Trigonometric Functions 6A

1 a 300° is in the 4th quadrant

$\sec 300° = \dfrac{1}{\cos 300°}$

In 4th quadrant cos is +ve,

so $\sec 300°$ is +ve.

b 190° is in the 3rd quadrant

$\text{cosec}\,190° = \dfrac{1}{\sin 190°}$

In 3rd quadrant sin is $-$ ve,

so $\text{cosec}\,190°$ is $-$ ve.

c 110° is in the 2nd quadrant

$\cot 110° = \dfrac{1}{\tan 110°}$

In the 2nd quadrant tan is $-$ ve,

so $\cot 110°$ is $-$ ve.

d 200° is in the 3rd quadrant

tan is +ve in the 3rd quadrant,

so cot 200° is+ve.

e 95° is in the 2nd quadrant

cos is $-$ ve in the 2nd quadrant,

so sec 95° is $-$ ve.

2 a $\sec 100° = \dfrac{1}{\cos 100°} = -5.76$ (3 s.f.)

b $\text{cosec}\,260° = \dfrac{1}{\sin 260°} = -1.02$ (3 s.f.)

c $\text{cosec}\,280° = \dfrac{1}{\sin 280°} = -1.02$ (3 s.f.)

d $\cot 550° = \dfrac{1}{\tan 550°} = 5.67$ (3 s.f.)

e $\cot \dfrac{4p}{3} = \dfrac{1}{\tan \frac{4p}{3}} = 0.577$ (3 s.f.)

f $\sec 2.4\,\text{rad} = \dfrac{1}{\cos 2.4\,\text{rad}} = -1.36$ (3 s.f.)

g $\text{cosec}\dfrac{11p}{10} = \dfrac{1}{\sin \frac{11p}{10}} = -3.24$ (3 s.f.)

h $\sec 6\,\text{rad} = \dfrac{1}{\cos 6\,\text{rad}} = 1.04$ (3 s.f.)

3 a $\text{cosec}\,90° = \dfrac{1}{\sin 90°} = \dfrac{1}{1} = 1$

(refer to graph of $y = \sin q$)

b $\cot 135° = \dfrac{1}{\tan 135°} = \dfrac{1}{-\tan 45°} = \dfrac{1}{-1} = -1$

c $\sec 180° = \dfrac{1}{\cos 180°} = \dfrac{1}{-1} = -1$

(refer to graph of $y = \cos q$)

d 240° is in the 3rd quadrant

$\sec 240° = \dfrac{1}{\cos 240°} = \dfrac{1}{-\cos 60°} = \dfrac{1}{-\frac{1}{2}} = -2$

e 300° is in the 4th quadrant

$\text{cosec}\,300° = \dfrac{1}{\sin 300°} = \dfrac{1}{-\sin 60°}$

$= \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\dfrac{2}{\sqrt{3}} = -\dfrac{2\sqrt{3}}{3}$

f $-45°$ is in the 4th quadrant

$\cot(-45°) = \dfrac{1}{\tan(-45°)} = \dfrac{1}{-\tan 45°}$

$= \dfrac{1}{-1} = -1$

g $\sec 60° = \dfrac{1}{\cos 60°} = \dfrac{1}{\frac{1}{2}} = 2$

h $-210°$ is in the 2nd quadrant

$\text{cosec}(-210°) = \dfrac{1}{\sin(-210°)}$

$= \dfrac{1}{\sin 30°} = \dfrac{1}{\frac{1}{2}} = 2$

3 i 225° is in the 3rd quadrant

$$\sec 225° = \frac{1}{\cos 225°} = \frac{1}{-\cos 45°}$$
$$= \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

j $\frac{4\pi}{3}$ is in the 3rd quadrant

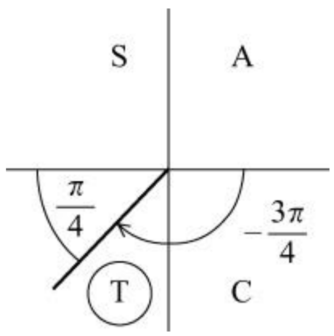
$$\cot \tfrac{4\pi}{3} = \frac{1}{\tan \frac{4\pi}{3}} = \frac{1}{\tan \frac{\pi}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

k $\frac{11\pi}{6} = 2\pi - \frac{\pi}{6}$ (in the 4th quadrant)

$$\sec \frac{11\pi}{6} = \frac{1}{\cos \frac{11\pi}{6}} = \frac{1}{\cos \frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

l $-\frac{3\pi}{4}$ is in the 3rd quadrant

$$\operatorname{cosec}\left(-\frac{3\pi}{4}\right) = \frac{1}{\sin\left(-\frac{3\pi}{4}\right)} = \frac{1}{-\sin \frac{\pi}{4}}$$
$$= \frac{1}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}$$



4 $\operatorname{cosec}(\pi - x) \equiv \frac{1}{\sin(\pi - x)}$
$$\equiv \frac{1}{\sin x}$$
$$\equiv \operatorname{cosec} x$$

5 $\cot 30° \sec 30° = \frac{1}{\tan 30°} \times \frac{1}{\cos 30°}$
$$= \frac{\sqrt{3}}{1} \times \frac{2}{\sqrt{3}}$$
$$= 2$$

6 $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$ (in the 2nd quadrant)

$$\operatorname{cosec}\left(\frac{2\pi}{3}\right) + \sec\left(\frac{2\pi}{3}\right) = \frac{1}{\sin\left(\frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\frac{2\pi}{3}\right)}$$
$$= \frac{1}{\sin\left(\frac{\pi}{3}\right)} + \frac{1}{-\cos\left(\frac{\pi}{3}\right)}$$
$$= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{1}{2}}$$
$$= -2 + \frac{2}{\sqrt{3}}$$
$$= -2 + \frac{2}{3}\sqrt{3}$$

Challenge

a Triangles *OPB* and *OAP* are right-angled triangles as line *AB* is a tangent to the unit circle at *P*.

Using triangle *OBP*, $OB\cos\theta = 1$

$$\Rightarrow OB = \frac{1}{\cos\theta} = \sec\theta$$

b $\angle POA = 90° - \theta \Rightarrow \angle OAP = \theta$

Using triangle *OAP*, $OA\sin\theta = 1$

$$\Rightarrow OA = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

c Using Pythagoras' theorem,

$AP^2 = OA^2 - OP^2$

So, $AP^2 = \operatorname{cosec}^2\theta - 1$
$$= \frac{1}{\sin^2\theta} - 1$$
$$= \frac{1 - \sin^2\theta}{\sin^2\theta}$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \cot^2\theta$$

Therefore $AP = \cot\theta$