Modelling in mechanics, Mixed Exercise 8

1 **a** $x = 2$ gives

$$h = \frac{1}{10}\left(24 \times 2 - 3 \times (2)^2\right)$$
$$= \frac{1}{10}\left(48 - 12\right) = 3.6$$

When it is 2 m horizontally from where it is hit, the ball is at a vertical height of 3.6 m.

b When $h = 2.1$,

$2.1 = \frac{1}{10}\left(24x - 3x^2\right)$ or rearranging, $0 = 24x - 21 - 3x^2$, and dividing by -3,

$0 = x^2 - 8x + 7 = (x - 1)(x - 7)$

So $x = 1$ or 7.

The ball is at a height of 2.1m when it is at a horizontal distance of 1 m and again at 7 m.

c Model becomes valid when the ball is hit, i.e. $x = 0$.

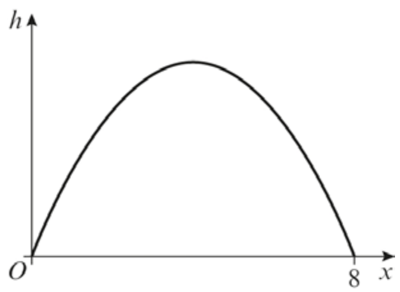
The model remains valid while the ball is above the ground, i.e. $h \geq 0$

$\frac{1}{10}\left(24x - 3x^2\right) \geq 0$ or factorising, $0.3x(8 - x) \geq 0$

We know that $x \geq 0$ for our region, so we need the bracket to be non-negative, i.e. $x \leq 8$.

The model is valid for $0 \leq x \leq 8$.

d The equation for this model produces a curve of the form shown:



Since the curve is symmetrical, maximum height occurs when $x = 4$, at which point:

$$h = \frac{1}{10}\left(24 \times 4 - 3 \times (4)^2\right)$$
$$= \frac{1}{10}\left(96 - 48\right) = 4.8$$

The maximum height of the ball is 4.8 m.

2 a $x = 2$ gives

$h = 10 - 0.58 \times (2)^2 = 7.68$

When the horizontal distance from the end of the board is 2 m, the diver is at a height of 7.68 m.

b When the diver hits the water, $h = 0$

$0 = 10 - 0.58x^2$

$0.58x^2 = 10$

$$x = \sqrt{\frac{10}{0.58}} \approx \sqrt{17.24}$$

The diver enters the water 4.15 m from the end of the board.

c By modelling the diver as a particle, we can ignore air resistance and the rotational effects of external forces. The mass of the diver is assumed to be concentrated at a single point.

d The forces acting on the body of the diver change when she enters the water. In particular, she will experience buoyancy which acts against her weight, and drag or water resistance which acts against her movement. Therefore the model is unlikely to be valid.

3 a Model the man on skis as a particle. This allows one to ignore the rotational effect of any forces that are acting on the man as well as any effects due to air resistance.

Consider the slope to be smooth: that there is no friction between the skis and the slope.

b Model the yo-yo as a particle. This allows one to ignore the rotation of the yo-yo and air resistance. We assume that the mass of the yo-yo is concentrated at a point.

Consider the string to be light and inextensible. This allows one to ignore the weight of the string and assume it does not stretch, thereby affecting the acceleration of attached objects.

Model the yo-yo as smooth, that is, assume there is no friction between the yo-yo and the string.

4 a 2.5 km per minute = $\frac{2.5\times1000}{60}$ m s^{-1} $= 41.7$ m s^{-1} (to 3 s.f.)

b 0.6 kg cm^{-2} = $\frac{0.6}{1\div(100\times100)}$ kg m^{-2} $= 6000$ kg m^{-2}

c 1.2×10^{-3} kg cm^{-3} = $\frac{1.2\times10^{-3}}{1\div(100\times100\times100)}$ kg m^{-3} $= 1200$ kg m^{-3}

5 a Model the ball as a particle.

