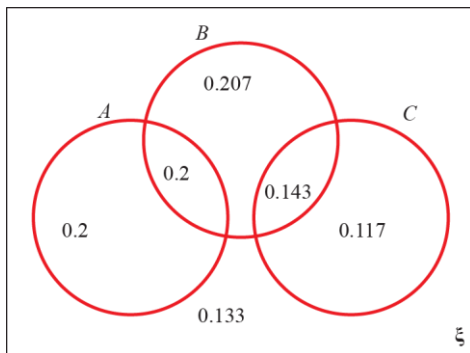


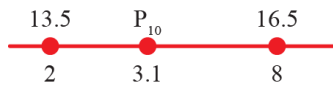
Exam-style practice: Paper 3, Section A: Statistics

- 1 a Use the cumulative binomial distribution tables, with $n = 40$ and $p = 0.52$. Then $P(X \geq 22) = 1 - P(X \leq 21) = 1 - 0.5867 = 0.4133$ (4 s.f.).
- b In order for the normal approximation to be used as an approximation to the binomial distribution the two conditions are: (i) n is large (>50); and (ii) p is close to 0.5.
- c The two conditions for the normal approximation to be a valid approximation are satisfied. $\mu = np = 250 \times 0.52 = 130$ and $\sigma = \sqrt{np(1-p)} = \sqrt{130 \times 0.48} = \sqrt{62.4} = 7.90$ (3 s.f.). Therefore $B(250, 0.52) \approx N(130, 7.9^2)$ so that $P(B \leq 120) \approx P(N \leq 120.5) = 0.1146$ (4 s.f.).
- d If the engineer's claim is true, then the observed result had a less than 12% chance of occurring. This would mean that there would be insufficient evidence to reject her claim with a two-tailed hypothesis test at the 10% level. Though it does provide some doubt as to the validity of her claim.
- 2 a Since A and C are mutually exclusive, $P(A \cap C) = 0$ and their intersection need not be represented on the Venn diagram. Since B and C are independent, $P(B \cap C) = P(B) \times P(C) = 0.55 \times 0.26 = 0.143$. Using the remaining information in the question allows for the other regions to be labelled. Therefore the completed Venn diagram should be:



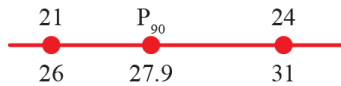
- b $P(A) \times P(B) = 0.4 \times 0.55 = 0.22 \neq 0.2 = P(A \cap B)$ and so the events are not independent.
- c $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.2}{0.2 + 0.117 + 0.133} = \frac{0.2}{0.45} = 0.444$ (3 s.f.)
- d $P(C|(A \cap B)') = \frac{P(C \cap (A \cap B)')}{P((A \cap B)')} = \frac{P(C)}{1 - P(A \cap B)} = \frac{0.26}{0.8} = 0.325$
- 3 a The variable t is continuous, since it can take any value between 12 and 26 (in degrees Celsius).
- b Estimated mean 19.419; estimated standard deviation 2.814 (3 d.p.).
- c Temperature is continuous and the data were given in a grouped frequency table.

- 3 d The 10th percentile is $\frac{31}{10} = 3.1$ th value. Using linear interpolation:



$$\frac{P_{10} - 13.5}{16.5 - 13.5} = \frac{3.1 - 2}{8 - 2} \Rightarrow \frac{P_{10} - 13.5}{3} = \frac{1.1}{6} \Rightarrow P_{10} = 3 \times \frac{1.1}{6} + 13.5 = 0.55 + 13.5 = 14.05$$

- The 90th percentile is $\frac{9 \times 31}{10} = 27.9$ th value. Using linear interpolation:



$$\frac{P_{90} - 21}{24 - 21} = \frac{27.9 - 26}{31 - 26} \Rightarrow \frac{P_{90} - 21}{3} = \frac{1.9}{5} \Rightarrow P_{90} = 3 \times \frac{1.9}{5} + 21 = 1.14 + 21 = 22.14$$

Therefore the 10th to 90th interpercentile range is $22.14 - 14.05 = 8.09$.

- e Since the meteorologist believes that there is positive correlation, the hypotheses are

$$H_0 : \rho = 0$$

$$H_1 : \rho > 0$$

The sample size is 8, and so the critical value (for a one-tailed test) is 0.6215.

Since $r = 0.612 < 0.6215$, there is not sufficient evidence to reject H_0 , and so there is not sufficient evidence, at the 5% significance level, to say that there is a positive correlation between the daily mean air temperature and the number of hours of sunshine.

- 4 a The value of 0.9998 is very close to 1, indicating that the plot of x against y is very close to being a linear relationship, and so the data should be well-modelled by an equation of the form $q = kt^n$.

