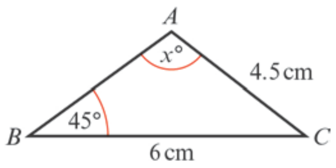
Trigonometric ratios 9C

1 a



$x > 45°$

So there are two possible results.

Using $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$$\frac{\sin x°}{6} = \frac{\sin 45°}{4.5}$$

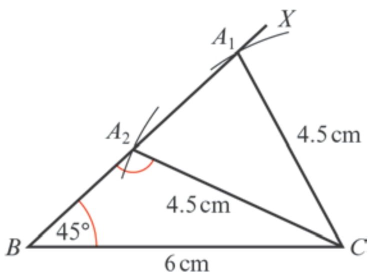
$$\sin x° = \frac{6\sin 45°}{4.5}$$

$$x° = \sin^{-1}\left(\frac{6\sin 45°}{4.5}\right) \text{ or}$$

$$x° = 180° - \sin^{-1}\left(\frac{6\sin 45°}{4.5}\right)$$

$$x° = 70.5° \,(3 \text{ s.f.}) \text{ or } x° = 109.5°$$

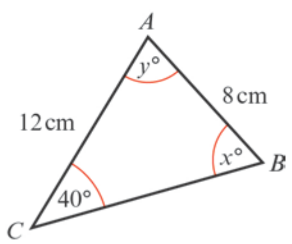
b



Draw *BC* = 6 cm.

Construct or draw an angle of 45° at *B* and extend the line as (*BX*).
Set the compasses to a radius of 4.5 cm.
Put the point on *C* and draw an arc.
The points where the arc meets *BX* are the two possible positions of *A*.

2 a



Using $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$$\frac{\sin x°}{12} = \frac{\sin 40°}{8}$$

$$\sin x° = \frac{12\sin 40°}{8}$$

$$x° = \sin^{-1}\left(\frac{12\sin 40°}{8}\right) \text{ or}$$

$$x° = 180° - \sin^{-1}\left(\frac{12\sin 40°}{8}\right)$$

$$x° = 74.6° \text{ or } x° = 105.4°$$

$$x = 74.6 \text{ or } 105 \,(3 \text{ s.f.})$$

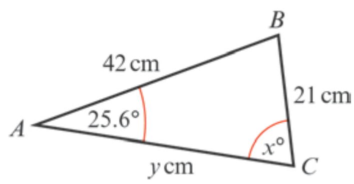
When $x = 74.6$:

$$y = 180 - (74.6 + 40)$$
$$= 180 - 114.6$$
$$= 65.4 \,(3 \text{ s.f.})$$

When $x = 105.4$:

$$y = 180 - (105.4 + 40)$$
$$= 180 - 145.4$$
$$= 34.6 \,(3 \text{ s.f.})$$

b



Using $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$$\frac{\sin x°}{42} = \frac{\sin 25.6°}{21}$$

$$\sin x° = \frac{42\sin 25.6°}{21}$$

$$x° = \sin^{-1}(2\sin 25.6°) \text{ or}$$

$$x° = 180° - \sin^{-1}(2\sin 25.6°)$$

$$x = 59.8 \text{ or } x = 120 \,(3 \text{ s.f.})$$

When $x = 59.8$:

angle $B = 180° - (59.8° + 25.6°) = 94.6°$

When $x = 120°$:

angle $B = 180° - (120.2° + 25.6°) = 34.2°$

2 b Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 94.6°} = \frac{21}{\sin 25.6°}$$

$$\Rightarrow y = \frac{21 \sin 94.6°}{\sin 25.6°}$$

$$= 48.4 \text{ (3 s.f.)}$$

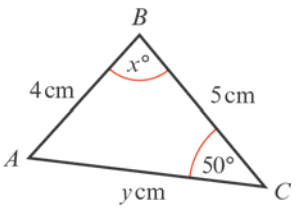
Using $\frac{b}{\sin B} = \frac{a}{\sin A}$

$$\frac{y}{\sin 34.2°} = \frac{21}{\sin 25.6°}$$

$$\text{So } y = \frac{21 \sin 34.2°}{\sin 25.6°}$$

$$= 27.3 \text{ (3 s.f.)}$$

c



Using $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin A}{5} = \frac{\sin 50°}{4}$$

$$\sin A = \frac{5 \sin 50°}{4}$$

$$A = \sin^{-1}\left(\frac{5 \sin 50°}{4}\right) \text{ or}$$

$$A = 180° - \sin^{-1}\left(\frac{5 \sin 50°}{4}\right)$$

$$A = 73.25 \text{ or } A = 106.75$$

When $A = 73.247$:

$$x = 180 - (50 + 73.247)$$

$$= 56.8 \text{ (3 s.f.)}$$

Using $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\frac{y}{\sin x°} = \frac{4}{\sin 50°}$$

$$\text{So } y = \frac{4 \sin x°}{\sin 50°}$$

$$= 4.37 \text{ (3 s.f.)}$$

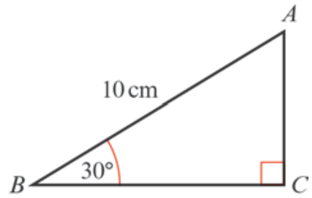
2 c When $A = 106.75$:

$$x = 180 - (50 + 106.75) = 23.2 \text{ (3 s.f.)}$$

As above:

$$y = \frac{4 \sin x°}{\sin 50°} = 2.06 \text{ (3 s.f.)}$$

3 a



The length of AC is least when it is at right angles to BC.