Assume the floor is smooth.

5 b i The velocity will be positive as the positive direction is defined as such.

ii In real life the acceleration would be negative, as the ball always slows down. However, if we assume there is no friction, then the ball would move at constant velocity.

6 a Velocity is positive, displacement is positive

b Velocity is negative, displacement is positive

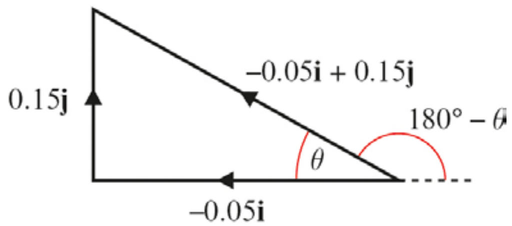
c Velocity is negative, displacement is negative

7 a $|\mathbf{a}| = \sqrt{0.05^2 + 0.15^2} = \sqrt{0.025} = 0.158$ (to 3 s.f.)

The magnitude of the acceleration is 0.158 m s^{-2}.

b Let the acute angle made with **i** be θ, then

$$\tan\theta = \frac{0.15}{0.05} = 3 \text{ so } \theta = 71.6° \text{ (to 3 s.f.)}$$



Angle required = 180° – θ = 180° – 71.6° = 108° (to 3 s.f.)

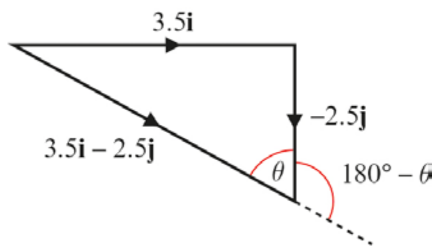
The direction of the acceleration is 108° from the **i** vector.

8 a $|\mathbf{v}| = \sqrt{3.5^2 + 2.5^2} = \sqrt{18.5} = 4.30$ (to 3 s.f.)

The speed of the toy car is 4.30 m s^{-1}.

b Let the acute angle made with **j** be θ, then

$$\tan\theta = \frac{3.5}{2.5} = 1.4 \text{ so } \theta = 54.5° \text{ (to 3 s.f.)}$$



Angle required = 180° – θ = 180° – 54.5° = 126° (to 3 s.f.)

The direction of the acceleration is 126° from the **j** vector.

9 a $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$$\overrightarrow{PR} = \begin{pmatrix} 100 \\ 80 \end{pmatrix} + \begin{pmatrix} 50 \\ -30 \end{pmatrix} = \begin{pmatrix} 150 \\ 50 \end{pmatrix}$$

$$\left|\overrightarrow{PR}\right| = \sqrt{150^2 + 50^2} = \sqrt{25000} = 158 \text{ (to 3 s.f.)}$$

The magnitude of the displacement is 158 m.

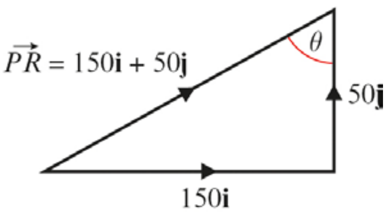
b $\left|\overrightarrow{PQ}\right| = \sqrt{100^2 + 80^2} = \sqrt{16400} = 128 \text{ (to 3 s.f.)}$

$$\left|\overrightarrow{QR}\right| = \sqrt{50^2 + 30^2} = \sqrt{3400} = 58.3 \text{ (to 3 s.f.)}$$

$$\left|\overrightarrow{PQ}\right| + \left|\overrightarrow{QR}\right| = \sqrt{16400} + \sqrt{3400} = 186 \text{ (to 3 s.f.)}$$

The plane travels a total distance of 186 m.

c Let the acute angle made with **j** be θ



$$\tan\theta = \frac{150}{50} = 3 \text{ so } \theta = 71.6° \text{ (to 3 s.f.)}$$

The direction of motion of the car is 71.6° from the **j** vector.