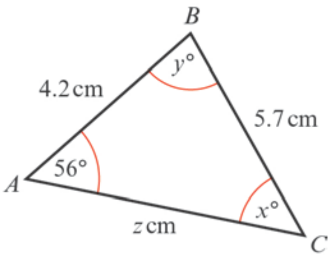
Trigonometric ratios 9E

1 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin x°}{4.2} = \frac{\sin 56°}{5.7}$$

$$\sin x° = \frac{4.2 \sin 56°}{5.7}$$

$$x° = \sin^{-1}\left(\frac{4.2 \sin 56°}{5.7}\right)$$

$$= 37.65\ldots°$$

$$x = 37.7 \text{ (3 s.f.)}$$

$$\text{So } y° = 180° - (56° + 37.7°)$$

$$= 86.3°$$

$$y = 86.3 \text{ (3 s.f.)}$$

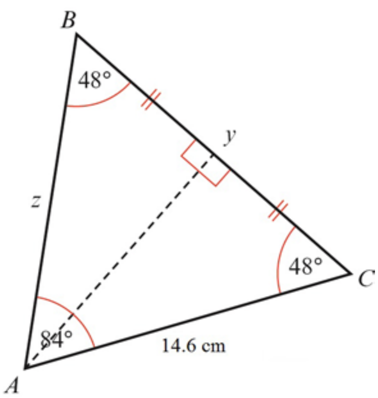
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{z}{\sin y°} = \frac{5.7}{\sin 56°}$$

$$\text{So } z = \frac{5.7 \sin y°}{\sin 56°}$$

$$= 6.86 \text{ (3 s.f.)}$$

b $x° = 180° - (48 + 84)°$
$x° = 48°$



As angle B = angle C, $z = 14.6$ cm.

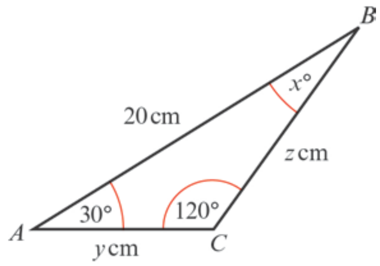
b Using the line of symmetry through A

$$\cos 48° = \frac{\frac{y}{2}}{14.6}$$

$$\text{So } y = 29.2 \cos 48°$$

$$= 19.5 \text{ (3 s.f.)}$$

c



$$x° = 180° - (120° + 30°)$$

$$= 30°$$

Using the line of symmetry through C

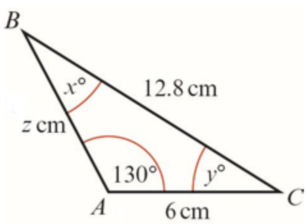
$$\cos 30° = \frac{10}{y}$$

$$\text{So } y = \frac{10}{\cos 30°}$$

$$= 11.5 \text{ (3 s.f.)}$$

Since ΔABC is isosceles with $AC = CB$
$z = 11.5$ (3 s.f.).

d



Using $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\frac{\sin 130°}{12.8} = \frac{\sin x°}{6}$$

$$\text{So } \sin x° = \frac{6 \sin 130°}{12.8}$$

$$= 0.359\,08\ldots$$

$$\Rightarrow x = 21.0 \text{ (3 s.f.)}$$

$$\text{So } y° = 180° - (130° + x°)$$

$$= 28.956\ldots°$$

1 d $\Rightarrow y = 29.0\ (3\text{ s.f.})$

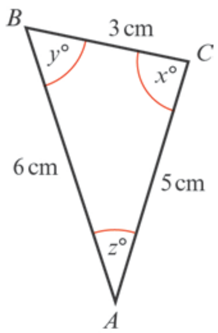
Using $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{z}{\sin y°} = \dfrac{12.8}{\sin 130°}$

So $z = \dfrac{12.8 \sin y°}{\sin 130°}$

$= 8.09\ (3\text{ s.f.})$

e



Using $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos x° = \dfrac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5}$

$= -0.0\dot{6}$

$x = 93.8\ (3\text{ s.f.})$

Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin y°}{5} = \dfrac{\sin x°}{6}$

