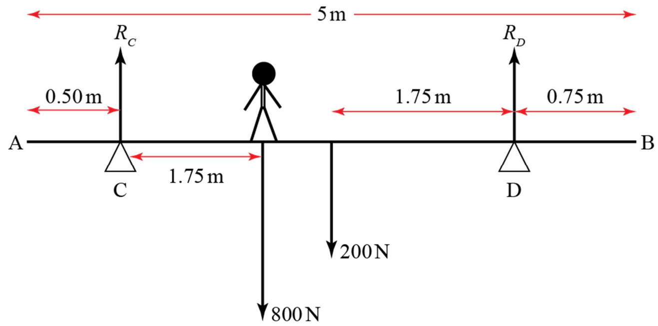
## Review exercise 2

**1 a** Let the reactions at $C$ and $D$ be $R_C$ and $R_D$ respectively.
Since the plank is in equilibrium, taking moments about $D$:
$(200\times1.75)+(800\times2.25)=R_C\times3.75$

$$R_C=\frac{350+1800}{3.75}=573.33...$$

The reaction at $C$ is 573 N (3 s.f.).




**b** Resolving vertically:
$R_C+R_D=200+800$
$R_D=1000-573.33...$
$R_D=426.66...$
The reaction at $C$ is 427 N (3 s.f.).

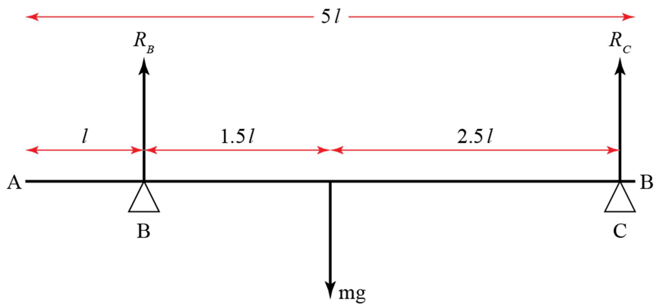
**c** By modelling the builder as a particle, we can assume all weight acts from a single point at his centre of gravity.

**2 a** Let the reactions at $B$ and $C$ be $R_B$ and $R_C$ respectively.
Since plank is in equilibrium, taking moments about $C$:

$$mg\times\frac{5}{2}l=R_B\times4l$$

$$R_B=\frac{5mg}{2\times4}$$

$$R_B=\frac{5mg}{8}\quad\text{as required.}$$




**b** Resolving vertically:

$$R_B+R_C=mg$$

$$\frac{5}{8}mg+R_C=mg\qquad\left(\text{Using } R_B=\frac{5mg}{8}\text{ from } \mathbf{a}\right)$$

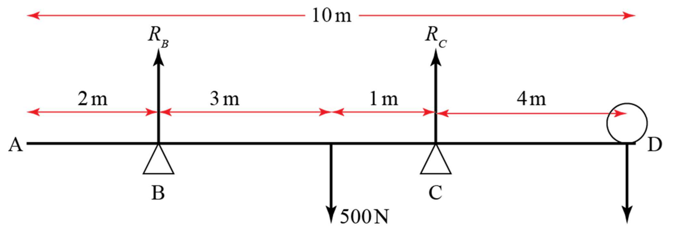
$$R_c=mg\left(1-\frac{5}{8}\right)$$

$$R_C=\frac{3mg}{8}\quad\text{as required.}$$

**c i** Assuming the plank is uniform allows us to assume the weight acts from its midpoint.

**ii** By assuming the plank is a rod, we ignore its width.

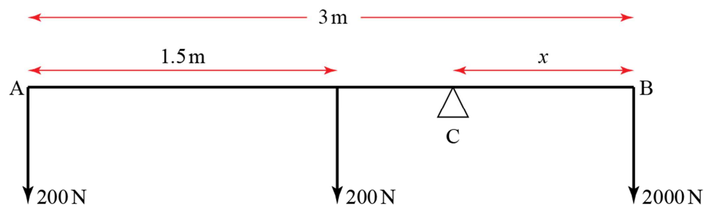
**3** Let the reactions at $B$ and $C$ be $R_B$ and $R_C$ respectively.




**a** Let the weight placed at $D$ be $W$.
As $W$ is increased, the rod will begin to tip about $C$ when $R_B = 0$.
Then, taking moments about $C$:
$W \times 4 = 500 \times 1$
$W = 125$
The largest weight that can be placed at $D$ before the rod tips is 125 N.

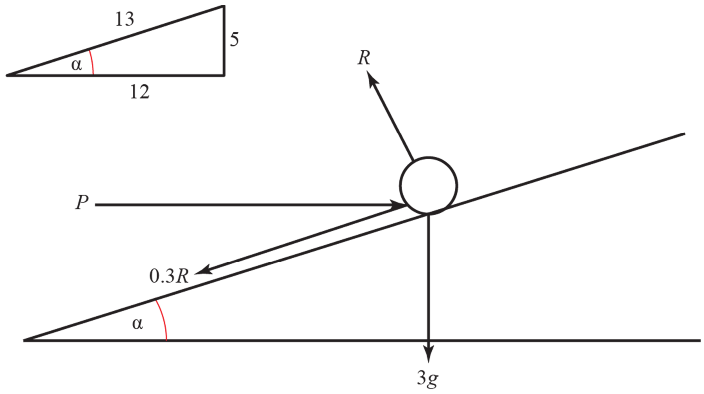
**b** Let the weight now placed at $A$ be $W$.
As $W$ is increased, the rod will begin to tip about $B$ when $R_C = 0$.
Then, taking moments about $B$:
$W \times 2 = 500 \times 3$
$W = 750$
The largest weight that can be placed at $A$ before the rod tips is 750 N.

