

Differentiation 9F

1 a $y = \tan 3x$

Using the result

$$y = \tan kx \Rightarrow \frac{dy}{dx} = k \sec^2 kx$$

$$\frac{dy}{dx} = 3 \sec^2 3x$$

b $y = 4 \tan^3 x$

Let $u = \tan x$; then $y = 4u^3$

$$\frac{du}{dx} = \sec^2 x \quad \text{and} \quad \frac{dy}{du} = 12u^2$$

Using the chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = 12u^2 \sec^2 x \\ &= 12 \tan^2 x \sec^2 x \end{aligned}$$

c $y = \tan(x-1)$

$$\frac{dy}{dx} = \sec^2(x-1)$$

d $y = x^2 \tan \frac{1}{2}x + \tan\left(x - \frac{1}{2}\right)$

The first term is a product with

$$u = x^2 \quad \text{and} \quad v = \tan \frac{1}{2}x$$

$$\frac{du}{dx} = 2x \quad \text{and} \quad \frac{dv}{dx} = \frac{1}{2} \sec^2 \frac{1}{2}x$$

Using the product rule for the first term:

$$\begin{aligned} \frac{dy}{dx} &= x^2 \left(\frac{1}{2} \sec^2 \frac{1}{2}x \right) + \tan \frac{1}{2}x \times 2x \\ &\quad + \sec^2 \left(x - \frac{1}{2} \right) \\ &= \frac{1}{2} x^2 \sec^2 \frac{1}{2}x + 2x \tan \frac{1}{2}x \\ &\quad + \sec^2 \left(x - \frac{1}{2} \right) \end{aligned}$$

2 a $y = \cot 4x$

Let $u = 4x$; then $y = \cot u$

$$\frac{du}{dx} = 4 \quad \text{and} \quad \frac{dy}{du} = -\operatorname{cosec}^2 u$$

Using the chain rule,

$$\frac{dy}{dx} = -\operatorname{cosec}^2 u \times 4 = -4 \operatorname{cosec}^2 4x$$

b $y = \sec 5x$

Let $u = 5x$; then $y = \sec u$

$$\frac{du}{dx} = 5 \quad \text{and} \quad \frac{dy}{du} = \sec u \tan u$$

Using the chain rule,

$$\frac{dy}{dx} = 5 \sec u \tan u = 5 \sec 5x \tan 5x$$

c $y = \operatorname{cosec} 4x$

Let $u = 4x$; then $y = \operatorname{cosec} u$

$$\frac{du}{dx} = 4 \quad \text{and} \quad \frac{dy}{du} = -\operatorname{cosec} u \cot u$$

Using the chain rule,

$$\begin{aligned} \frac{dy}{dx} &= -4 \operatorname{cosec} u \cot u \\ &= -4 \operatorname{cosec} 4x \cot 4x \end{aligned}$$

d $y = \sec^2 3x = (\sec 3x)^2$

Let $u = \sec 3x$; then $y = u^2$

$$\frac{du}{dx} = 3 \sec 3x \tan 3x \quad \text{and} \quad \frac{dy}{du} = 2u$$

Using the chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 2u \times 3 \sec 3x \tan 3x \\ &= 2 \sec 3x \times 3 \sec 3x \tan 3x \\ &= 6 \sec^2 3x \tan 3x \end{aligned}$$

e $y = x \cot 3x$

This is a product, so let

$$u = x \quad \text{and} \quad v = \cot 3x$$

and use the product rule.

$$\frac{du}{dx} = 1 \quad \text{and} \quad \frac{dv}{dx} = -3 \operatorname{cosec}^2 3x$$

$$\begin{aligned} \frac{dy}{dx} &= x(-3 \operatorname{cosec}^2 3x) + \cot 3x \times 1 \\ &= \cot 3x - 3x \operatorname{cosec}^2 3x \end{aligned}$$

f $y = \frac{\sec^2 x}{x}$

This is a quotient, so let

$$u = \sec^2 x \quad \text{and} \quad v = x$$

and use the quotient rule.

$$\frac{du}{dx} = 2 \sec x (\sec x \tan x) \quad \text{and} \quad \frac{dv}{dx} = 1$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{x(2 \sec^2 x \tan x) - \sec^2 x \times 1}{x^2} \\ &= \frac{\sec^2 x (2x \tan x - 1)}{x^2} \end{aligned}$$

g $y = \operatorname{cosec}^3 2x$

Let $u = \operatorname{cosec} 2x$; then $y = u^3$

$$\frac{du}{dx} = -2 \operatorname{cosec} 2x \cot 2x \quad \text{and} \quad \frac{dy}{du} = 3u^2$$