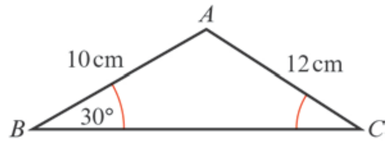
Using $\sin B = \frac{AC}{AB}$

$$\sin 30° = \frac{AC}{10}$$

$$AC = 10 \sin 30° = 5$$

$$AC = 5 \text{ cm}$$

b



Using $\frac{\sin C}{c} = \frac{\sin B}{b}$

$$\frac{\sin C}{10} = \frac{\sin 30°}{12}$$

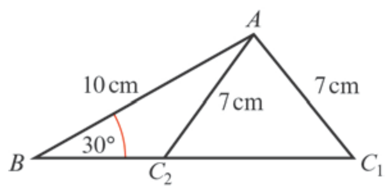
$$\sin C = \frac{10 \sin 30°}{12}$$

$$C = \sin^{-1}\left(\frac{10 \sin 30°}{12}\right)$$

$$= 24.62°$$

$$\angle ABC = 24.6° \text{ (3 s.f.)}$$

c



As 7 cm < 10 cm, $\angle ACB > 30°$.

3 c There are two possible results.
Using 7 cm instead of 12 cm in (b):

$$\sin C = \frac{10\sin 30°}{7}$$

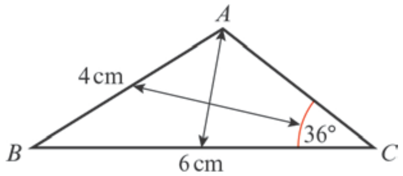
$$C = \sin^{-1}\left(\frac{10\sin 30°}{7}\right) \text{ or}$$

$$C = 180° - \sin^{-1}\left(\frac{10\sin 30°}{7}\right)$$

$$C = 45.58° \text{ or } 134.4°$$

$$\angle ABC = 45.6° (3 \text{ s.f.}) \text{ or } 134° \ (3 \text{ s.f.})$$

4



As 4 < 6, 36° < $\angle BAC$, so there are two possible values for angle A.

$$\text{Using } \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{6} = \frac{\sin 36°}{4}$$

$$\sin A = \frac{6\sin 36°}{4}$$

$$A = \sin^{-1}\left(\frac{6\sin 36°}{4}\right) \text{ or}$$

$$A = 180° - \sin^{-1}\left(\frac{6\sin 36°}{4}\right)$$

$$A = 61.845\ldots° \text{ or } A = 118.154\ldots°$$

When $A = 118.154\ldots°$:

$$\angle ABC = 180° - (36° + 118.154\ldots°)$$

$$= 25.8° \ (3 \text{ s.f.})$$

Using this value for $\angle ABC$ with

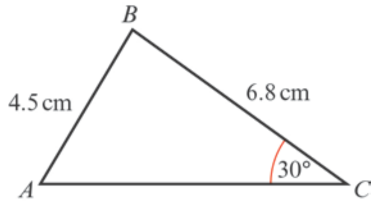
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{AC}{\sin 25.8°} = \frac{4}{\sin 36°}$$

$$\text{So } AC = \frac{4\sin 25.8°}{\sin 36°}$$

$$= 2.96 \text{ cm } (3 \text{ s.f.})$$

5



As 6.8 > 4.5, angle A > 30° and so there are two possible values for A.

$$\text{Using } \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{6.8} = \frac{\sin 30°}{4.5}$$

$$A = \sin^{-1}\left(\frac{6.8\sin 30°}{4.5}\right) \text{ or}$$

$$A = 180° - \sin^{-1}\left(\frac{6.8\sin 30°}{4.5}\right)$$

$$A = 49.07\ldots° \text{ or } 130.926\ldots°$$

When A = 49.07…°, B is the largest angle.

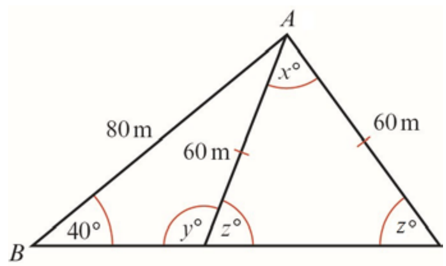
$$\angle ABC = 180° - (30° + 49.07\ldots°)$$

$$= 101° \ (3 \text{ s.f.})$$

When A = 130.926…°, this the largest angle.

$$\angle BAC = 131° (3 \text{ s.f.})$$

6 a



Using the sine rule:

$$\frac{\sin y}{80} = \frac{\sin 40°}{60}$$

$$\sin y = \frac{80\sin 40°}{60}$$

y = 59° or 121°
y is obtuse, therefore, y = 121°
z = 59°
x = 180° − 2 × 59° = 62°
x = 62°

b The assumption is that the ball swings symmetrically.