$\sin y° = \dfrac{5 \sin x°}{6}$

$y° = \sin^{-1}\left(\dfrac{5 \sin x°}{6}\right)$

$= 56.25\ldots°$

$y = 56.3\ (3\text{ s.f.})$

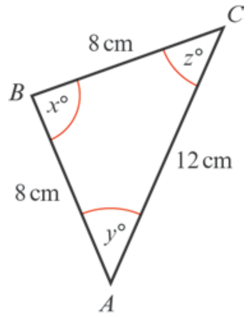
Using the angle sum for a triangle

$z° = 180° - (x + y)°$

$= 29.926\ldots°$

$z = 29.9\ (3\text{ s.f.})$

f



Using the line of symmetry through B

$\cos y° = \frac{6}{8}$

$= \frac{3}{4}$

$y° = \cos^{-1}\left(\frac{3}{4}\right)$

$= 41.40\ldots$

$y = 41.4\ (3\text{ s.f.})$

As the triangle is isosceles

$z = y$

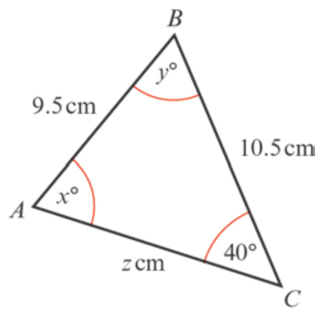
$= 41.4\ (3\text{ s.f.})$

So $x° = 180° - (y + z)°$

$= 97.2°$

$x = 97.2\ (3\text{ s.f.})$

g



Using $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\dfrac{\sin x°}{10.5} = \dfrac{\sin 40°}{9.5}$

$\sin x° = \dfrac{10.5 \sin 40°}{9.5}$

$x° = \sin^{-1}\left(\dfrac{10.5 \sin 40°}{9.5}\right)$ or

$x° = 180° - \sin^{-1}\left(\dfrac{10.5 \sin 40°}{9.5}\right)$

$x° = 45.27°$ or $x° = 134.728\ldots°$

$x = 45.3\ (3\text{ s.f.})$ or $x = 135\ (3\text{ s.f.})$

1 g Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{z}{\sin y°} = \frac{9.5}{\sin 40°}$$

$$z = \frac{9.5 \sin y°}{\sin 40°}$$

When $x = 45.3$

$$y° = 180° - (40 + 45.3)°$$
$$= 94.7°$$

So $y = 94.7$ (3 s.f.)

$$z = \frac{9.5 \sin y°}{\sin 40°}$$
$$= 14.7 \text{ (3 s.f.)}$$

When $x = 134.728\ldots$

$$y° = 180° - (40 + 134.72\ldots)°$$
$$= 5.27°$$

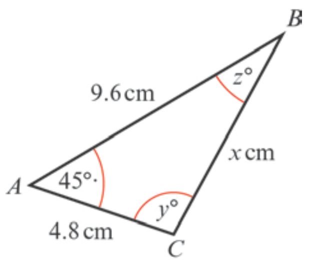
So $y = 5.27$ (3 s.f.)

$$z = \frac{9.5 \sin y°}{\sin 40°}$$
$$= 1.36 \text{ (3 s.f.)}$$

So $x = 45.3,\ y = 94.7,\ z = 14.7$

or $x = 135,\ y = 5.27,\ z = 1.36$

h



Using $a^2 = b^2 + c^2 - 2bc \cos A$

$$x^2 = 4.8^2 + 9.6^2 - 2 \times 4.8 \times 9.6 \times \cos 45°$$
$$= 50.03\ldots$$
$$x = 7.07 \text{ (3 s.f.)}$$

Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$$\frac{\sin y°}{9.6} = \frac{\sin 45°}{x}$$

$$\sin y° = \frac{9.6 \sin 45°}{x}$$

$$y° = \sin^{-1}\left(\frac{9.6 \sin 45°}{x}\right)$$

h $y° = 73.68\ldots°$

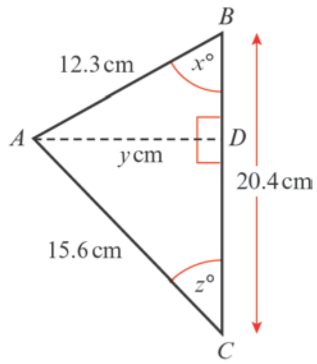
$y = 73.7$ (3 s.f.)

