

Variable acceleration 11C

1 a $s = 0.4t^3 - 0.3t^2 - 1.8t + 5$

$$v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$$

$$\frac{dv}{dt} = 2.4t - 0.6$$

$$\frac{dv}{dt} = 0 \text{ when } 2.4t = 0.6$$

$$t = 0.25$$

P is moving with minimum velocity at $t = 0.25$ s.

b When $t = 0.25$

$$\begin{aligned} s &= 0.4(0.25)^3 - 0.3(0.25)^2 - 1.8(0.25) + 5 \\ &= 4.54 \text{ (3 s.f.)} \end{aligned}$$

When P is moving with minimum velocity, the displacement is 4.54 m.

c $v = \frac{ds}{dt} = 1.2t^2 - 0.6t - 1.8$

$$\begin{aligned} \text{When } t = 0.25, v &= 1.2 \times 0.25^2 - 0.6 \times 0.25 - 1.8 \\ &= -1.88 \text{ (3 s.f.)} \end{aligned}$$

2 a $s = 4t^3 - t^4$

$$\begin{aligned} \text{When } t = 4, \\ s &= 4(4)^3 - 4^4 = 0 \end{aligned}$$

The body returns to its starting position 4 s after leaving it.

b $s = 4t^3 - t^4 = t^3(4 - t)$

Since $t \geq 0$, t^3 is always positive.

Since $t \leq 4$, $(4 - t)$ is always positive.

So for $0 \leq t \leq 4$, s is always non-negative.

c $\frac{ds}{dt} = 12t^2 - 4t^3$

$$\frac{ds}{dt} = 0 \text{ when}$$

$$12t^2 - 4t^3 = 0$$

$$4t^2(3 - t) = 0$$

$$t = 0 \text{ or } 3$$

At $t = 0$, the body is at $s = 0$, so maximum displacement occurs when $t = 3$.

When $t = 3$, using factorised form of the equation of motion:

$$s = 3^3(4 - 3) = 27$$

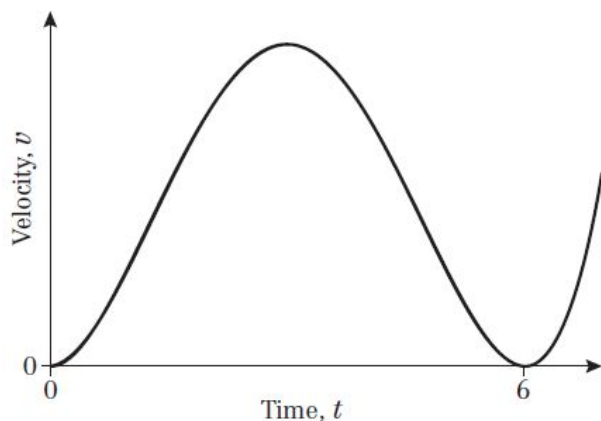
The maximum displacement of the body from its starting point is 27 m.

$$3 \text{ a } v = t^2(6 - t)^2$$

Velocity is zero when $t = 0$ and $t = 6$.

The graph touches the time axis at $t = 0$ and $t = 6$.

Graph only shown for $t \geq 0$, as this is the range over which equation is valid.



$$\begin{aligned} 3 \text{ b } v &= t^2(6 - t)^2 \\ &= t^2(36 - 12t + t^2) \\ &= 36t^2 - 12t^3 + t^4 \end{aligned}$$

$$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$$

$$\frac{dv}{dt} = 0 \text{ when}$$

$$72t - 36t^2 + 4t^3 = 0$$

$$4t(18 - 9t + t^2) = 0$$

$$4t(3 - t)(6 - t) = 0$$

The turning points are at $t = 0$, $t = 3$ and $t = 6$.

$v = 0$ when $t = 0$ and $t = 6$, therefore the maximum velocity occurs when $t = 3$.

When $t = 3$,

$$v = 3^2(6 - 3)^2 = 9 \times 9 = 81$$

The maximum velocity is 81 m s^{-1} and the body reaches this 3 s after leaving O .

$$4 \text{ a } v = 2t^2 - 3t + 5$$

For this particle to come to rest, v must be 0 for some positive value of t .