- b Rearranging the equation

$$y = 0.07601 + 2.1317x$$

$$\Rightarrow \log q = 0.07601 + 2.1317 \log t$$

$$\Rightarrow q = 10^{0.07601 + 2.1317 \log t} = 10^{0.07601} \times 10^{2.1317 \log t}$$

$$\Rightarrow q = 10^{0.07601} \times 10^{\log t^{2.1317}} = 10^{0.07601} \times t^{2.1317}$$

Therefore $k = 10^{0.0761} = 1.19$ (3 s.f.) and $n = 2.1317$.

- c It would not be sensible to use the model to predict the amount of substance produced when $t = 85^\circ \text{C}$, since this is considerably outside the range of the provided data (extrapolation).

- 5 a $P(Z < a) = 0.025 \Rightarrow a = -1.96$ and $P(Z > a) = 0.05 \Rightarrow a = 1.645$. Therefore, for the given distribution, $\frac{3.416 - \mu}{\sigma} = -1.96$ and $\frac{4.858 - \mu}{\sigma} = 1.645$. Rearranging these equations:

$$\frac{3.416 - \mu}{\sigma} = -1.96 \Rightarrow 3.416 - \mu = -1.96\sigma \text{ and } \frac{4.858 - \mu}{\sigma} = 1.645 \Rightarrow 4.858 - \mu = 1.645\sigma.$$

Now subtract the second equation from the first to obtain:

$4.858 - \mu - (3.416 - \mu) = 1.645\sigma - (-1.96\sigma) \Rightarrow 1.442 = 3.605\sigma \Rightarrow \sigma = 0.4$ and so, using the first equation, $3.416 - \mu = -1.96 \times 0.4 \Rightarrow \mu = 3.416 + 0.784 = 4.2$. Using these values within the normal distribution, $P(3.5 < X < 4.6) = P(4.6) - P(3.5) = 0.84134 - 0.04006 = 0.8013$ (4 s.f.) of the cats will be of the standard weight.

- b Using the binomial distribution, $P(B \geq 10) = 1 - P(B \leq 9) = 1 - 0.0594 = 0.9406$ (4 s.f.).

- c Assume the mean is 4.5kg and standard deviation is 0.51. Then the sample \bar{X} should be normally distributed with $\bar{X} \sim N\left(4.5, \frac{0.51^2}{12}\right)$. The hypothesis test should determine whether it is statistically significant, at the 10% level, that the mean is not 4.5kg. Therefore the test should be 2-tailed with

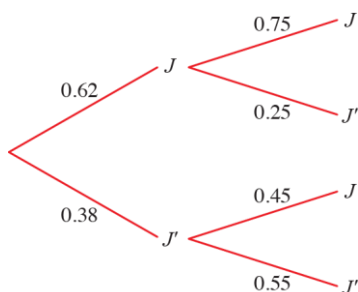
$$H_0 : \mu = 4.5$$

$$H_1 : \mu \neq 4.5$$

The critical region therefore consists of values greater than a where $P(\bar{X} > a) = 0.05$ and so $a = 4.742$ (4 s.f.) and values less than b where $P(\bar{X} < b) = 0.05$ and so $b = 4.258$ (4 s.f.).

Since the observed mean is 4.73 and $4.73 < a = 4.742$, there is not enough evidence, at the 10% significance level, to reject H_0 i.e. there is not sufficient evidence to say, at the 10% level, that the mean weight of all cats in the town is different from 4.5kg.

- 6 It is first worth displaying the information in a tree diagram. Let J denote the event that Jemima wins a game of tennis and J' be the event that Jemima loses a game of tennis. Since Jemima either wins or loses each game of tennis, $P(J) + P(J') = 1$. This allows for the other probabilities on the tree diagram to be filled in. Therefore the completed tree diagram should be:



The required probability is then:

$$\begin{aligned} & P(\text{wins both games} \mid \text{wins second game}) \\ &= \frac{P(\text{wins both games})}{P(\text{wins second game})} = \frac{0.62 \times 0.75}{0.62 \times 0.75 + 0.38 \times 0.45} = \frac{0.465}{0.465 + 0.171} = 0.731 \text{ (3 s.f.)} \end{aligned}$$

Exam-style practice: Paper 3, Section B: Mechanics

$$\begin{aligned}
 7 \quad \mathbf{r} &= \int \mathbf{v} \, dt \\
 &= \int (2 - 6t^2)\mathbf{i} - t\mathbf{j} \, dt \\
 &= \left(2t - \frac{6}{3}t^3\right)\mathbf{i} - \frac{t^2}{2}\mathbf{j} + \mathbf{c}
 \end{aligned}$$

At $t = 1$ s, $\mathbf{r} = 5\mathbf{i}$ m $\Rightarrow 5\mathbf{i} = (2 - 2)\mathbf{i} - \frac{1}{2}\mathbf{j} + \mathbf{c}$

$$\mathbf{c} = 5\mathbf{i} + \frac{1}{2}\mathbf{j}$$

$$\therefore \mathbf{r} = (2t - 2t^3 + 5)\mathbf{i} + \frac{1}{2}(1 - t^2)\mathbf{j}$$

When $t = 3$ s,

$$\mathbf{r} = (6 - 54 + 5)\mathbf{i} + \frac{1}{2}(1 - 9)\mathbf{j}$$

$$\mathbf{r} = -43\mathbf{i} - 4\mathbf{j}$$

$$s = |\mathbf{r}| = \sqrt{43^2 + 4^2} = 43.185\dots$$

At $t = 3$ s, P is 43.2 m from O (3 s.f.).