Then

$$z° = 180° - (45 + 73.68\ldots)°$$
$$= 61.32\ldots°$$
$$z = 61.3 \text{ (3 s.f.)}$$

So $x = 7.07,\ y = 73.7,\ z = 61.3$

i



Using $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$$\cos x° = \frac{20.4^2 + 12.3^2 - 15.6^2}{2 \times 20.4 \times 12.3}$$
$$= 0.6458\ldots$$
$$x = 49.77\ldots°$$
$$x = 49.8 \text{ (3 s.f.)}$$

In right-angled triangle *ABD*

$$\sin x° = \frac{y}{12.3}$$

So $y = 12.3 \sin x°$

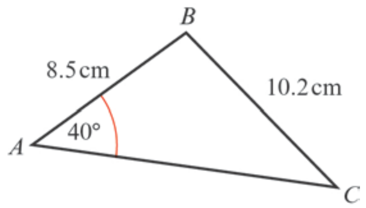
$$= 9.39 \text{ (3 s.f.)}$$

In right-angled triangle *ACD*

$$\sin z° = \frac{y}{15.6}$$
$$= 0.60199\ldots$$
$$z° = 37.01\ldots°$$
$$z = 37.0 \text{ (3 s.f.)}$$

So $x = 49.8,\ y = 9.39,\ z = 37.0$

2 a



Using $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{8.5} = \frac{\sin 40°}{10.2}$

$\sin C = \frac{8.5 \sin 40°}{10.2}$

$C = \sin^{-1}\left(\frac{8.5 \sin 40°}{10.2}\right)$

$= 32.388...°$

$= 32.4° \text{ (3 s.f.)}$

$B = 180° - (40 + C)°$

$= 107.6...°$

$B = 108° \text{ (3 s.f.)}$

Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{10.2 \sin B}{\sin 40°}$

$= 15.1 \text{ cm (3 s.f.)}$

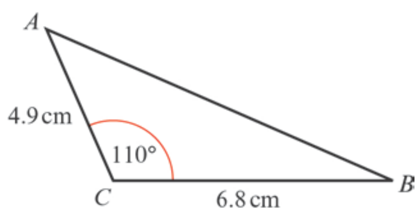
Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 10.2 \times 8.5 \times \sin 108°$

$= 41.228$

$= 41.2 \text{ cm}^2 \text{ (3 s.f.)}$

b



Using $c^2 = a^2 + b^2 - 2ab\cos C$

$AB^2 = 6.8^2 + 4.9^2 - 2 \times 6.8 \times 4.9 \times \cos 110°$

$= 93.04...$

$AB = 9.6458...$

$= 9.65 \text{ cm (3 s.f.)}$

Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

2 b $\sin A = \frac{6.8 \sin 110°}{AB}$

$= 0.662\,45...$

$A = 41.49°$

$= 41.5° \text{ (3 s.f.)}$

So $B = 180° - (110 + A)°$

$= 28.5° \text{ (3 s.f.)}$

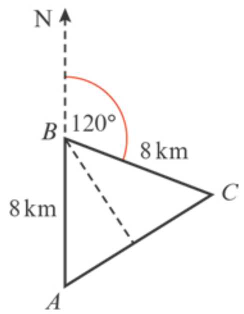
Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 6.8 \times 4.9 \times \sin 110°$

$= 15.655...$

$= 15.7 \text{ cm}^2$ (3 s.f.)

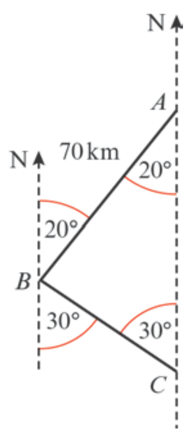
3



a Angle ABC = 180° – 120°
= 60°
As $\angle A = \angle C$, all angles are 60°.
It is an equilateral triangle.
So AC = 8 km.

b As $\angle BAC = 60°$,
the bearing of C from A is 060°.

