Constant acceleration 9D

1 $a = 2.5,\ u = 3,\ s = 8,\ v = ?$

$v^2 = u^2 + 2as = 3^2 + 2\times 2.5\times 8 = 9 + 40 = 49$
$v = \sqrt{49} = 7$

The velocity of the particle as it passes through B is $7\,\text{m s}^{-1}$.

2 $u = 8,\ t = 6,\ s = 60,\ a = ?$

$s = ut + \frac{1}{2}at^2$
$60 = 8\times 6 + \frac{1}{2}\times a\times 6^2 = 48 + 18a$
$a = \dfrac{60-48}{18} = \dfrac{2}{3}$

The acceleration of the car is $0.667\,\text{m s}^{-2}$ (to 3 s.f.)

3 $u = 12,\ v = 0,\ s = 36,\ a = ?$

$v^2 = u^2 + 2as$
$0^2 = 12^2 + 2\times a\times 36 = 144 + 72a$
$a = -\frac{144}{72} = -2$

The deceleration is $2\ \text{m s}^{-2}$.

4 $u = 15,\ v = 20,\ s = 500,\ a = ?$ $\quad 54\,\text{km h}^{-1} = \dfrac{54\times 1000}{3600}\text{m s}^{-1} = 15\,\text{m s}^{-1}$

$72\,\text{km h}^{-1} = \dfrac{72\times 1000}{3600}\text{m s}^{-1} = 20\,\text{m s}^{-1}$

$v^2 = u^2 + 2as$
$20^2 = 15^2 + 2\times a\times 500$
$400 = 225 + 1000a$
$a = \dfrac{400-225}{1000} = 0.175$

The acceleration of the train is $0.175\,\text{m s}^{-2}$.

5 a $s = 48,\ u = 4,\ v = 16,\ a = ?$
$v^2 = u^2 + 2as$
$16^2 = 4^2 + 2\times a\times 48$

5 a $256 = 16 + 96a$

$$a = \frac{256 - 16}{96} = 2.5$$

The acceleration of the particle is $2.5\,\text{m}\,\text{s}^{-2}$.

b $u = 4,\ v = 16,\ a = 2.5,\ t = ?$

$$v = u + at$$
$$16 = 4 + 2.5t$$
$$t = \frac{16 - 4}{2.5} = 4.8$$

The time taken to move from A to B is 4.8 s.

6 a $a = 3$, $s = 38$, $t = 4$, $u = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$38 = 4u + \tfrac{1}{2} \times 3 \times 4^2 = 4u + 24$$
$$u = \frac{38 - 24}{4} = 3.5$$

The initial velocity of the particle is $3.5\,\text{m}\,\text{s}^{-1}$.

b $a = 3$, $t = 4$, $u = 3.5$, $v = ?$

$$v = u + at = 3.5 + 3 \times 4 = 15.5$$

The final velocity of the particle is $15.5\,\text{m}\,\text{s}^{-1}$.

7 a $u = 18,\ v = 0,\ a = -3,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 18^2 + 2 \times (-3) \times s = 324 - 6s$$
$$s = \tfrac{324}{6} = 54$$

The distance travelled as the car decelerates is 54 m.

b $u = 18,\ v = 0,\ a = -3,\ t = ?$

$$v = u + at$$
$$0 = 18 - 3t$$
$$t = \tfrac{18}{3} = 6$$

The time taken for the car to decelerate is 6 s.

8 a $u = 12,\ v = 0,\ a = -0.8,\ s = ?$

$$v^2 = u^2 + 2as$$
$$0^2 = 12^2 + 2\times(-0.8)\times s = 144 - 1.6s$$
$$s = \tfrac{144}{1.6} = 90$$

The distance moved by the stone is 90 m.

b Half the distance in **a** is 45 m.

$u = 12,\ a = -0.8,\ s = 45,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2\times(-0.8)\times 45 = 144 - 72 = 72$$
$$v = \sqrt{72} = 8.49 \text{ (to 3 s.f.)}$$

The speed of the stone is $8.49\,\text{m s}^{-1}$.

9 a $a = 2.5$, $u = 8$, $s = 40$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$40 = 8t + 1.25t^2$$
$$0 = 1.25t^2 + 8t - 40$$
$$t = \frac{-(8) \pm \sqrt{(8)^2 - 4\times(1.25)\times(-40)}}{2\times(1.25)}$$
$$t = \frac{-8 + \sqrt{264}}{2.5} = 3.30 \text{ (to 3 s.f.)}$$

The time taken for the particle to move from O to A is 3.30 s.

b $a = 2.5,\ u = 8,\ s = 40,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 8^2 + 2\times 2.5\times 40 = 264$$
$$v = \sqrt{264} = 16.2 \text{ (to 3 s.f.)}$$

The speed of the particle at A is $16.2\,\text{m s}^{-1}$.

10 a $a = -2,\ s = 32,\ u = 12,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$32 = 12t - t^2$$
$$t^2 - 12t + 32 = (t-4)(t-8) = 0$$

So $t = 4$ or $t = 8$.

10 b When $t = 4$,

$v = u + at = 12 - 2 \times 4 = 4$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{AB}$.

When $t = 8$,

$v = u + at = 12 - 2 \times 8 = -4$

The velocity is $4\,\text{m s}^{-1}$ in the direction $\overrightarrow{BA}$.

11 a $a = -5,\ u = 12,\ s = 8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$8 = 12t - 2.5t^2$$
$$2.5t^2 - 12t + 8 = 0$$
$$5t^2 - 24t + 16 = (5t - 4)(t - 4) = 0$$

So $t = 0.8$ or $t = 4$.

b $a = -5,\ u = 12,\ s = -8,\ v = ?$

$$\begin{aligned} v^2 &= u^2 + 2as \\ &= 12^2 + 2 \times (-5) \times (-8) \\ &= 144 + 80 = 224 \\ v &= \sqrt{224} = 15.0 \text{ (to 3 s.f.)} \end{aligned}$$

The velocity at $x = -8$ is $15.0\,\text{m s}^{-1}$.

