

Quadratics 2E

$$\begin{aligned} 1 \text{ a } f(1) &= 5(1) + 3 \\ &= 5 + 3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{b } g(3) &= 3^2 - 2 \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{c } h(8) &= \sqrt{8+1} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{d } f(1.5) &= 5(1.5) + 3 \\ &= 7.5 + 3 \\ &= 10.5 \end{aligned}$$

$$\begin{aligned} \text{e } g(\sqrt{2}) &= (\sqrt{2})^2 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{f } h(-1) &= \sqrt{-1+1} \\ &= \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{g } f(4) + g(2) &= 5(4) + 3 + 2^2 - 2 \\ &= 20 + 3 + 4 - 2 \\ &= 25 \end{aligned}$$

$$\begin{aligned} \text{h } f(0) + g(0) + h(0) &= 5(0) + 3 + 0^2 - 2 \\ &\quad + \sqrt{0+1} \\ &= 0 + 3 + 0 - 2 + 1 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{i } \frac{g(4)}{h(3)} &= \frac{4^2 - 2}{\sqrt{3+1}} \\ &= \frac{16-2}{\sqrt{4}} \\ &= \frac{14}{2} \\ &= 7 \end{aligned}$$

$$\begin{aligned} 2 \quad f(a) &= a^2 - 2a = 8 \\ a^2 - 2a - 8 &= 0 \\ (a - 4a + 2) &= 0 \\ a = 4 \text{ and } a &= -2 \end{aligned}$$

$$\begin{aligned} 3 \text{ a } \quad f(x) &= 0 \\ 10 - 15x &= 0 \\ 5(2 - 3x) &= 0 \\ x &= \frac{2}{3} \\ \text{The root of } f(x) &\text{ is } \frac{2}{3}. \end{aligned}$$

$$\begin{aligned} \text{b } \quad g(x) &= 0 \\ (x + 9)(x - 2) &= 0 \\ (x + 9x - 2) &= 0 \\ x = -9 \text{ or } x &= 2 \\ \text{The roots of } g(x) &\text{ are } -9 \text{ and } 2. \end{aligned}$$

$$\begin{aligned} \text{c } \quad h(x) &= 0 \\ x^2 + 6x - 40 &= 0 \\ (x + 10)(x - 4) &= 0 \\ x = -10 \text{ or } x &= 4 \\ \text{The roots of } h(x) &\text{ are } -10 \text{ and } 4. \end{aligned}$$

$$\begin{aligned} \text{d } \quad j(x) &= 0 \\ 144 - x^2 &= 0 \\ (12 + 12x - x) &= 0 \\ x = -12 \text{ or } 12 \\ \text{The roots of } j(x) &\text{ are } 12 \text{ and } -12. \end{aligned}$$

$$\begin{aligned} \text{e } \quad k(x) &= 0 \\ x(x + 5)(x + 7) &= 0 \\ x(x + 5x + 7) &= 0 \\ x = 0, x = -5 \text{ or } x &= -7 \\ \text{The roots of } k(x) &\text{ are } 0, -5 \text{ and } -7. \end{aligned}$$

$$\begin{aligned} \text{f } \quad m(x) &= 0 \\ x^3 + 5x^2 - 24x &= 0 \\ x(x^2 + 5x - 24) &= 0 \\ x(x + 8x - 3) &= 0 \\ x = 0, x = -8 \text{ or } x &= 3 \\ \text{The roots of } m(x) &\text{ are } 0, -8 \text{ and } 3. \end{aligned}$$

$$\begin{aligned} 4 \quad p(x) &= q(x) \\ x^2 - 3x &= 2x - 6 \\ x^2 - 5x + 6 &= 0 \\ (x - 3)(x - 2) &= 0 \\ x = 3 \text{ and } x &= 2 \end{aligned}$$

$$\begin{aligned} 5 \quad f(x) &= g(x) \\ 2x^3 + 30x &= 17x^2 \\ 2x^3 - 17x^2 + 30x &= 0 \\ x(2x^2 - 17x + 30) &= 0 \\ x(2x - 5x - 6) &= 0 \\ x = 0, x = \frac{5}{2} \text{ and } x &= 6 \end{aligned}$$

6 a $f(x) = x^2 - 2x + 2$
 $= (x - 1)^2 - 1^2 + 2$
 $= (x - 1)^2 + 1$
 $p = -1$ and $q = 1$

b $(x - 1)^2$ is a squared term so is always ≥ 0 .
 Therefore, the minimum value of
 $f(x) = 0 + 1 = 1$, so $f(x) > 0$.