4



From the diagram

$\angle ABC = 180° - (20 + 30)°$

$= 130°$

4 Using $\frac{b}{\sin B}=\frac{c}{\sin C}$

$$\frac{AC}{\sin 130°}=\frac{70}{\sin 30°}$$

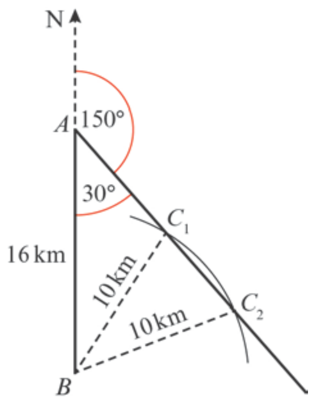
$$AC=\frac{70\sin 130°}{\sin 30°}$$

$$=107.246\ldots$$

$$AC=107\text{ km }(3\text{ s.f.})$$

From the diagram, the bearing of C from A is 180˚.

5



Using the sine rule

$$\frac{\sin C}{c}=\frac{\sin A}{a}$$

$$\frac{\sin C}{16}=\frac{\sin 30°}{10}$$

$$\sin C=\frac{16\sin 30°}{10}$$

$$=0.8$$

$$C=\sin^{-1}(0.8)\text{ or }C=180°-\sin^{-1}(0.8)$$

$$C=53.1°\text{ or }C=126.9°$$

$$\angle AC_2B=53.1°,\ \angle AC_1B=127°\ (3\text{ s.f.})$$

(Store the correct values; these are not required answers.)

Triangle BC_1C_2 is isosceles, so C_1C_2 can be found using this triangle, without finding AC_1 and AC_2.

Use the line of symmetry through B

$$\cos\angle C_1C_2B=\frac{\frac{1}{2}C_1C_2}{10}$$

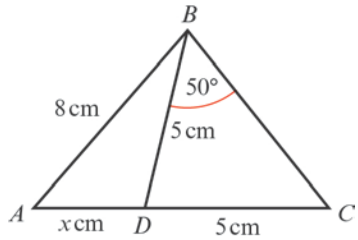
$$\Rightarrow C_1C_2=20\cos\angle C_1C_2B$$

$$=20\cos\angle AC_2B$$

$$=20\cos 53.1°$$

$$\Rightarrow C_1C_2=12\text{ km}$$

6 a



In the isosceles ΔBDC

$$\angle BDC=180°-(50+50)°$$

$$=80°$$

$$\text{So }\angle BDA=180°-80°$$

$$=100°$$

Using the sine rule in ΔABD

$$\frac{\sin A}{a}=\frac{\sin D}{d}$$

$$\Rightarrow\frac{\sin A}{5}=\frac{\sin 100°}{8}$$

$$\Rightarrow\sin A=\frac{5\sin 100°}{8}$$

$$\text{So }A=\sin^{-1}\left(\frac{5\sin 100°}{8}\right)$$

$$=37.9886\ldots$$

$$\angle ABD=180°-(100+A)°$$

$$=42.01\ldots°$$

Using $\frac{b}{\sin B}=\frac{d}{\sin D}$

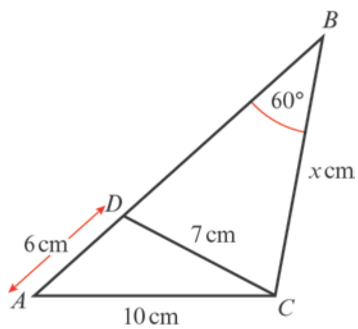
$$\frac{x}{\sin B}=\frac{8}{\sin 100°}$$

$$x=\frac{8\sin B}{\sin 100°}$$

$$=5.436\ldots$$

$$x=5.44\ (3\text{ s.f.})$$

b



In ΔADC, using $\cos A=\frac{c^2+d^2-a^2}{2cd}$

6 b $\cos A = \frac{6^2 + 10^2 - 7^2}{2 \times 6 \times 10}$

$= 0.725$

So $A = 43.53\ldots°$

Using the sine rule in ΔABC

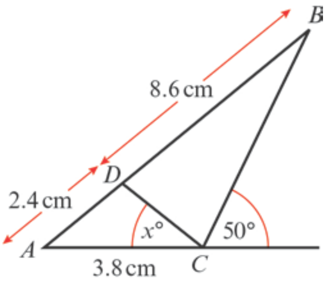
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

So $\frac{x}{\sin A} = \frac{10}{\sin 60°}$

$\Rightarrow x = \frac{10 \sin A}{\sin 60°}$

So $x = 7.95$ (3 s.f.)