**4** Let $CB = x$ m
Taking moments about $C$, since lever is in equilibrium:
$2000x = 200(3 - x) + 200(1.5 - x)$
$2000x = (200 \times 4.5) - 400x$
$2400x = 900$
$x = 0.375$
The length $CB$ is 0.375 m.



**5** Since the particle is moving at constant velocity, the forces acting on it are balanced.

$\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$




$R(\nwarrow)$:

$R = 3g\cos\alpha + P\sin\alpha$

$R = \frac{3g \times 12}{13} + \frac{5P}{13}$

$R = \frac{36g + 5P}{13}$

$R(\nearrow)$:

$P\cos\alpha = \mu R + 3g\sin\alpha$

$\frac{12}{13}P = \frac{1}{5}\left(\frac{36g + 5P}{13}\right) + \frac{3g \times 5}{13}$

$12P = \frac{36g}{5} + P + 15g$

$11P = \left(\frac{36}{5} + 15\right)g$

$P = \left(\frac{36}{5} + 15\right) \times \frac{9.8}{11}$

$= 19.778...$

$P$ is 19.8 N (to 3s.f.).

**6** $m = 2$ kg, $a = 2$ ms$^{-2}$

Using Newton's second law of motion and resolving up the slope:

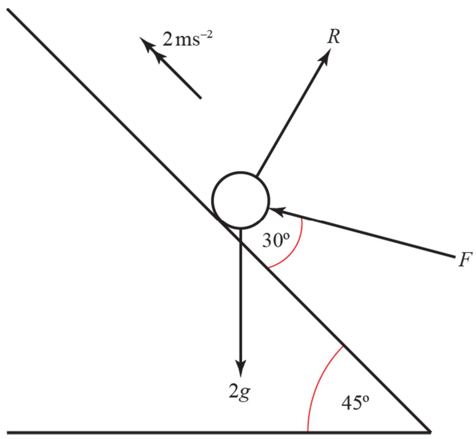
$$F = ma$$

$$F\cos 30^\circ - 2g\sin 45^\circ = 2 \times 2$$

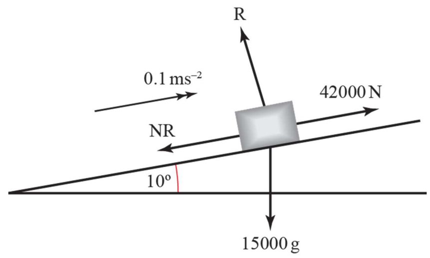
$$\frac{\sqrt{3}}{2}F - \frac{2g}{\sqrt{2}} = 4$$

$$\frac{\sqrt{3}}{2}F = 4 + \sqrt{2}g$$

$$F = \frac{2}{\sqrt{3}}\left(4 + \sqrt{2}g\right)$$ as required.



**7** $m = 15\,000$ kg, $a = 0.1$ ms$^{-2}$




**a** $R(\nwarrow)$ :

$R = 15000g\cos 10^\circ$

$R = 15000 \times 9.8\cos 10^\circ = 144\,767$

To the nearest whole newton, the reaction between the container and the slope is 144 767 N.

**b** Using Newton's second law of motion and resolving up the slope:

$$F = ma$$

$$42\,000 - \mu R - 15\,000g\sin 10^\circ = 15\,000 \times 0.1$$

$$\mu \times 144\,767 = 42\,000 - 1500 - \left(15\,000 \times 9.8\sin 10^\circ\right)$$

$$\mu = \frac{40500 - 25526.2}{144\,767}$$

$$= 0.103\,433...$$

The coefficient of friction between the container and the slope is 0.103 (3s.f.).

**c** Using Newton's second law of motion and resolving down the slope after winch stops working:

$$F = ma$$

$$\mu R + 15\,000g\sin 10^\circ = 15\,000a$$

$$144\,767 \times 0.103\,433 + 15\,000g\sin 10^\circ = 15\,000a \quad \text{(using results from } \mathbf{a} \text{ and } \mathbf{b}\text{)}$$

$$a = \frac{40\,500}{15\,000}$$

$$= 2.7$$

So the container accelerates down the slope at $2.7$ ms$^{-2}$

So: $u = -2$ ms$^{-1}$, $v = 0$ ms$^{-1}$, $a = 2.7$ ms$^{-2}$, $t = ?$

$v = u + at$

$0 = -2 + 2.7t$

$t = \dfrac{2}{2.7}$

$= 0.74074...$

The container takes 0.740 s (3s.f.) to come to rest.

**d** Once the container comes to rest, the container will tend to move down the slope and hence the frictional force will act up the slope. The container will therefore move back down if the component of weight down the slope is greater than the frictional force; i.e. if

$$mg\sin 10^\circ > \mu R$$

$$15\,000g\sin 10^\circ > 144\,767 \times 0.103\,433$$

$$25\,526 > 14\,974$$

Since this inequality is true, the container will start to slide back down the slope.