8 $R(\rightarrow): u_x = 100 \cos 30^\circ = 50\sqrt{3}$

$R(\uparrow): u_y = 100 \sin 30^\circ = 50$

a $R(\uparrow): u_y = 50 \text{ ms}^{-1}, s = 0 \text{ m}, a = g = -9.8 \text{ ms}^{-2}, t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$0 = 50t - 4.9t^2$$

$$4.9t^2 = 50t$$

The solution $t = 0$ corresponds to the time the arrow is fired and can therefore be ignored.

$$\therefore t = \frac{50}{4.9} = 10.204\dots$$

The arrow reaches the ground after 10.2 s (3 s.f.).

b At maximum height, $v_y = 0$

$R(\uparrow): u_y = 50 \text{ ms}^{-1}, v_y = 0 \text{ m}, a = g = -9.8 \text{ ms}^{-2}, s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 50^2 - 19.6s$$

$$19.6s = 2500$$

$$s = \frac{2500}{19.6} = 127.55\dots$$

The maximum height reached by the arrow is 128 m (3s.f.).

8 c At $t = 3$ s,

$$R(\rightarrow): v_x = u_x = 50\sqrt{3} \text{ ms}^{-1} \text{ since horizontal speed remains constant.}$$

$$R(\uparrow): u_y = 50 \text{ ms}^{-1}, t = 3 \text{ s}, a = g = -9.8 \text{ ms}^{-2}, v_y = ?$$

$$v = u + at$$

$$v_y = 50 - (3 \times 9.8) = 20.6$$

The speed at $t = 3$ s is given by:

$$v^2 = v_x^2 + v_y^2$$

$$v^2 = (50\sqrt{3})^2 + (20.6)^2$$

$$v = \sqrt{7500 + 424.36} = 89.018\dots$$

The speed of the arrow after 3 s is 89.0 ms^{-1} (3 s.f).

9 a $\mathbf{u} = 2\mathbf{i} \text{ ms}^{-1}$, $t = 10$ s, $\mathbf{a} = 0.2\mathbf{i} - 0.8\mathbf{j} \text{ ms}^{-2}$, $\mathbf{r} = ?$

$$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = 20\mathbf{i} + \frac{100}{2}(0.2\mathbf{i} - 0.8\mathbf{j})$$

$$\mathbf{r} = 20\mathbf{i} + 10\mathbf{i} - 40\mathbf{j}$$

After 10 s, the position vector of the cyclist is $(30\mathbf{i} - 40\mathbf{j}) \text{ m}$.

b $s = |\mathbf{r}|$

$$s = \sqrt{30^2 + 40^2} = 50$$

After 10 s, the cyclist is 50 m from A.

c For $t > 10$ s, $\mathbf{v} = 5\mathbf{i} \text{ ms}^{-1}$ and $\mathbf{a} = 0$

The position vector is now given by:

$$\mathbf{r} = (30\mathbf{i} - 40\mathbf{j}) + \mathbf{v}(t - 10)\mathbf{i}$$

$$\mathbf{r} = 30\mathbf{i} - 40\mathbf{j} + 5(t - 10)\mathbf{i}$$

$$\mathbf{r} = (5t - 20)\mathbf{i} - 40\mathbf{j}$$

The cyclist will be south-east of A when the coefficient of \mathbf{i} is positive and coefficient of \mathbf{j} is negative, but both have equal magnitude.

$$5t - 20 = 40$$

$$5t = 60$$

$$t = \frac{60}{5} = 12$$

The cyclist is directly south-east of A after 12 s.

9 d First, work out the position vector of B from A :

$$\mathbf{r} = (5t - 20)\mathbf{i} - 40\mathbf{j}$$

Cyclist reaches B when $t = 12 + 30 = 42$ s

$$\mathbf{r} = ((5 \times 42) - 20)\mathbf{i} - 40\mathbf{j}$$

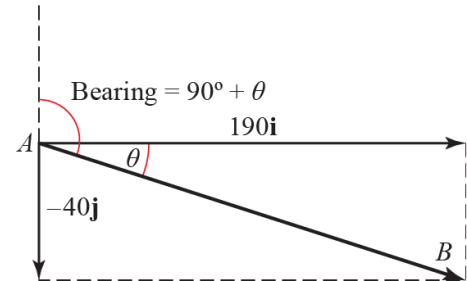
$$\mathbf{r} = 190\mathbf{i} - 40\mathbf{j}$$

Let θ be the acute angle between the horizontal and B (as shown in the diagram).

$$\text{Then } \tan \theta = \frac{40}{190}$$

$$\theta = 11.888\dots$$

To the nearest degree, the bearing of B from A is $90 + 12 = 102^\circ$.



