

Integration 13D

$$\begin{aligned}
 \mathbf{1 \ a} \quad \int_2^5 x^3 dx &= \left[\frac{x^4}{4} \right]_2^5 \\
 &= \left(\frac{5^4}{4} \right) - \left(\frac{2^4}{4} \right) \\
 &= \frac{609}{4} \\
 &= 152\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \int_1^3 x^4 dx &= \left[\frac{x^5}{5} \right]_1^3 \\
 &= \left(\frac{3^5}{5} \right) - \left(\frac{1^5}{5} \right) \\
 &= \frac{242}{5} \\
 &= 48\frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \int_0^4 \sqrt{x} dx &= \int_0^4 x^{\frac{1}{2}} dx \\
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= \left[\frac{2x^{\frac{3}{2}}}{3} \right]_0^4 \\
 &= \left(\frac{2(4)^{\frac{3}{2}}}{3} \right) - \left(\frac{2(0)^{\frac{3}{2}}}{3} \right) \\
 &= \frac{16}{3} \\
 &= 5\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad \int_1^3 \frac{3}{x^2} dx &= \int_1^3 3x^{-2} dx \\
 &= \left[\frac{3x^{-1}}{-1} \right]_1^3 \\
 &= \left[-\frac{3}{x} \right]_1^3 \\
 &= \left(-\frac{3}{3} \right) - \left(-\frac{3}{1} \right) \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2 \ a} \quad \int_1^2 \left(\frac{2}{x^3} + 3x \right) dx &= \int_1^2 (2x^{-3} + 3x) dx \\
 &= \left(\frac{2x^{-2}}{-2} + \frac{3x^2}{2} \right) \Big|_1^2 \\
 &= \left(-x^{-2} + \frac{3}{2}x^2 \right) \Big|_1^2 \\
 &= \left(-\frac{1}{4} + \frac{3}{2} \times 4 \right) - \left(-1 + \frac{3}{2} \right) \\
 &= \left(-\frac{1}{4} + 6 \right) - \frac{1}{2} \\
 &= 5\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \int_0^2 (2x^3 - 4x + 5) dx &= \left(\frac{2x^4}{4} - \frac{4x^2}{2} + 5x \right) \Big|_0^2 \\
 &= \left(\frac{x^4}{2} - 2x^2 + 5x \right) \Big|_0^2 \\
 &= \left(\frac{16}{2} - 2 \times 4 + 10 \right) - (0) \\
 &= 8 - 8 + 10 \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \int_4^9 \left(\sqrt{x} - \frac{6}{x^2} \right) dx &= \int_4^9 \left(x^{\frac{1}{2}} - 6x^{-2} \right) dx \\
 &= \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^{-1}}{-1} \right) \Big|_4^9 \\
 &= \left(\frac{2}{3}x^{\frac{3}{2}} + 6x^{-1} \right) \Big|_4^9 \\
 &= \left(\frac{2}{3} \times 3^3 + \frac{2}{3} \right) - \left(\frac{2}{3} \times 2^3 + \frac{3}{2} \right) \\
 &= 18 + \frac{2}{3} - \frac{16}{3} - \frac{3}{2} \\
 &= 16\frac{1}{2} - \frac{14}{3} \\
 &= 11\frac{5}{6}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \mathbf{d} \quad \int_1^8 \left(x^{\frac{1}{3}} + 2x - 1 \right) dx &= \left(\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{2x^2}{2} - x \right)_1^8 \\
 &= \left(\frac{3}{2}x^{\frac{2}{3}} + x^2 - x \right)_1^8 \\
 &= \left(\frac{3}{2} \times 2^2 + 64 - 8 \right) - \left(\frac{3}{2} + 1 - 1 \right) \\
 &= 62 - \frac{3}{2} \\
 &= 60\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \mathbf{a} \quad \int_1^3 \left(\frac{x^3 + 2x^2}{x} \right) dx \\
 &= \int_1^3 (x^2 + 2x) dx \\
 &= \left(\frac{x^3}{3} + x^2 \right)_1^3 \\
 &= \left(\frac{27}{3} + 9 \right) - \left(\frac{1}{3} + 1 \right) \\
 &= 18 - \frac{4}{3} \\
 &= 16\frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad \int_3^6 \left(x - \frac{3}{x} \right)^2 dx &= \int_3^6 \left(x^2 - 6 + \frac{9}{x^2} \right) dx \\
 &= \int_3^6 (x^2 - 6 + 9x^{-2}) dx \\
 &= \left(\frac{x^3}{3} - 6x + \frac{9x^{-1}}{-1} \right)_3^6 \\
 &= \left(\frac{x^3}{3} - 6x - 9x^{-1} \right)_3^6 \\
 &= \left(\frac{216}{3} - 36 - \frac{9}{6} \right) - \left(\frac{27}{3} - 18 - \frac{9}{3} \right) \\
 &= 72 - 36 - \frac{3}{2} - 9 + 18 + 3 \\
 &= 48 - \frac{3}{2} \\
 &= 46\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \int_0^1 x^2 \left(\sqrt{x} + \frac{1}{x} \right) dx &= \int_0^1 \left(x^{\frac{5}{2}} + x \right) dx \\
 &= \left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{x^2}{2} \right)_0^1 \\
 &= \left(\frac{2}{7} + \frac{1}{2} \right) - (0) \\
 &= \frac{11}{14}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \mathbf{d} \quad \int_1^4 \left(\frac{2 + \sqrt{x}}{x^2} \right) dx &= \int_1^4 \left(\frac{2}{x^2} + \frac{1}{x^{\frac{3}{2}}} \right) dx \\
 &= \int_1^4 \left(2x^{-2} + x^{-\frac{3}{2}} \right) dx \\
 &= \left(\frac{2x^{-1}}{-1} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} \right)_1^4 \\
 &= \left(-2x^{-1} - 2x^{-\frac{1}{2}} \right)_1^4 \\
 &= \left(-\frac{2}{4} - \frac{2}{2} \right) - \left(-2 - 2 \right) \\
 &= -1\frac{1}{2} + 4 \\
 &= 2\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \int_1^4 (6\sqrt{x} - A) dx &= A^2 \\
 \int_1^4 (6x^{\frac{1}{2}} - A) dx &= A^2 \\
 \left[\frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - Ax \right]_1^4 &= A^2 \\
 \left[4x^{\frac{3}{2}} - Ax \right]_1^4 &= A^2 \\
 \left(4(4)^{\frac{3}{2}} - A(4) \right) - \left(4(1)^{\frac{3}{2}} - A(1) \right) &= A^2 \\
 (32 - 4A) - (4 - A) &= A^2 \\
 28 - 3A &= A^2 \\
 A^2 + 3A - 28 &= 0 \\
 (A + 7)(A - 4) &= 0 \\
 A &= -7 \text{ or } A = 4
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \int_1^9 (2x - 3\sqrt{x}) dx &= \int_1^9 (2x - 3x^{\frac{1}{2}}) dx \\
 &= \left[\frac{2x^2}{2} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^9 \\
 &= \left[x^2 - 2x^{\frac{3}{2}} \right]_1^9
 \end{aligned}$$

