$

$$k = \pm \frac{1}{2}$$

**Challenge**

**1 a**  $x^2 - 10x + 9 = 0$   
 $(x - 1)(x - 9) = 0$   
 $x = 1$  or  $x = 9$

**b**  $3^{x-2}(3^x - 10) = -1$   
 $3^{2x-2} - 10 \times 3^{x-2} + 1 = 0$   
 Multiply by  $3^2$   
 $3^{2x} - 10 \times 3^x + 9 = 0$   
 Let  $y = 3^x$   
 $y^2 - 10y + 9 = 0$   
 Using your answers from part **a**  
 $y = 1$  or  $9$   
 $3^x = 1$  or  $3^x = 9$   
 $x = 0$  or  $2$

**2** Let  $x$  and  $y$  be the length and width of the rectangle.  
 Area =  $xy = 6$   
 Perimeter =  $2x + 2y = 8\sqrt{2}$   
 $2y = 8\sqrt{2} - 2x$   
 $y = 4\sqrt{2} - x$   
 Solving simultaneously  
 $x(4\sqrt{2} - x) = 6$   
 $x^2 - 4\sqrt{2}x + 6 = 0$

Using the quadratic formula

$$x = \frac{4\sqrt{2} \pm \sqrt{(4\sqrt{2})^2 - 4(1)(6)}}{2(1)}$$

$$= \frac{4\sqrt{2} \pm \sqrt{8}}{2}$$

$$= \frac{4\sqrt{2} \pm 2\sqrt{2}}{2}$$

$x = \sqrt{2}$  or  $x = 3\sqrt{2}$

When  $x = \sqrt{2}$ ,  $y = 3\sqrt{2}$

When  $x = 3\sqrt{2}$ ,  $y = \sqrt{2}$

The dimensions of the rectangle are  $\sqrt{2}$  cm and  $3\sqrt{2}$  cm.

**3** Solving simultaneously  
 $3x^3 + x^2 - x = 2x(x - 1)(x + 1)$   
 $3x^3 + x^2 - x = 2x(x^2 - 1)$   
 $3x^3 + x^2 - x = 2x^3 - 2x$   
 $x^3 + x^2 + x = 0$   
 $x(x^2 + x + 1) = 0$

The discriminant of  $x^2 + x + 1$   
 $b^2 - 4ac = 1^2 - 4(1)(1) = -3$   
 $-3 < 0$  so there are no real solutions for  $x^2 + x + 1$

The only solution is when  $x = 0$  at  $(0, 0)$ .

**4**  $f(x) = (x^2 + x - 20)(x^2 + x - 2)$   
 $= (x + 5)(x - 4)(x + 2)(x - 1)$   
 when  $f(x) = 0$   
 $x = -5, -2, 1$  or  $4$

$g(x - k) = (x - k + 5)(x - k - 4)$   
 $(x - k + 2)(x - k - 1)$   
 When  $k = 3$ ,  
 $g(x - 3) = (x + 2)(x - 7)(x - 1)(x - 4)$   
 $(x + 2)$ ,  $(x - 1)$  and  $(x - 4)$  match  
 When  $k = -3$ ,  $g(x + 3)$   
 $= (x + 8)(x - 1)(x + 5)(x + 2)$

$(x - 1)$ ,  $(x + 5)$  and  $(x + 2)$  match  
 So  $k = -3$  or  $3$ .