**8 a** $R(\downarrow)$:

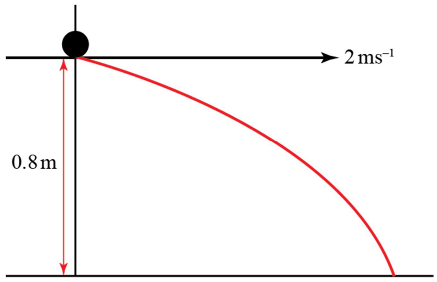
$s = 0.8$ m, $u = 0$ ms$^{-1}$, $a = 9.8$ ms$^{-2}$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0.8 = 0 + \frac{9.8}{2}t^2$$

$$t = \sqrt{\frac{0.8}{4.9}} = 0.40406...$$

The ball reaches the ground after 0.404 s (3s.f.).




**b** $R(\rightarrow)$:

$v = 2$ ms$^{-1}$, $t = 0.404$ s, $s = ?$

$$s = vt$$

$$s = 2 \times 0.40406... = 0.80812...$$

The ball lands 0.808 m from the table edge (3s.f.).

**9 a** First resolve vertically to find time of flight, then resolve horizontally to find initial velocity.

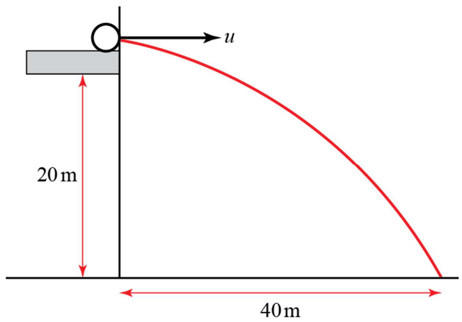
$R(\downarrow)$:

$s = 20$ m, $u = 0$ ms$^{-1}$, $a = 9.8$ ms$^{-2}$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$20 = 0 + \frac{9.8}{2}t^2$$

$$t = \sqrt{\frac{20}{4.9}} = \frac{10\sqrt{2}}{7}$$




$R(\rightarrow)$:

$t = \frac{10\sqrt{2}}{7}$ s, $s = 40.0$ m, $u = ?$

$$s = vt$$

$$40 = u \times \frac{10\sqrt{2}}{7}$$

$$u = \frac{4 \times 7}{\sqrt{2}} = 19.798...$$

The initial horizontal speed of the ball is 19.8 ms$^{-1}$ (3s.f.).

**b** Assumptions made are that the ball behaves as a particle (i.e. that there is negligible air resistance) and that the acceleration due to gravity remains constant over the distance fallen.

**10** Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = 150\cos 10°$

$R(\uparrow)\ u_y = 150\sin 10°$

**a** $R(\uparrow)$

$u = 150\sin 10°,\quad v = 0,\quad a = -9.8,\quad t = ?$

$v = u + at$

$0 = 150\sin 10° - 9.8t$

$t = \dfrac{150\sin 10°}{9.8} = 2.657$

The time taken to reach the projectile's highest point is 2.7 s (2 s.f.).

**b** By symmetry, the time of flight is $(2.657... \times 2)\text{s} = 5.315\text{s}$

[Note that you could also find the time of flight by resolving vertically with $s = 0$, but since you have already found *half* the time of flight in part **a**, it is simpler just to double this.]

Now find the range by resolving horizontally:

$R(\rightarrow)$:

$u = 150\cos 10°,\ t = 5.315,\ s = ?$

$s = ut$

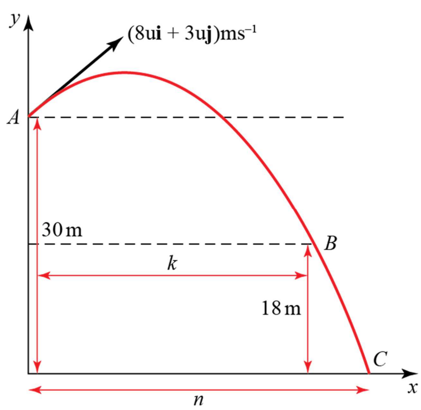
$= 150\cos 10° \times 5.315$

$= 785.250$

The range of the projectile is 790 m (2 s.f.).

**11** $\mathbf{u} = (8u\mathbf{i} + 3u\mathbf{j})\ \text{ms}^{-1}$, $\mathbf{a} = -9.8\mathbf{j}\ \text{ms}^{-2}$, $t = 3$, $\mathbf{s} = (k\mathbf{i} + 18\mathbf{j}) - 30\mathbf{j}$

$= (k\mathbf{i} - 12\mathbf{j})$




$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$k\mathbf{i} - 12\mathbf{j} = 3(8u\mathbf{i} + 3u\mathbf{j}) - \left(\frac{9}{2} \times 9.8\mathbf{j}\right)$$

$$k\mathbf{i} - 12\mathbf{j} = 24u\mathbf{i} + (9u - 44.1)\mathbf{j}$$

**a** Considering **j** components only:

$-12 = 9u - 44.1$

$u = \dfrac{44.1 - 12}{9} = 3.5666...$

The value of $u$ is 3.6 (to 2s.f.).

**b** Considering **i** components only:

$k = 24u$

$k = 24 \times 3.567 = 85.6$

The value of $k$ is 86 (to 2s.f.).

**11 c** At C, let $\mathbf{v} = (v_x\mathbf{i} - v_y\mathbf{j})$ ms$^{-1}$

Now $v_x = 8 \times 3.567 = 28.533$ since there is no horizontal acceleration.