c



In ΔABC, $c = 11$ cm, $b = 3.8$ cm,
$\angle ACB = 130°$, $(180° - 50°)$

Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\sin B = \frac{3.8 \sin 130°}{11}$

$= 0.2646\ldots$

$B = 15.345\ldots°$

So $A = 180° - (130 + B)°$

$= 34.654\ldots°$

In ΔADC, $c = 2.4$ cm, $d = 3.8$ cm,
$A = 34.654\ldots°$

Using the cosine rule:

$a^2 = c^2 + d^2 - 2cd \cos A$

So $DC^2 = 2.4^2 + 3.8^2 - 2 \times 2.4 \times 3.8 \times \cos A$

$= 5.1959\ldots$

$\Rightarrow DC = 2.279\ldots$cm

Using the sine rule:

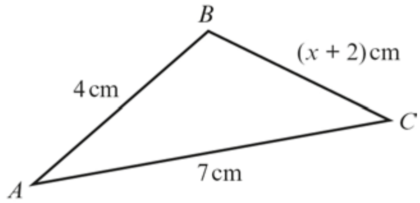
$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\sin x° = \frac{2.4 \sin A}{DC}$

$= 0.598\,69\ldots$

$x = 36.8$ (3 s.f.)

7



a As $AB + BC > AC$

$4 + (x + 2) > 7$

$\Rightarrow x + 2 > 3$

$\Rightarrow x > 1$

As $AB + AC > BC$

$4 + 7 > x + 2$

$\Rightarrow 9 > x$

So $1 < x < 9$

b Using $b^2 = a^2 + c^2 - 2ac \cos B$

i $7^2 = (x+2)^2 + 4^2 - 2(x+2) \times 4 \times \cos 60°$

$49 = x^2 + 4x + 4 + 16 - 4(x + 2)$

$49 = x^2 + 4x + 4 + 16 - 4x - 8$

So $x^2 = 37$

$\Rightarrow x = 6.08$ (3 s.f.)

Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 8.08 \times 4 \times \sin 60°$

$= 13.9949\ldots$

$= 14.0$ cm^2 (3 s.f.)

ii $7^2 = (x + 2)^2 + 4^2$
$- 2 \times (x + 2) \times 4 \times \cos 45°$

$49 = x^2 + 4x + 4 + 16$
$- (8 \cos 45°)x - 16 \cos 45°$

So:

$x^2 + (4 - 8 \cos 45°)x$
$- (29 + 16 \cos 45°) = 0$

or $x^2 + 4(1 - \sqrt{2})x$
$- (29 + 8\sqrt{2}) = 0$

Use the quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1$

$b = 4 - 8 \cos 45°$

$= 4(1 - \sqrt{2})$

$= -1.6568\ldots$

7 b ii $c = -\left(29 + 16\cos 45°\right)$

$= -\left(29 + 8\sqrt{2}\right)$

$= -40.313\ldots$

$x = 7.23\,(3 \text{ s.f.})$

(The other value of x is less than –2.)

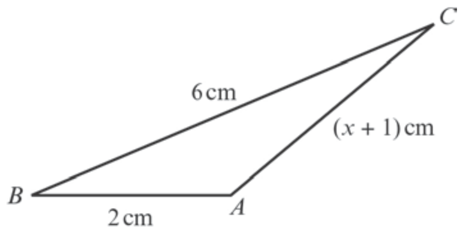
Area $= \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 4 \times 9.23 \times \sin 45°$

$= 13.05\ldots$

$= 13.1\text{ cm}^2$ (3 s.f.)

8 a



Using $b^2 = a^2 + c^2 - 2ac\cos B$ where $\cos B = \frac{5}{8}$

$(x+1)^2 = 6^2 + 2^2 - 2 \times 6 \times 2 \times \frac{5}{8}$

$x^2 + 2x + 1 = 36 + 4 - 15$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

So $x = 4\,(x > -1)$

b Use identity, $\cos^2 x + \sin^2 x = 1$.