Using the chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 3u^2 (-2 \operatorname{cosec} 2x \cot 2x) \\ &= -6 \operatorname{cosec}^2 2x \operatorname{cosec} 2x \cot 2x \\ &= -6 \operatorname{cosec}^3 2x \cot 2x \end{aligned}$$

h $y = \cot^2 (2x - 1)$

Let $u = \cot(2x - 1)$; then $y = u^2$

$$\frac{du}{dx} = -2 \operatorname{cosec}^2 (2x - 1) \quad \text{and} \quad \frac{dy}{du} = 2u$$

Using the chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 2u (-2 \operatorname{cosec}^2 (2x - 1)) \\ &= -4 \cot(2x - 1) \operatorname{cosec}^2 (2x - 1) \end{aligned}$$

3 a $f(x) = (\sec x)^{\frac{1}{2}}$

Using the chain rule,

$$\begin{aligned} f'(x) &= \frac{1}{2} (\sec x)^{-\frac{1}{2}} \times \sec x \tan x \\ &= \frac{1}{2} (\sec x)^{\frac{1}{2}} \tan x \end{aligned}$$

3 b $f(x) = \sqrt{\cot x} = (\cot x)^{\frac{1}{2}}$

Using the chain rule,

$$\begin{aligned} f'(x) &= \frac{1}{2}(\cot x)^{-\frac{1}{2}} \times (-\operatorname{cosec}^2 x) \\ &= -\frac{1}{2}(\cot x)^{-\frac{1}{2}} \operatorname{cosec}^2 x \end{aligned}$$

c $f(x) = \operatorname{cosec}^2 x = (\operatorname{cosec} x)^2$

Using the chain rule,

$$\begin{aligned} f'(x) &= 2(\operatorname{cosec} x)^1(-\operatorname{cosec} x \cot x) \\ &= -2 \operatorname{cosec}^2 x \cot x \end{aligned}$$

d $f(x) = \tan^2 x = (\tan x)^2$

Using the chain rule,

$$f'(x) = 2 \tan x \times \sec^2 x = 2 \tan x \sec^2 x$$

e $f(x) = \sec^3 x = (\sec x)^3$

Using the chain rule,

$$f'(x) = 3(\sec x)^2 \sec x \tan x = 3 \sec^3 x \tan x$$

f $f(x) = \cot^3 x = (\cot x)^3$

Using the chain rule,

$$\begin{aligned} f'(x) &= 3(\cot x)^2(-\operatorname{cosec}^2 x) \\ &= -3 \cot^2 x \operatorname{cosec}^2 x \end{aligned}$$

4 a $f(x) = x^2 \sec 3x$

Let $u = x^2$ and $v = \sec 3x$

$$\frac{du}{dx} = 2x \quad \text{and} \quad \frac{dv}{dx} = 3 \sec 3x \tan 3x$$

Using the product rule,

$$f'(x) = 3x^2 \sec 3x \tan 3x + 2x \sec 3x$$

b $f(x) = \frac{\tan 2x}{x}$

Let $u = \tan 2x$ and $v = x$

$$\frac{du}{dx} = 2 \sec^2 2x \quad \text{and} \quad \frac{dv}{dx} = 1$$

Using the quotient rule,

$$f'(x) = \frac{2x \sec^2 2x - \tan 2x}{x^2}$$

c $f(x) = \frac{x^2}{\tan x}$

Let $u = x^2$ and $v = \tan x$

$$\frac{du}{dx} = 2x \quad \text{and} \quad \frac{dv}{dx} = \sec^2 x$$

Using the quotient rule,

$$f'(x) = \frac{2x \tan x - x^2 \sec^2 x}{\tan^2 x}$$

d $f(x) = e^x \sec 3x$

Let $u = e^x$ and $v = \sec 3x$

$$\frac{du}{dx} = e^x \quad \text{and} \quad \frac{dv}{dx} = 3 \sec 3x \tan 3x$$

Using the product rule,

$$\begin{aligned} f'(x) &= 3e^x \sec 3x \tan 3x + e^x \sec 3x \\ &= e^x \sec 3x(3 \tan 3x + 1) \end{aligned}$$

e $f(x) = \frac{\ln x}{\tan x}$

Let $u = \ln x$ and $v = \tan x$

$$\frac{du}{dx} = \frac{1}{x} \quad \text{and} \quad \frac{dv}{dx} = \sec^2 x$$

Using the quotient rule,

$$\begin{aligned} f'(x) &= \frac{\left(\frac{1}{x}\right) \tan x - \ln x \sec^2 x}{\tan^2 x} \\ &= \frac{\tan x - x \ln x \sec^2 x}{x \tan^2 x} \end{aligned}$$

f $f(x) = \frac{e^{\tan x}}{\cos x}$

Let $u = e^{\tan x}$ and $v = \cos x$

$$\frac{du}{dx} = e^{\tan x} \sec^2 x \quad \text{and} \quad \frac{dv}{dx} = -\sin x$$