7 a $f(x) = 0$
 $x^6 + 9x^3 + 8 = 0$
 $(x^3)^2 + 9(x^3) + 8 = 0$
 $(x^3 + 1)(x^3 + 8) = 0$
 So $x^3 = -1$ or $x^3 = -8$
 $x^3 = -1 \Rightarrow x = -1$
 $x^3 = -8 \Rightarrow x = -2$
 The roots of $f(x)$ are -1 and -2 .

b $g(x) = 0$
 $x^4 - 12x^2 + 32 = 0$
 $(x^2)^2 - 12(x^2) + 32 = 0$
 $(x^2 - 4)(x^2 - 8) = 0$
 So $x^2 = 4$ or $x^2 = 8$
 $x^2 = 4 \Rightarrow x = \pm 2$
 $x^2 = 8 \Rightarrow x = \pm\sqrt{8} = \pm\sqrt{4 \times 2} = \pm 2\sqrt{2}$
 The roots of $g(x)$ are $-2, 2, -2\sqrt{2}$ and $2\sqrt{2}$

c $h(x) = 0$
 $27x^6 + 26x^3 - 1 = 0$
 $27(x^3)^2 + 26(x^3) - 1 = 0$
 $(27x^3 - 1)(x^3 + 1) = 0$
 $x^3 = \frac{1}{27} \Rightarrow x = \frac{1}{3}$
 $x^3 = -1 \Rightarrow x = -1$
 The roots of $h(x)$ are -1 and $\frac{1}{3}$.

d $j(x) = 0$
 $32x^{10} - 33x^5 + 1 = 0$
 $32(x^5)^2 - 33(x^5) + 1 = 0$
 $(32x^5 - 1)(x^5 - 1) = 0$
 So $x^5 = \frac{1}{32}$ or $x^5 = 1$
 $x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$
 $x^5 = 1 \Rightarrow x = 1$
 The roots of $j(x)$ are $\frac{1}{2}$ and 1 .

e $k(x) = 0$
 $x - 7\sqrt{x} + 10 = 0$
 $\left(x^{\frac{1}{2}}\right)^2 - 7\left(x^{\frac{1}{2}}\right) + 10 = 0$

7 e $\left(x^{\frac{1}{2}} - 2\right)\left(x^{\frac{1}{2}} - 5\right) = 0$

So $x^{\frac{1}{2}} = 2$ or $x^{\frac{1}{2}} = 5$

$x^{\frac{1}{2}} = 2 \Rightarrow x = 4$

The roots of $k(x)$ are 4 and 25 .

f $m(x) = 0$
 $2x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 12 = 0$

$\left(x^{\frac{1}{3}}\right)^2 + \left(x^{\frac{1}{3}}\right) - 6 = 0$

$\left(x^{\frac{1}{3}} - 2\right)\left(x^{\frac{1}{3}} + 3\right) = 0$

So $x^{\frac{1}{3}} = 2$ or $x^{\frac{1}{3}} = -3$

$x^{\frac{1}{3}} = 2 \Rightarrow x = 8$

$x^{\frac{1}{3}} = -3 \Rightarrow x = -27$

The roots of $m(x)$ are 8 and -27 .

8 a $3^{2x} - 28(3^x) + 27 = (3^x)^2 - 28(3^x) + 27$
 $= (3^x - 27)(3^x - 1)$

b $f(x) = 0$
 $(3^x - 27)(3^x - 1) = 0$
 $3^x = 27 \Rightarrow x = 3$
 $3^x = 1 \Rightarrow x = 0$
 The roots of $f(x)$ are 0 and 3 .