$2t^2 - 3t + 5 = 0$ must have real, positive roots.

$$b^2 - 4ac = (-3)^2 - 4(2)(5)$$

$$= 9 - 40$$

$$= -31 < 0$$

The equation therefore has no real roots, so v is never zero.

$$4 \text{ b } v = 2t^2 - 3t + 5$$

$$\frac{dv}{dt} = 4t - 3$$

$$\frac{dv}{dt} = 0 \text{ when } 4t = 3$$

$$t = 0.75$$

Minimum velocity is when $t = 0.75$.

$$\begin{aligned} \text{When } t = 0.75, v &= 2(0.75)^2 - 3(0.75) + 5 \\ &= 1.125 - 2.25 + 5 \\ &= 3.875 \\ &= 3.88 \text{ (3 s.f.)} \end{aligned}$$

The minimum velocity of the particle is 3.88 m s^{-1} .

$$5 \text{ a } s = \frac{9t^2}{2} - t^3$$

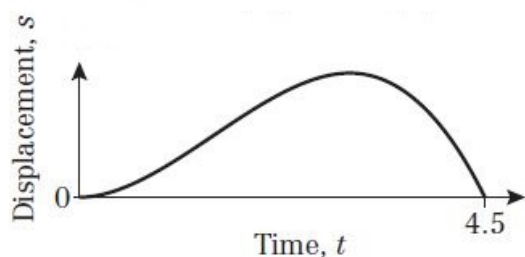
$$= t^2(4.5 - t)$$

Displacement is zero when $t = 0$ and $t = 4.5$.

The graph touches the time axis at $t = 0$ and crosses it at $t = 4.5$.

Graph only shown for $0 \leq t \leq 4.5$, as this is range over which equation is valid.

The curve is cubic, so not symmetrical.



b For values of $t > 4.5$, s is negative. However s is a distance and can only be positive.

$$c \ s = \frac{9t^2}{2} - t^3$$

$$\frac{ds}{dt} = 9t - 3t^2$$

$$\frac{ds}{dt} = 0 \text{ when}$$

$$9t - 3t^2 = 0$$

$$3t(3 - t) = 0$$

The turning points are at $t = 0$ and $t = 3$.

$s = 0$ when $t = 0$, so maximum distance occurs when $t = 3$.

When $t = 3$, using factorised form of the equation of motion:

$$s = 3^2(4.5 - 3) = 9 \times 1.5 = 13.5$$

The maximum distance of P from O is 13.5 m .

$$5 \quad d \quad v = \frac{ds}{dt} = 9t - 3t^2$$

$$a = \frac{dv}{dt} = 9 - 6t$$

When $t = 3$,

$$a = 9 - 6 \times 3 = -9$$

The magnitude of the acceleration of P at the maximum distance is 9 m s^{-2} .

$$6 \quad s = 3.6t + 1.76t^2 - 0.02t^3$$

$$\frac{ds}{dt} = 3.6 + 3.52t - 0.06t^2$$

Maximum distance occurs when $\frac{ds}{dt} = 0$.

$$\frac{ds}{dt} = 0 \text{ when}$$

$$3.6 + 3.52t - 0.06t^2 = 0$$

$$3t^2 - 176t - 180 = 0$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{176 \pm \sqrt{(-176)^2 + (4)(3)(180)}}{2 \times 3} \\ &= \frac{176 \pm \sqrt{33136}}{6} \\ &= -1.005 \text{ or } 59.67 \end{aligned}$$

$t > 0$, so maximum distance occurs when $t = 59.67$.

$$\begin{aligned} \text{When } t = 59.67, s &= 3.6(59.67) + 1.76(59.67)^2 - 0.02(59.67)^3 \\ &= 2230 \text{ (3 s.f.)} \end{aligned}$$

The maximum distance from the start of the track is 2230 m or 2.23 km. Since this is less than 4 km, the train never reaches the end of the track.