Considering the **j** components only:

$u = 3 \times 3.567,\ v = v_y,\ s = -30,\ a = -9.8$

$v^2 = u^2 + 2as$

$v_y{}^2 = (3 \times 3.6)^2 + (2 \times (-9.8) \times (-30))$

$v_y = \sqrt{114.49 + 588}$

$v_y = 26.504$

$\tan\alpha = \dfrac{v_y}{v_x}$

$\tan\alpha = \dfrac{26.504}{28.533}$

$\alpha = 42.888$

At $C$, the velocity of $P$ makes an angle of 43° (2s.f.) with the $x$-axis.

**12 a** Then $\mathbf{u} = (12\mathbf{i} + 24\mathbf{j}),\ \mathbf{a} = -9.8\mathbf{j},\ t = 3,\ \mathbf{s} = ?$

$\mathbf{s} = \mathbf{u}t + \dfrac{1}{2}\mathbf{a}t^2$

$= 3 \times (12\mathbf{i} + 24\mathbf{j}) - 3^2 \times 4.9\mathbf{j}$

$= 36\mathbf{i} + 27.9\mathbf{j}$

The position vector of $P$ after 3 s is $(36\mathbf{i} + 27.9\mathbf{j})$ m

**b** $\mathbf{u} = (12\mathbf{i} + 24\mathbf{j}),\ \mathbf{a} = -9.8\mathbf{j},\ t = 3,\ \mathbf{v} = ?$

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$= (12\mathbf{i} + 24\mathbf{j}) - 3 \times 9.8\mathbf{j}$

$= 12\mathbf{i} - 5.4\mathbf{j}$

Let the speed of $P$ after 3 s be $V$ m s$^{-1}$

$V^2 = 12^2 + (-5.4)^2$

$= 173.16$

$V = \sqrt{173.16}$

$= 13.159$

The speed of $P$ after 3 s is 13 m s$^{-1}$ (2 s.f.).

**13** Resolving the initial velocity horizontally and vertically

$R(\rightarrow)\ u_x = u\cos\alpha$

$R(\uparrow)\ u_y = u\sin\alpha$

**a** $R(\uparrow)$:

$u_y = u\sin\alpha,\ s = 0,\ a = -g,\ t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = u\sin\alpha t - \frac{1}{2}gt^2$$

$$0 = t(u\sin\alpha - \frac{1}{2}gt)$$ ($t = 0$ corresponds to the point of projection)

$$\frac{1}{2}gt = u\sin\alpha$$

$$\Rightarrow t = \frac{2u\sin\alpha}{g},$$ as required

**b** $R(\rightarrow)$:

$u_x = u\cos\alpha,\ t = \dfrac{2u\sin\alpha}{g},\ s = ?$

$$s = u\cos\alpha \times \frac{2u\sin\alpha}{g}$$

$$= \frac{u^2 \times 2\sin\alpha\cos\alpha}{g}$$

$$= \frac{u^2\sin 2\alpha}{g}$$ (Using $\sin 2\alpha = 2\sin\alpha\cos\alpha$)

$$\therefore R = \frac{u^2\sin 2\alpha}{g},$$ as required

**c** The greatest possible value of $\sin 2\alpha$ is 1, which is when $2\alpha = 90° \Rightarrow \alpha = 45°$.
Hence, for a fixed $u$, the greatest possible range is when $\alpha = 45°$.

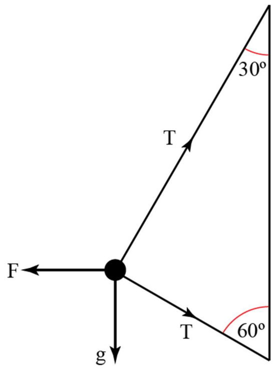
**d** $\dfrac{2u^2}{5g} = \dfrac{u^2\sin 2\alpha}{g} \Rightarrow \sin 2\alpha = \dfrac{2}{5}$

$2\alpha = 23.578°, 156.422°$

$\alpha = 11.79°,\ 78.21°$

The two possible angles of elevation are
$12°$ and $78°$ (nearest degree).

**14** The system is in equilibrium.




**a** Resolving vertically:

$$T\cos 60^\circ + g = T\cos 30^\circ$$

$$\frac{T}{2} + g = \frac{T\sqrt{3}}{2}$$

$$2g = T\left(\sqrt{3}-1\right)$$

$$T = \frac{2g}{\sqrt{3}-1} \quad \text{as required.}$$

**b** Resolving horizontally:

$$F = T\sin 60^\circ + T\sin 30^\circ$$

$$F = T\left(\sin 60^\circ + \sin 30^\circ\right)$$

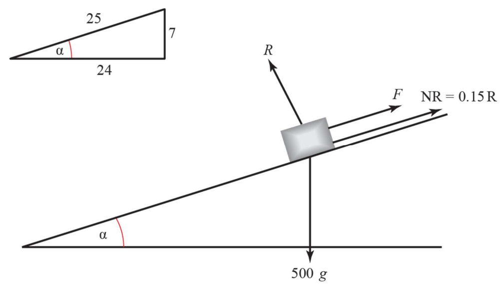
$$F = \left(\frac{2g}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{2}\right)$$

$$F = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)g \quad \text{as required.}$$

**c** We model the bead as smooth in order to assume there is no friction between it and the string.

**15** $\tan\alpha = \frac{7}{24} \Rightarrow \sin\alpha = \frac{7}{25}$ and $\cos\alpha = \frac{24}{25}$

The system is in equilibrium.