$\cos B = \frac{5}{8}$

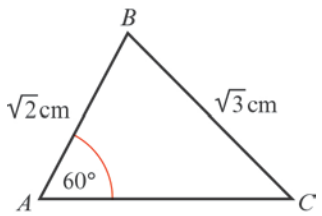
so $\sin \angle ABC = \frac{\sqrt{39}}{8}$

Area $= \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 6 \times 2 \times \frac{\sqrt{39}}{8}$

$= 4.68\text{ cm}^2$ (3 s.f.)

9



9 Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\sin C = \dfrac{\sqrt{2}\sin 60°}{\sqrt{3}}$

$= 0.7071\ldots$

$C = \sin^{-1}\left(\dfrac{\sqrt{2}\sin 60°}{\sqrt{3}}\right)$

$= 45°$

$B = 180° - (60 + 45)°$

$= 75°$

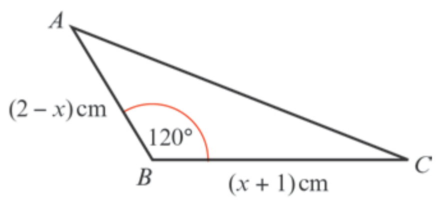
Using $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{AC}{\sin 75°} = \dfrac{\sqrt{3}}{\sin 60°}$

So $AC = \dfrac{\sqrt{3}\sin 75°}{\sin 60°}$

$= 1.93\text{ cm}\ (3 \text{ s.f.})$

10



a Using the cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos B$

$AC^2 = (x+1)^2 + (2-x)^2$
$\quad - 2(x+1)(2-x)\cos 120°$

$AC^2 = (x^2 + 2x + 1) + (4 - 4x + x^2)$
$\quad + (x+1)(2-x)$

$AC^2 = x^2 + 2x + 1 + 4 - 4x + x^2$
$\quad - x^2 + 2x - x + 2$

$AC^2 = x^2 - x + 7$

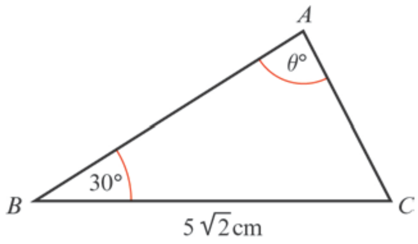
b Completing the square:

$x^2 - x + 7 \equiv \left(x - \frac{1}{2}\right)^2 + 7 - \frac{1}{4}$

$\equiv \left(x - \frac{1}{2}\right)^2 + 6\frac{3}{4}$

This is a minimum when $x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$.

11



Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{AC}{\sin 30°} = \frac{5\sqrt{2}}{\sin \theta°}$$

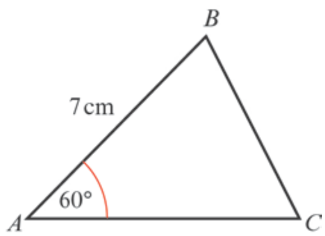
$$AC = \frac{5\sqrt{2}\sin 30°}{\left(\frac{\sqrt{5}}{8}\right)}$$

$$AC = \frac{5\sqrt{2}\sin 30° \times 8}{\sqrt{5}}$$

$$= \left(\sqrt{5}\sqrt{2}\right)\left(8\sin 30°\right)$$

$$= 4\sqrt{10} \text{ cm}$$

12



Using the cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

with $a = x$, $b = (8 - x)$, $c = 7$ and $A = 60°$

$$x^2 = (8-x)^2 + 49 - 2(8-x) \times 7 \times \cos 60°$$

$$x^2 = 64 - 16x + x^2 + 49 - 7(8-x)$$

$$x^2 = 64 - 16x + x^2 + 49 - 56 + 7x$$

$$\Rightarrow 9x = 57$$

$$\Rightarrow x = \tfrac{57}{9} = \tfrac{19}{3} = 6\tfrac{1}{3}$$

So $BC = 6\frac{1}{3}$ cm and

$$AC = \left(8 - 6\tfrac{1}{3}\right)\text{cm}$$

$$= 1\tfrac{2}{3}\text{cm}$$

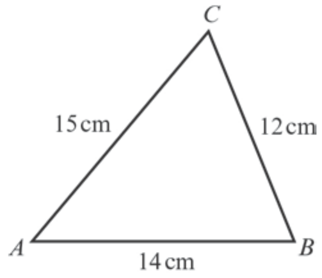
$$\text{Area} = \tfrac{1}{2}bc\sin A$$

$$= \tfrac{1}{2} \times 7 \times \tfrac{5}{3} \times \sin 60°$$

$$= 5.0518\ldots$$

$$= 5.05 \text{ cm}^2 \text{ (3 s.f.)}$$

13 a



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{12^2 + 15^2 - 14^2}{2(12)(15)}$$

$$\cos C = \frac{144 + 225 - 196}{360}$$

$C = 61.278\ldots$

$C = 61.3°$ (3 s.f.)