$$\begin{aligned}
 5 \int_1^9 (2x - 3\sqrt{x}) \, dx &= \left(9^2 - 2(9)^{\frac{3}{2}} \right) - \left(1^2 - 2(1)^{\frac{3}{2}} \right) \\
 &= (81 - 54) - (1 - 2) \\
 &= 28
 \end{aligned}$$

$$\begin{aligned}
 6 \int_4^{12} \left(\frac{2}{\sqrt{x}} \right) dx &= \int_4^{12} (2x^{-\frac{1}{2}}) dx \\
 &= \left[\frac{2x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^{12} \\
 &= \left[4x^{\frac{1}{2}} \right]_4^{12} \\
 &= \left(4(12)^{\frac{1}{2}} \right) - \left(4(4)^{\frac{1}{2}} \right) \\
 &= 4\sqrt{12} - 8 \\
 &= 4\sqrt{4 \times 3} - 8 \\
 &= -8 + 8\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 7 \int_1^k \frac{1}{\sqrt{x}} dx &= 3 \\
 \int_1^k x^{-\frac{1}{2}} dx &= 3 \\
 \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^k &= 3 \\
 [2\sqrt{x}]_1^k &= 3 \\
 2\sqrt{k} - 2\sqrt{1} &= 3 \\
 2\sqrt{k} &= 5 \\
 \sqrt{k} &= \frac{5}{2} \\
 k &= \frac{25}{4}
 \end{aligned}$$

$$\begin{aligned}
 8 \, s &= \int_0^{10} (20 + 5t) dt \\
 &= \left[20t + \frac{5t^2}{2} \right]_0^{10} \\
 &= \left(20(10) + \frac{5(10)^2}{2} \right) - \left(20(0) + \frac{5(0)^2}{2} \right) \\
 &= 450 \text{ m}
 \end{aligned}$$

Challenge

$$\begin{aligned}
 \int_k^{3k} \frac{3x+2}{8} dx &= 7 \\
 \int_k^{3k} \left(\frac{3x}{8} + \frac{1}{4} \right) dx &= 7 \\
 \left[\frac{1}{2} \frac{3x^2}{8} + \frac{x}{4} \right]_k^{3k} &= 7 \\
 \left[\frac{3x^2}{16} + \frac{x}{4} \right]_k^{3k} &= 7 \\
 \left(\frac{3(3k)^2}{16} + \frac{3k}{4} \right) - \left(\frac{3k^2}{16} + \frac{k}{4} \right) &= 7 \\
 \left(\frac{27k^2}{16} + \frac{3k}{4} \right) - \left(\frac{3k^2}{16} + \frac{k}{4} \right) &= 7 \\
 \frac{24k^2}{16} + \frac{k}{2} &= 7 \\
 24k^2 + 8k - 112 &= 0 \\
 3k^2 + k - 14 &= 0 \\
 (3k+7)(k-2) &= 0 \\
 k = -\frac{7}{3} \text{ or } k = 2 \\
 \text{As } k > 0, k &= 2
 \end{aligned}$$