**a** $R(\nwarrow)$:

$$R = 500g\cos\alpha$$

$$R = \frac{24}{25}\times 500g = 480g$$

The normal reaction of the hill on the crate is $480g$ N, as required.

**b** Minimum value of F occurs when the crate is on the point of sliding down the hill. Frictional force then acts up the hill.

$R(\nearrow)$:

$$F + \mu R = 500g\sin\alpha$$

$$F = \left(\frac{7}{25}\times 500g\right) - \left(\frac{3}{20}\times 480g\right) \qquad \left(\text{Using } \mu = \frac{3}{20}, \text{ and } R = 480g \text{ from } \mathbf{a}\right)$$

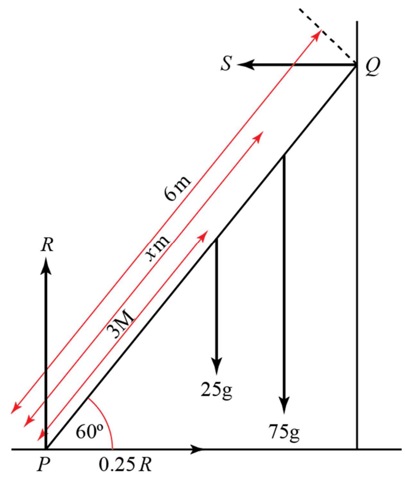
$$F = (140-72)g$$

$$= 68g$$

The minimum value of $F$ required to maintain equilibrium is $68g$ N.

**16** Let:
$R$ be the normal reaction of the floor on the ladder at $P$,
$S$ be the normal reaction of the wall on the ladder at $Q$,
$F$ be the friction between the floor and the ladder at $P$
$x$ be the max. distance up the ladder from $P$ that the builder can stand before the ladder begins to slip

$$R(\uparrow): \ R = 75g + 25g = 100g$$




$$R(\rightarrow): F = S$$

The ladder is in limiting equilibrium, so $F = \mu R$
Hence $S = \mu R$

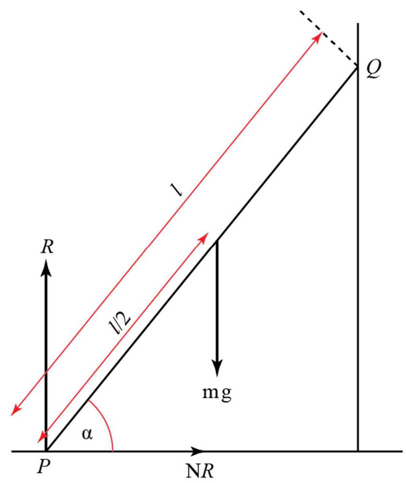
$$= 0.25 \times 100g$$
$$= 25g$$

Taking moments about $P$:

$$S \times 6\sin 60^\circ = 75g \times x\cos 60^\circ + 25g \times 3\cos 60^\circ$$
$$25g \times 6\sin 60^\circ = 75(x+1)g\cos 60^\circ$$
$$x + 1 = \frac{25g \times 6\sin 60^\circ}{75g\cos 60^\circ}$$
$$x + 1 = 2\tan 60^\circ$$
$$x = 2\sqrt{3} - 1$$
$$x = 2.4641\ldots$$

The maximum distance the builder can climb up the ladder before it slips is 2.46 m (3s.f.).

**17** Let:
$R$ be the normal reaction of the floor on the ladder at $P$,
$S$ be the normal reaction of the wall on the ladder at $Q$,
$F$ be the friction between the floor and the ladder at $P$




$$R(\uparrow): \ R = mg$$

$$R(\rightarrow): F = S$$

The ladder is in limiting equilibrium, so $F = \mu R$
Hence $S = \mu R$

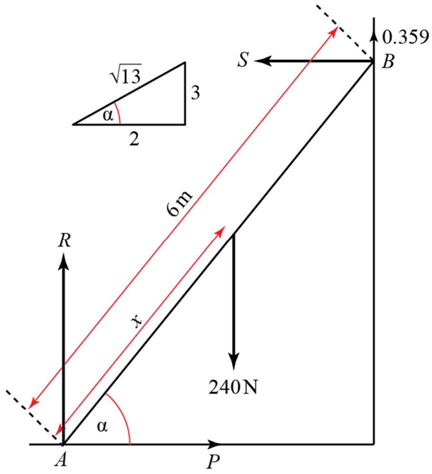
$$= \mu mg$$

Taking moments about $P$:

$$S \times l\sin\alpha = mg \times \frac{l}{2}\cos\alpha$$
$$\mu mgl\sin\alpha = \frac{mgl}{2}\cos\alpha$$
$$\mu = \frac{\cos\alpha}{2\sin\alpha} = \frac{1}{2\tan\alpha}$$

The coefficient of friction, $\mu$, is given by $\dfrac{1}{2\tan\alpha}$.

**18** $\tan\alpha = \frac{3}{2} \Rightarrow \sin\alpha = \frac{3}{\sqrt{13}}$ and $\cos\alpha = \frac{2}{\sqrt{13}}$




Let:
$R$ be the normal reaction of the floor on the ladder at $A$,
$S$ be the normal reaction of the wall on the ladder at $B$,
$F$ be the friction between the wall and the ladder at $B$

$R(\rightarrow): P = S$

The ladder is in limiting equilibrium, so $F = \mu S$

$$= \mu P$$
$$= 0.3P$$

Taking moments about $A$:

$$240 \times 2\cos\alpha = (S \times 6\sin\alpha) + (F \times 6\cos\alpha)$$
$$240 \times 2\cos\alpha = (P \times 6\sin\alpha) + (0.3P \times 6\cos\alpha)$$
$$\frac{480 \times 2}{\sqrt{13}} = \frac{6P \times 3}{\sqrt{13}} + \frac{1.8P \times 2}{\sqrt{13}}$$
$$960 = (18 + 3.6)P$$
$$P = \frac{960}{21.6} = 44.444...$$

The minimum value of $P$ is therefore 44.4 N (3s.f.).