b Use the formula.

$$\text{Area} = \tfrac{1}{2}ab\sin C$$

$$= \tfrac{1}{2} \times 12 \times 15 \times \sin 61.3°$$

$$= 78.943\ldots$$

$$= 78.9 \text{ cm}^2 \text{ (3 s.f.)}$$

14 a $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\cos A = \frac{2.1^2 + 4.2^2 - 5.9^2}{2(2.1)(4.2)}$$

$$\cos A = \frac{4.41 + 17.64 - 34.81}{17.64}$$

A = 136.33...

∴ Angle DAB = 136.3° (1 d.p.)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{3.5^2 + 7.5^2 - 5.9^2}{2(3.5)(7.5)}$$

$$\cos C = \frac{12.25 + 56.25 - 34.81}{52.5}$$

$C = 50.080\ldots$

∴ Angle BCD = 50.1°

b Area $ABD = \frac{1}{2}bc\sin A$

$= \frac{1}{2} \times 2.1 \times 4.2 \times \sin 136.3°$

$= 3.046\ 79\ldots$

14 b Area $BCD = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 3.5 \times 7.5 \times \sin 50.1°$

$= 10.069\ 04...$

Total area = 3.046 79 + 10.069 04

= 13.11583

$\therefore$ The area of the flower bed is 13.1 m^2.

c First find angle *ADB*:

$$\cos D = \frac{a^2 + b^2 - d^2}{2ab}$$

$$\cos D = \frac{5.9^2 + 2.1^2 - 4.2^2}{2(5.9)(2.1)}$$

$$\cos D = \frac{34.81 + 4.41 - 17.64}{24.78}$$

So $D = 29.440\ 849...$

Now find angle *BDC*:

$$\cos D = \frac{b^2 + c^2 - d^2}{2bc}$$

$$\cos D = \frac{3.5^2 + 5.9^2 - 7.5^2}{2(3.5)(5.9)}$$

$$\cos D = \frac{12.25 + 34.81 - 56.25}{41.3}$$

So D = 102.856 97...

Angle $ADC = 29.440849 + 102.85697$

$= 132.298°$

Now find the length *AC*:

$$d^2 = a^2 + c^2 - 2ac\cos D$$

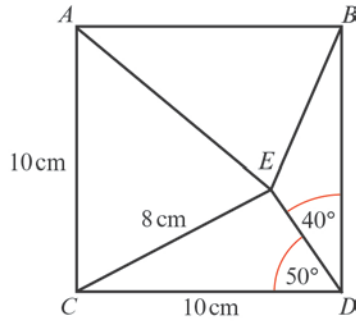
$$d^2 = 3.5^2 + 2.1^2 - 2 \times 3.5 \times 2.1 \times \cos 132.298°$$

$$d^2 = 12.25 + 4.41 + 9.8929$$

So $d = 5.15$

So the length of *AC* is 5.15 m.

15



Use the sine rule to work out angle *CED*.

$$\frac{\sin E}{e} = \frac{\sin D}{d}$$

$$\frac{\sin E}{10} = \frac{\sin 50°}{8}$$

$$\sin E = \frac{10\sin 50°}{8}$$

$E = 73.246\ 86°$ or $106.753\ 14°$

The angle is obtuse so

Angle $CED = 106.753\ 14°$

Angle $ECD = 180° - 50° - 106.753\ 14°$

$= 23.25°$

Use trigonometry to work out the height of triangle *CDE*.

$$\sin 23.25° = \frac{\text{height}}{8}$$

Height = 3.1575 cm

The height of triangle *ABE* = 10 − 3.1575

= 6.84 cm

Area of triangle = $\frac{1}{2} \times 10 \times 6.84 = 34.2$

$\therefore$ Area of the shaded triangle is 34.2 cm^2.