Using the quotient rule,

$$\begin{aligned} f'(x) &= \frac{e^{\tan x} \sec^2 x \cos x - e^{\tan x}(-\sin x)}{\cos^2 x} \\ &= \frac{e^{\tan x} \sec x + e^{\tan x} \sin x}{\cos^2 x} \\ &= \frac{e^{\tan x}(\sec x + \sin x)}{\cos^2 x} \\ &= e^{\tan x}(\sec^3 x + \sec x \tan x) \\ &= e^{\tan x} \sec x(\sec^2 x + \tan x) \end{aligned}$$

5 a $y = \frac{1}{\cos x \sin x} = \sec x \operatorname{cosec} x$

Let $u = \sec x$ and $v = \operatorname{cosec} x$

$$\frac{du}{dx} = \sec x \tan x \quad \text{and} \quad \frac{dv}{dx} = -\operatorname{cosec} x \cot x$$

Using the product rule,

$$\begin{aligned} \frac{dy}{dx} &= \sec x(-\operatorname{cosec} x \cot x) \\ &\quad + \operatorname{cosec} x(\sec x \tan x) \\ &= -\frac{\cos x}{\cos x \sin x \sin x} + \frac{\sin x}{\sin x \cos x \cos x} \\ &= -\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \end{aligned}$$

Alternative solution:

$$y = \frac{1}{\cos x \sin x} = \frac{2}{\sin 2x} = 2 \operatorname{cosec} 2x$$

(because $\sin 2x = 2 \sin x \cos x$)

$$\frac{dy}{dx} = -4 \operatorname{cosec} 2x \cot 2x$$

b At stationary points $\frac{dy}{dx} = 0$

$$\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = 0$$

$$\frac{1}{\cos^2 x} = \frac{1}{\sin^2 x}$$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

In the interval $0 < x \leq \pi$

there are two solutions, $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

So the number of stationary points is 2.

Alternative solution:

$$-4 \operatorname{cosec} 2x \cot 2x = 0$$

$$\operatorname{cosec} 2x \neq 0$$

but $\cot 2x = 0$ has two solutions

in the interval $0 < x \leq \pi$.

So there are 2 stationary points.

c When $x = \frac{\pi}{3}$,

$$y = \frac{1}{\cos \frac{\pi}{3} \sin \frac{\pi}{3}} = \frac{1}{\frac{1}{2} \times \frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

$$\frac{dy}{dx} = -\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\left(\frac{1}{2}\right)^2} = -\frac{4}{3} + 4 = \frac{8}{3}$$

or, using the alternative expression,

$$\frac{dy}{dx} = -4 \times \frac{2}{\sqrt{3}} \times \left(-\frac{1}{\sqrt{3}}\right) = \frac{8}{3}$$

Equation of tangent is

$$y - \frac{4\sqrt{3}}{3} = \frac{8}{3} \left(x - \frac{\pi}{3}\right)$$

$$3y - 4\sqrt{3} = 8x - \frac{8\pi}{3}$$

$$24x - 9y + 12\sqrt{3} - 8\pi = 0$$

This is in the required form $ax + by + c = 0$

With $a = 24$, $b = -9$ and $c = 12\sqrt{3} - 8\pi$.