12 a $a = -4,\ u = 14,\ s = 22.5,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$22.5 = 14t - 2t^2$$
$$2t^2 - 14t + 22.5 = 0$$
$$4t^2 - 28t + 45 = (2t - 5)(2t - 9) = 0$$

The difference between the times is $(4.5 - 2.5)\text{ s} = 2\text{ s}$.

b The maximum distance is reached when P reverses direction.
$a = -4,\ u = 14,\ v = 0,\ t = ?$

$$v = u + at$$
$$0 = 14 - 4t \Rightarrow t = \tfrac{14}{4} = 3.5$$

12 b Find the displacement when $t = 3.5$.

$$s = ut + \tfrac{1}{2}at^2$$
$$= 14 \times 3.5 - 2 \times 3.5^2 = 24.5$$

Between $t = 2.5$ and $t = 4.5$ the particle moves back and forward.

Hence total distance travelled $= 2 \times (24.5 - 22.5)$ m $= 4$ m.

13 a From B to C, $u = 14$, $v = 20$, $s = 300$, $a = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = 14^2 + 2 \times a \times 300$$
$$a = \frac{20^2 - 14^2}{600} = 0.34$$

The acceleration of the car is $0.34\ \text{m s}^{-2}$.

b From A to C, $v = 20$, $s = 400$, $a = 0.34$, $u = ?$

$$v^2 = u^2 + 2as$$
$$20^2 = u^2 + 2 \times 0.34 \times 400 = u^2 + 272$$
$$u^2 = 400 - 272 = 128$$
$$u = \pm\sqrt{128} = \pm 8\sqrt{2}$$

Assuming the car is not in reverse at A, $u = +8\sqrt{2}$

$$v = u + at$$
$$20 = 8\sqrt{2} + 0.34t$$
$$t = \frac{20 - 8\sqrt{2}}{0.34} = 25.5 \text{ (to 3 s.f.)}$$

The time taken for the car to travel from A to C is 25.5 s.

14 a For P, $a = 2$, $u = 4$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 4t + \tfrac{1}{2} \times 2t^2 = 4t + t^2$$

The displacement of P is $(4t + t^2)$ m.

For Q, $a = 3.6$, $u = 3$

14 a Q has been moving for $(t-1)$ seconds since passing through A, so

$$s = u(t-1) + \tfrac{1}{2}a(t-1)^2$$
$$= 3(t-1) + 1.8(t-1)^2 = 1.8t^2 - 0.6t - 1.2$$

The displacement of Q is $(1.8t^2 - 0.6t - 1.2)$ m.

b P and Q meet when $s_P = s_Q$, so, from **a**:

$$4t + t^2 = 1.8t^2 - 0.6t - 1.2$$
$$0.8t^2 - 4.6t - 1.2 = 0$$

Divide throughout by 0.2:

$$4t^2 - 23t - 6 = 0$$
$$(t-6)(4t+1) = 0$$

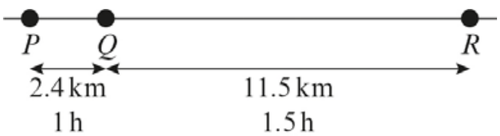
Rejecting a negative solution for time, $t = 6$.

c Substitute $t = 6$ into the equation for one of the displacements (here P):

$$s = 4t + t^2 = 4\times 6 + 6^2 = 60$$

The distance of A from the point where the particles meet is 60 m.

15



a Let the velocity as the competitor passes point Q be v_Q

For PQ, $s = 2.4$, $t = 1$, $v = v_Q$

$$s = vt - \tfrac{1}{2}at^2$$
$$2.4 = v_Q \times 1 - \tfrac{1}{2}(a \times 1^2) = v_Q - \tfrac{1}{2}a$$
$$v_Q = 2.4 + 0.5a$$

For QR, $s = 11.5$, $t = 1.5$, $u = v_Q$

$$s = ut + \tfrac{1}{2}at^2$$
$$11.5 = v_Q \times 1.5 + \tfrac{1}{2}a \times 1.5^2 = 1.5v_Q + 1.125a$$

15 a Substituting for v_Q:

$$\begin{aligned}11.5 &= 1.5(2.4+0.5a)+1.125a\\ &= 3.6+0.75a+1.125a\\ 11.5-3.6 &= (0.75 + 1.125)a\\ a &= \frac{11.5-3.6}{0.75+1.125} = \frac{7.9}{1.875} = 4.21 \text{ (to 3 s.f.)}\end{aligned}$$

The acceleration is 4.21 km h^{-2}.

$$4.21 \text{ km h}^{-2} = \frac{4.21\times1000}{3600\times3600} \text{ m s}^{-2} = 3.25\times10^{-4} \text{ m s}^{-2} \text{ (to 3 s.f.)}$$

So her acceleration is 3.25 x 10^{-4} m s^{-2}.

b For *PQ*, s = 2.4, t = 1, a = 4.21, u = ?, using exact figures

$$\begin{aligned}s &= ut+\tfrac{1}{2}at^2\\ 2.4 &= u\times1+\tfrac{1}{2}\times\frac{7.9}{1.875}\times1^2\\ u &= 0.293 \text{ (to 3 s.f.)}\end{aligned}$$

$$0.293 \text{ km h}^{-1} = \frac{0.293\times1000}{3600} \text{ m s}^{-1} = 0.0815 \text{ m s}^{-1} \text{ (to 3 s.f.)}$$