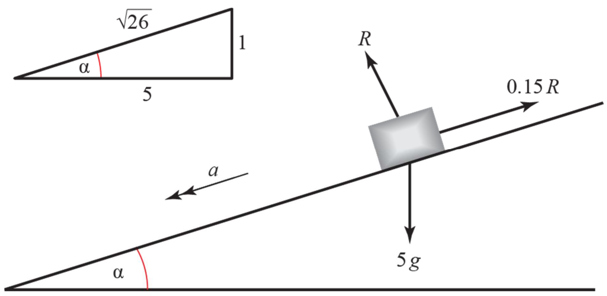
**19** $\tan\alpha = \frac{1}{5} \Rightarrow \sin\alpha = \frac{1}{\sqrt{26}}$ and $\cos\alpha = \frac{5}{\sqrt{26}}$

Resolving at right angles to the hill:

$$R = 5g\cos\alpha$$
$$R = \frac{5g \times 5}{\sqrt{26}} = \frac{25g}{\sqrt{26}}$$

The sled slides down the hill; the frictional force therefore acts up the hill.
Resolving down the hill and using Newton's second law of motion:

$ma = F$




$$5a = 5g\sin\alpha - \mu R$$
$$5a = \frac{5g}{\sqrt{26}} - \frac{0.15 \times 25g}{\sqrt{26}}$$
$$5a = \left(\frac{5 - 3.75}{\sqrt{26}}\right)g$$
$$a = \frac{5g}{4\sqrt{26}}$$

**19 (Cont.)**

Consider motion down the hill:

$u=0,\ a=\dfrac{5g}{4\sqrt{26}},\ s=200,\ t=?$

$s=ut+\dfrac{1}{2}at^2$

$200=0+\left(\dfrac{1}{2}\times\dfrac{5g}{4\sqrt{26}}t^2\right)$

$t^2=\dfrac{200\times8\sqrt{26}}{5g}$

$t=\sqrt{\dfrac{1600\sqrt{26}}{5\times9.8}}=12.903...$

To 3 s.f., the sled takes 12.9 s to travel 200 m down the hill.

**20** At 10 am:

$\mathbf{r}_{P_0}=(400\mathbf{i}+200\mathbf{j})$ km

$\mathbf{r}_{Q_0}=(500\mathbf{i}-100\mathbf{j})$ km

$\mathbf{v}_P=(300\mathbf{i}+250\mathbf{j})$ kmh$^{-1}$

$\mathbf{v}_Q=(600\mathbf{i}-200\mathbf{j})$ kmh$^{-1}$

**a** $\mathbf{r}=\mathbf{r}_0+\mathbf{v}t$

$\mathbf{r}_P=((400+300t)\mathbf{i}+(200+250t)\mathbf{j})$km

$\mathbf{r}_Q=((500+600t)\mathbf{i}-(100+200t)\mathbf{j})$km

**b** $\mathbf{r}_{QP}=\mathbf{r}_Q-\mathbf{r}_P$

$\mathbf{r}_{QP}=500\mathbf{i}-100\mathbf{j}-(400\mathbf{i}+200\mathbf{j})$

$\mathbf{r}_{QP}=100\mathbf{i}-300\mathbf{j}$

**c** At noon, $t=2$ h

$\mathbf{r}_P=(400+(300\times2))\mathbf{i}+(200+(250\times2))\mathbf{j}$

$\mathbf{r}_Q=(500+(600\times2))\mathbf{i}-(100+(200\times2))\mathbf{j}$

$\mathbf{r}_{QP}=1700\mathbf{i}-500\mathbf{j}-(1000\mathbf{i}+700\mathbf{j})$

$\mathbf{r}_{QP}=700\mathbf{i}-1200\mathbf{j}$

$|\mathbf{r}_{QP}|=\sqrt{700^2+1200^2}=1389.2...$

At noon, the two aeroplanes are 1390 km apart (3s.f.).

**21 a** $x = \left(3t - \dfrac{2k}{2t-1}\right)$m,

$$v = \frac{dx}{dt}$$

$$v = 3 + \frac{4k}{(2t-1)^2}$$

$$t = 0 \Rightarrow v = 10 \text{ ms}^{-1}$$

$$10 = 3 + \frac{4k}{-1^2}$$

$k = \dfrac{10-3}{4} = \dfrac{7}{4}$ as required.

**b** $t = 2$ s

$$x = (3\times 2) - \frac{2\times\frac{7}{4}}{(2\times 2)-1}$$

$$x = 6 - \frac{7}{2\times 3} = \frac{29}{6}$$

After 2 s, $P$ is $\dfrac{29}{6}$ m from $O$.