10 a Considering Q and using Newton's second law of motion:

$$a = 0.5 \text{ ms}^{-2}, m = 2 \text{ kg}$$

$$F = ma$$

$$2g - T = 2 \times 0.5$$

$$T = (2 \times 9.8) - 1 = 18.6$$

The tension in the string immediately after the particles begin to move is 18.6 N.

b Considering P :

$$\text{Resolving vertically } \Rightarrow R = 3g$$

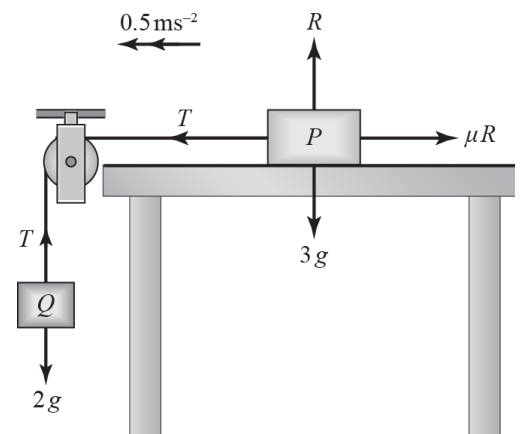
Resolving horizontally and using Newton's second law of motion with $a = 0.5 \text{ ms}^{-2}$ and $m = 3 \text{ kg}$:

$$T - \mu R = 3 \times 0.5$$

$$3\mu g = T - 1.5$$

$$\mu = \frac{18.6 - 1.5}{3 \times 9.8} = 0.58163\dots$$

The coefficient of friction is 0.582 (3 s.f.), as required.



10 c Consider P before string breaks: $u = 0 \text{ ms}^{-1}$, $t = 2 \text{ s}$, $a = 0.5 \text{ ms}^{-2}$, $v = ?$

$$v = u + at$$

$$v = 0 + (0.5 \times 2) = 1$$

After string breaks, the only force acting on P is a frictional force of magnitude $F = \mu R = 3\mu g$

Using Newton's Second Law for P ,

$$F = ma$$

$$3\mu g = 3a$$

$$a = \mu g$$

$$a = 9.8 \times 0.58163\dots$$

$$= 5.7$$

The acceleration is in the opposite direction to the motion of P , hence

$$u = 1 \text{ ms}^{-1}, v = 0 \text{ ms}^{-1}, a = -0.5 \text{ ms}^{-2}, t = ?$$

$$v = u + at$$

$$0 = 1 - 5.7t$$

$$t = \frac{1}{5.7} = 0.17543\dots$$

P takes 0.175 s (3 s.f.) to come to rest.

d The information that the string is inextensible has been used in assuming that the tension is the same in all parts of the string and that the acceleration of P and Q are identical while they are connected.

11 a The rod is in equilibrium so resultant force and moment are both zero.

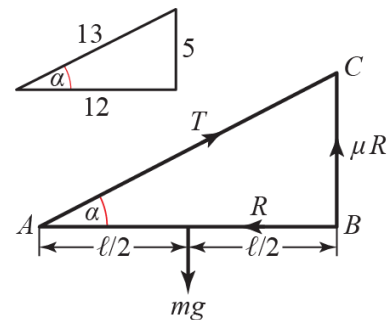
$$\tan \alpha = \frac{5}{12} \Rightarrow \sin \alpha = \frac{5}{13} \text{ and } \cos \alpha = \frac{12}{13}$$

Taking moments about B:

$$mg \frac{l}{2} = (T \sin \alpha) \times l$$

$$T = \frac{mg}{2 \sin \alpha}$$

$$T = \frac{mg}{2 \times \frac{5}{13}} = \frac{13mg}{10} \text{ as required.}$$



b Resolving horizontally:

$$R = T \cos \alpha$$

$$R = \frac{13mg}{10} \times \frac{12}{13} = \frac{6mg}{5}$$

Resolving vertically:

$$T \sin \alpha + \mu R = mg$$

$$\left(\frac{13mg}{10} \times \frac{5}{13} \right) + \mu \frac{6mg}{5} = mg$$

$$\frac{6}{5} \mu = 1 - \frac{1}{2}$$

$$\mu = \frac{5}{12}$$

The coefficient of friction between the rod and the wall is $\frac{5}{12}$.