6 $y = \sec x = \frac{1}{\cos x}$

Let $u = 1$ and $v = \cos x$

$$\frac{du}{dx} = 0 \quad \text{and} \quad \frac{dv}{dx} = -\sin x$$

Using the quotient rule,

$$\frac{dy}{dx} = \frac{\cos x \times 0 - 1 \times (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

7 $y = \cot x = \frac{1}{\tan x}$

Let $u = 1$ and $v = \tan x$

$$\frac{du}{dx} = 0 \quad \text{and} \quad \frac{dv}{dx} = \sec^2 x$$

Using the quotient rule,

$$\frac{dy}{dx} = \frac{\tan x \times 0 - 1 \times \sec^2 x}{\tan^2 x} = -\frac{\sec^2 x}{\tan^2 x}$$

$$= -\frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

8 a Let $y = \arccos x$

So $x = \cos y$

$$\frac{dx}{dy} = -\sin y$$

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{1 - x^2}}$$

b Let $y = \arctan x$

So $x = \tan y$

$$\frac{dx}{dy} = \sec^2 y$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

9 a Let $y = \arccos 2x$

Let $t = 2x$; then $y = \arccos t$

$$\frac{dt}{dx} = 2 \quad \text{and} \quad \frac{dy}{dt} = \frac{-1}{\sqrt{1 - t^2}}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-1}{\sqrt{1 - t^2}} \times 2$$

$$= \frac{-2}{\sqrt{1 - 4x^2}}$$

b Let $y = \arctan \frac{x}{2}$

Let $t = \frac{x}{2}$; then $y = \arctan t$

$$\frac{dt}{dx} = \frac{1}{2} \quad \text{and} \quad \frac{dy}{dt} = \frac{1}{1 + t^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{1}{1 + t^2} \times \frac{1}{2} = \frac{1}{2 \left(1 + \frac{x^2}{4} \right)} = \frac{1}{2 + \frac{x^2}{2}}$$

$$= \frac{2}{4 + x^2}$$

c Let $y = \arcsin 3x$

So $\sin y = 3x$

$$x = \frac{\sin y}{3}$$

$$\frac{dx}{dy} = \frac{\cos y}{3}$$

$$\frac{dy}{dx} = \frac{3}{\cos y}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - (3x)^2} = \sqrt{1 - 9x^2}$$

$$\therefore \frac{dy}{dx} = \frac{3}{\sqrt{1 - 9x^2}}$$

d Let $y = \operatorname{arccot} x$

So $x = \cot y$

$$\frac{dx}{dy} = -\operatorname{cosec}^2 y$$

$$\frac{dy}{dx} = \frac{-1}{\operatorname{cosec}^2 y}$$

$$= \frac{-1}{1 + \cot^2 y} = \frac{-1}{1 + x^2}$$

9 e Let $y = \operatorname{arcsec} x$

So $x = \sec y$

$$\frac{dx}{dy} = \sec y \tan y$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\tan y = \sqrt{\sec^2 y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$$

f Let $y = \operatorname{arccosec} x$

So $x = \operatorname{cosec} y$

$$\frac{dx}{dy} = -\operatorname{cosec} y \cot y$$

$$\frac{dy}{dx} = -\frac{1}{\operatorname{cosec} y \cot y}$$

$$1 + \cot^2 y = \operatorname{cosec}^2 y$$

$$\cot y = \sqrt{\operatorname{cosec}^2 y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{-1}{x\sqrt{x^2 - 1}}$$

g Let $y = \arcsin\left(\frac{x}{x-1}\right)$

$$\text{Let } u = \frac{x}{x-1}$$

$$\text{then } y = \arcsin u, \quad \frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$$

Using the quotient rule,

$$\frac{du}{dx} = \frac{(x-1) \times 1 - x \times 1}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times \frac{-1}{(x-1)^2}$$

$$= \frac{1}{\sqrt{1 - \frac{x^2}{(x-1)^2}}} \times \frac{-1}{(x-1)^2}$$

$$= \frac{1}{\sqrt{\frac{(x-1)^2 - x^2}{(x-1)^2}}} \times \frac{-1}{(x-1)^2}$$

$$= \frac{1}{\frac{\sqrt{1-2x}}{x-1}} \times \frac{-1}{(x-1)^2} = \frac{-1}{(x-1)\sqrt{1-2x}}$$

h Let $y = \arccos x^2$

Let $t = x^2$ and $y = \arccos t$

$$\frac{dt}{dx} = 2x \quad \text{and} \quad \frac{dy}{dt} = \frac{-1}{\sqrt{1-t^2}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\ &= \frac{-1}{\sqrt{1-t^2}} \times 2x = \frac{-2x}{\sqrt{1-x^4}} \end{aligned}$$

i Let $y = e^x \arccos x$

Using the product rule,

$$\begin{aligned} \frac{dy}{dx} &= e^x \left(\frac{-1}{\sqrt{1-x^2}} \right) + e^x \arccos x \\ &= e^x \left(\arccos x - \frac{1}{\sqrt{1-x^2}} \right) \end{aligned}$$