**22 a** $\mathbf{r} = \left(\left(\frac{1}{3}t^3 + 2t\right)\mathbf{i} + \left(\frac{1}{2}t^2 - 1\right)\mathbf{j}\right)$ m

$$\mathbf{v} = \dot{\mathbf{r}}$$

$$\mathbf{v} = (t^2 + 2)\mathbf{i} + t\mathbf{j}$$

**b** $t = 5$ s

$$\mathbf{v} = (5^2 + 2)\mathbf{i} + 5\mathbf{j}$$

$$\mathbf{v} = 27\mathbf{i} + 5\mathbf{j}$$

$$|\mathbf{v}| = \sqrt{27^2 + 5^2} = 27.459...$$

At $t = 5$ s, the speed of $P$ is 27.5 $\text{ms}^{-1}$ (3s.f.).

**c**

$$\mathbf{a} = \dot{\mathbf{v}}$$

$$\mathbf{a} = 2t\mathbf{i} + \mathbf{j}$$

$$t = 2s \Rightarrow$$

$$\mathbf{a} = 4\mathbf{i} + \mathbf{j}$$

$$|\mathbf{a}| = \sqrt{17} = 4.1231...$$

$$\tan\alpha = \frac{1}{4}$$

$$\alpha = 14.036...$$

At $t = 2$ s, $P$ is accelerating at 4.12 $\text{ms}^{-2}$ at an angle of $14.0^{\circ}$ to the **i** vector (both values to 3s.f.).

**23 a** $\mathbf{r} = \left(\left(4t^2+1\right)\mathbf{i} + \left(2t^2-3\right)\mathbf{j}\right)$ m

$\mathbf{v} = \dot{\mathbf{r}}$

$\mathbf{v} = 8t\mathbf{i} + 4t\mathbf{j}$

$t = 3s \Rightarrow$

$\mathbf{v} = 24\mathbf{i} + 12\mathbf{j}$

At $t = 3$ s, the velocity of the particle is $\left(24\mathbf{i}+12\mathbf{j}\right)$ ms$^{-1}$.

**b** $\mathbf{a} = \dot{\mathbf{v}}$

$\mathbf{a} = 8\mathbf{i} + 4\mathbf{j}$

Since all terms in this expression are independent of $t$, the acceleration is constant.

**24** $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t = \int -2t\mathbf{i} + 3t^{\frac{1}{2}}\mathbf{j}$

$= -t^2\mathbf{i} + 3 \times \frac{2}{3}t^{\frac{3}{2}}\mathbf{j} + c$

$= -t^2\mathbf{i} + 2\sqrt{t^3}\,\mathbf{j} + c$

When $t = 0$, $\mathbf{r} = 2\mathbf{j}$ m $\Rightarrow$

$2\mathbf{j} = 0\mathbf{i} + 0\mathbf{j} + c \Rightarrow c = 2\mathbf{j}$

$\therefore \mathbf{r} = -t^2\mathbf{i} + 2\left(\sqrt{t^3}+1\right)\mathbf{j}$

When $t = 4$ s

$\mathbf{r} = -16\mathbf{i} + 2\left(8+1\right)\mathbf{j}$

$\mathbf{r} = -16\mathbf{i} + 18\mathbf{j}$

$\left|\mathbf{r}\right| = \sqrt{16^2+18^2} = 24.083...$

After 4 s, $P$ is 24 m from $O$ (2s.f.).

**25 a** $\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int \left(2t^2-3t^3\right)\mathbf{i} - 4\left(2t+1\right)\mathbf{j}\,\mathrm{d}t$

$\mathbf{v} = \left(t^2 - \frac{3}{4}t^4\right)\mathbf{i} - 4\left(t^2+t\right)\mathbf{j} + c$

$t = 0 \Rightarrow \mathbf{v} = \left(3\mathbf{i}+\mathbf{j}\right)$ ms$^{-1}$

$3\mathbf{i} + \mathbf{j} = 0\mathbf{i} - 4(0)\mathbf{j} + c$

$c = 3\mathbf{i} + \mathbf{j}$

$\Rightarrow \mathbf{v} = \left(t^2 - \frac{3}{4}t^4 + 3\right)\mathbf{i} - \left(4t^2+4t-1\right)\mathbf{j}$

**25 b** If $P$ is moving in the direction of **i**, the coefficient of **j** in the velocity vector is 0.

$$0 = 4t^2 + 4t - 1$$

$$t = \frac{-4 \pm \sqrt{16 - (4 \times 4 \times (-1))}}{8}$$

$$t = \frac{-1 \pm \sqrt{2}}{2}$$

The negative solution can be ignored as it is outside the range over which the equation applies.

$P$ is moving in the direction of **i** after $\left(\frac{\sqrt{2}-1}{2}\right)$ s (0.207 s to 3 s.f.).

**26 a** $\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t = \int (-4t\mathbf{i} - 2\mathbf{j})\,\mathrm{d}t$

$$\mathbf{v} = -2t^2\mathbf{i} - 2t\mathbf{j} + c$$

$$t = 0 \Rightarrow \mathbf{v} = 8\mathbf{i}\ \mathrm{ms}^{-1}$$

$$8\mathbf{i} = 0\mathbf{i} - 0\mathbf{j} + c$$

$$c = 8\mathbf{i}$$

$$\Rightarrow \mathbf{v} = 2(4 - t^2)\mathbf{i} - 2t\mathbf{j}$$

**b** When the windsurfer is moving due south, the coefficient of **i** in the velocity vector is 0.

$$0 = 2(4 - t^2)$$

$$t^2 = 4$$

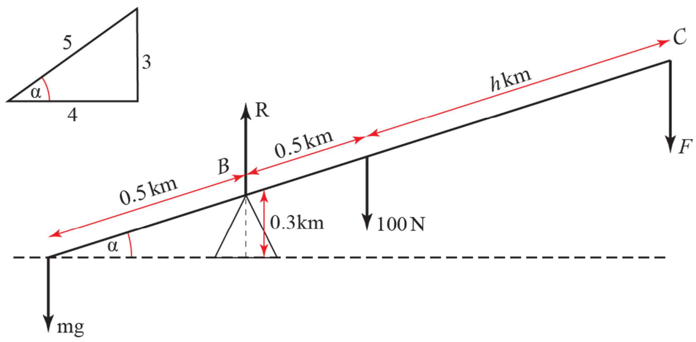
$$t = \pm 2$$

The negative solution can be ignored as it is before the time the windsurfer starts to move.
The windsurfer is moving due south after 2 s.

**Challenge**

**1** The rod makes an angle of $\alpha^{\circ}$ with the horizontal where

$$\sin\alpha = \frac{0.3}{0.5} = \frac{3}{5} \Rightarrow \cos\alpha = \frac{4}{5}$$




To lift the mass, total clockwise moments about $B$ must exceed total anticlockwise moments about $B$:

$$(F \times 1.5k\cos\alpha) + (100 \times 0.5k\cos\alpha) > mg \times 0.5k\cos\alpha$$

$$1.5F + 50 > 0.5mg$$

$$\frac{3}{2}F > \frac{1}{2}mg - 50$$

$$F > \frac{1}{3}(mg - 100) \quad \text{as required.}$$

**Challenge**

**2** $v = 3\sin kt + \cos kt \;\; \text{ms}^{-1}$

$a = \dot{v}$

$a = 3k\cos kt - \sin kt$

$t = 0 \Rightarrow a = 1.5 \text{ ms}^{-2}$

$\therefore 1.5 = 3k - 0$

$\therefore k = 0.5$

$$s = \int v\,\mathrm{d}t = \int 3\sin kt + \cos kt \;\mathrm{d}t$$

$$= -\frac{3\cos kt}{k} + \frac{\sin kt}{k} + c$$

$$s = -6\cos\frac{t}{2} + 2\sin\frac{t}{2} + c \qquad \text{(substituting } k = \frac{1}{2}\text{)}$$

$t = 0 \Rightarrow s = 0$

$0 = -6 + 0 + c \Rightarrow c = 6$

$$s = 2\left(3 - 3\cos\frac{t}{2} + \sin\frac{t}{2}\right)$$

Maximum displacement occurs when $v = 0$

$0 = 3\sin kt + \cos kt$

$0 = 3\tan kt + 1$

$\tan kt = -\frac{1}{3}$

$0.5t = 161.565...$

$t = 323.13...$

Maximum displacement is therefore

$$s = 2\left(3 - 3\cos(161.56...)^\circ + \sin(161.56...)^\circ\right)$$

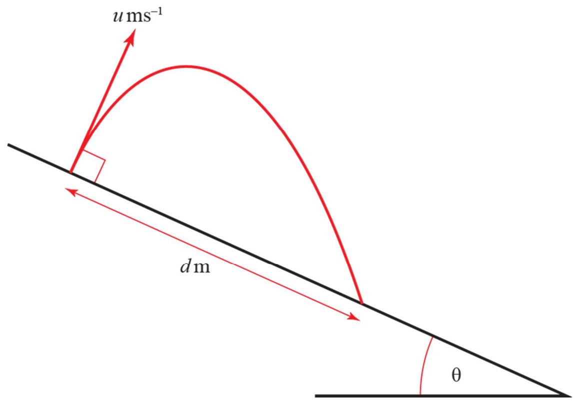
$s = 2(3 + 2.8460... + 0.31622..)$

$s = 12.324...$

The maximum displacement of 12.3 m first occurs at 323 s (both to 3s.f.).

**Challenge**

**3** There is no change in the horizontal component of the velocity.




$R(\rightarrow)$:

$v = u_x = u\sin\theta,\ s = d\cos\theta,\ t = ?$

$s = vt$

$$t = \frac{s}{v} = \frac{d\cos\theta}{u\sin\theta}$$

$R(\uparrow)$

$$u = u_y = u\cos\theta,\ s = -d\sin\theta,\ a = -g,\ t = \frac{d\cos\theta}{u\sin\theta}$$

$$s = ut + \frac{1}{2}at^2$$

$$-d\sin\theta = u\cos\theta\left(\frac{d\cos\theta}{u\sin\theta}\right) - \frac{g}{2}\left(\frac{d\cos\theta}{u\sin\theta}\right)^2$$

$$-d\sin\theta = \frac{d\cos^2\theta}{\sin\theta} - \frac{gd^2\cos^2\theta}{2u^2\sin^2\theta}$$

$$-\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\sin\theta} - \frac{gd\cos\theta}{2u^2\sin^2\theta}$$

$$\frac{gd\cos\theta}{2u^2\sin^2\theta} = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$\frac{gd}{2u^2} = \frac{\sin^2\theta}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$\frac{gd}{2u^2} = \tan\theta\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right)$$

$$d = \frac{2u^2}{g}\tan\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}\right)$$

$$d = \frac{2u^2}{g}\tan\theta\left(\frac{1}{\cos\theta}\right)$$

$$d = \frac{2u^2}{g}\tan\theta\sec\theta \quad \text{as required.}$$