j Let $y = \arcsin x \cos x$

Using the product rule,

$$\begin{aligned} \frac{dy}{dx} &= \arcsin x (-\sin x) + \frac{1}{\sqrt{1-x^2}} \cos x \\ &= \frac{\cos x}{\sqrt{1-x^2}} - \sin x \arcsin x \end{aligned}$$

k Let $y = x^2 \arccos x$

Using the product rule,

$$\begin{aligned} \frac{dy}{dx} &= x^2 \times \frac{-1}{\sqrt{1-x^2}} + 2x \arccos x \\ &= 2x \arccos x - \frac{x^2}{\sqrt{1-x^2}} \\ &= x \left(2 \arccos x - \frac{x}{\sqrt{1-x^2}} \right) \end{aligned}$$

9 1 Let $y = e^{\arctan x}$

Let $u = \arctan x$; then $y = e^u$

$$\frac{dy}{du} = e^u \quad \text{and} \quad \frac{du}{dx} = \frac{1}{1+x^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{e^u}{1+x^2} = \frac{e^{\arctan x}}{1+x^2} \end{aligned}$$

10 a $y = \frac{\arctan 2x}{x}$

Let $u = \arctan 2x$ and $v = x$

$$\frac{du}{dx} = \frac{2}{1+4x^2} \quad \text{and} \quad \frac{dv}{dx} = 1$$

Using the quotient rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{x \left(\frac{2}{1+4x^2} \right) - 1 \times \arctan 2x}{x^2} \\ &= \frac{2}{x(1+4x^2)} - \frac{\arctan 2x}{x^2} \end{aligned}$$

When $x = \frac{\sqrt{3}}{2}$,

$$\begin{aligned} \frac{dy}{dx} &= \frac{2}{\frac{\sqrt{3}}{2} \left(1 + 4 \left(\frac{\sqrt{3}}{2} \right)^2 \right)} - \frac{\arctan \sqrt{3}}{\left(\frac{\sqrt{3}}{2} \right)^2} \\ &= \frac{2}{\frac{\sqrt{3}}{2} \times 4} - \frac{\frac{\pi}{3}}{\frac{3}{4}} \\ &= \frac{1}{\sqrt{3}} - \frac{4\pi}{9} = \frac{3\sqrt{3} - 4\pi}{9} \end{aligned}$$

b When $x = \frac{\sqrt{3}}{2}$,

$$y = \frac{\arctan \sqrt{3}}{\frac{\sqrt{3}}{2}} = \frac{\frac{\pi}{3}}{\frac{\sqrt{3}}{2}} = \frac{2\pi}{3\sqrt{3}} = \frac{2\sqrt{3}\pi}{9}$$

Equation of normal is

$$y - \frac{2\sqrt{3}\pi}{9} = \left(\frac{-9}{3\sqrt{3} - 4\pi} \right) \left(x - \frac{\sqrt{3}}{2} \right)$$

$$y = - \left(\frac{9}{3\sqrt{3} - 4\pi} \right) x + \frac{9\sqrt{3}}{6\sqrt{3} - 8\pi} + \frac{2\sqrt{3}\pi}{9}$$

11 $x = (\arccos y)^2$

$$\sqrt{x} = \arccos y$$

$$y = \cos(\sqrt{x})$$

Let $u = \sqrt{x} = x^{\frac{1}{2}}$; then $y = \cos u$

$$\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad \text{and} \quad \frac{dy}{du} = -\sin u$$

Using the chain rule,

$$\frac{dy}{dx} = -\sin u \times \frac{1}{2\sqrt{x}} = -\frac{\sin \sqrt{x}}{2\sqrt{x}}$$

$$\sin^2 \sqrt{x} + \cos^2 \sqrt{x} = 1$$

$$\sin \sqrt{x} = \sqrt{1 - \cos^2 \sqrt{x}}$$

$$\therefore \frac{dy}{dx} = -\frac{\sqrt{1 - \cos^2 \sqrt{x}}}{2\sqrt{x}}$$

12 a $x = \operatorname{cosec} 5y$

$$\frac{dx}{dy} = -5 \operatorname{cosec} 5y \cot 5y$$

$$\frac{dy}{dx} = -\frac{1}{5 \operatorname{cosec} 5y \cot 5y}$$

b $1 + \cot^2 5y = \operatorname{cosec}^2 5y$

$$\cot 5y = \sqrt{\operatorname{cosec}^2 5y - 1} = \sqrt{x^2 - 1}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{5x\sqrt{x^2 - 1}}$$