Binomial expansion 4B

1 a i $\sqrt{(4+2x)}$ Write in index form.

$=(4+2x)^{\frac{1}{2}}$ Take out a factor of 4

$=\left(4\left(1+\frac{2x}{4}\right)\right)^{\frac{1}{2}}$ Remember to put the 4 to the power $\frac{1}{2}$

$=4^{\frac{1}{2}}\left(1+\frac{x}{2}\right)^{\frac{1}{2}}$ $4^{\frac{1}{2}}=2$

$=2\left(1+\frac{x}{2}\right)^{\frac{1}{2}}$ Use the expansion with $n=\frac{1}{2}$ and $x=\frac{x}{2}$

$$=2\left(1+\left(\frac{1}{2}\right)\left(\frac{x}{2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{x}{2}\right)^2}{2!}+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{x}{2}\right)^3}{3!}+\ldots\right)$$

$=2\left(1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\ldots\right)$ Multiply by the 2

$=2+\frac{x}{2}-\frac{x^2}{16}+\frac{x^3}{64}+\ldots$

ii Valid if $\left|\frac{x}{2}\right|<1\Rightarrow|x|<2$

b i $\frac{1}{2+x}$ Write in index form

$=(2+x)^{-1}$ Take out a factor of 2

$=\left(2\left(1+\frac{x}{2}\right)\right)^{-1}$ Remember to put 2 to the power -1

$=2^{-1}\left(1+\frac{x}{2}\right)^{-1}$, $2^{-1}=\frac{1}{2}$. Use the binomial expansion with $n=-1$ and $x=\frac{x}{2}$

$$=\frac{1}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-1)(-2)(-3)}{3!}\left(\frac{x}{2}\right)^3+\ldots\right)$$

$=\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\ldots\right)$ Multiply by the $\frac{1}{2}$

$=\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}+\ldots$

ii Valid if $\left|\frac{x}{2}\right|<1\Rightarrow|x|<2$

1 c i $\frac{1}{(4-x)^2}$ Write in index form

$=(4-x)^{-2}$ Take 4 out as a factor

$=\left(4\left(1-\frac{x}{4}\right)\right)^{-2}$

$=4^{-2}\left(1-\frac{x}{4}\right)^{-2}$, $4^{-2}=\frac{1}{16}$. Use the binomial expansion with $n=-2$ and $x=-\frac{x}{4}$

$=\frac{1}{16}\left(1+(-2)\left(-\frac{x}{4}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{x}{4}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{x}{4}\right)^3+\ldots\right)$

$=\frac{1}{16}\left(1+\frac{x}{2}+\frac{3x^2}{16}+\frac{x^3}{16}+\ldots\right)$ Multiply by $\frac{1}{16}$

$=\frac{1}{16}+\frac{x}{32}+\frac{3x^2}{256}+\frac{x^3}{256}+\ldots$

ii Valid if $\left|\frac{x}{4}\right|<1\Rightarrow|x|<4$

d i $\sqrt{9+x}$ Write in index form

$=(9+x)^{\frac{1}{2}}$ Take 9 out as a factor

$=\left(9\left(1+\frac{x}{9}\right)\right)^{\frac{1}{2}}$

$=9^{\frac{1}{2}}\left(1+\frac{x}{9}\right)^{\frac{1}{2}}$, $9^{\frac{1}{2}}=3$. Use binomial expansion with $n=\frac{1}{2}$ and $x=\frac{x}{9}$

$=3\left(1+\left(\frac{1}{2}\right)\left(\frac{x}{9}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{9}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{9}\right)^3+\ldots\right)$

$=3\left(1+\frac{x}{18}-\frac{x^2}{648}+\frac{x^3}{11\,664}+\ldots\right)$ Multiply by 3

$=3+\frac{x}{6}-\frac{x^2}{216}+\frac{x^3}{3888}+\ldots$

ii Valid for $\left|\frac{x}{9}\right|<1\Rightarrow|x|<9$

1 e i $\dfrac{1}{\sqrt{2+x}}$ Write in index form

$= (2+x)^{-\frac{1}{2}}$ Take out a factor of 2

$= \left(2\left(1+\dfrac{x}{2}\right)\right)^{-\frac{1}{2}}$

$= 2^{-\frac{1}{2}}\left(1+\dfrac{x}{2}\right)^{-\frac{1}{2}}$, $2^{-\frac{1}{2}} = \dfrac{1}{2^{\frac{1}{2}}} = \dfrac{1}{\sqrt{2}}$. Use binomial expansion with $n = -\dfrac{1}{2}$ and $x = \dfrac{x}{2}$

$$= \frac{1}{\sqrt{2}}\left(1+\left(-\frac{1}{2}\right)\left(\frac{x}{2}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(\frac{x}{2}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{x}{2}\right)^3+\ldots\right)$$

$= \dfrac{1}{\sqrt{2}}\left(1-\dfrac{x}{4}+\dfrac{3x^2}{32}-\dfrac{5x^3}{128}+\ldots\right)$ Multiply by $\dfrac{1}{\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}-\dfrac{x}{4\sqrt{2}}+\dfrac{3x^2}{32\sqrt{2}}-\dfrac{5x^3}{128\sqrt{2}}+\ldots$ Rationalise surds

$= \dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}x}{8}+\dfrac{3\sqrt{2}x^2}{64}-\dfrac{5\sqrt{2}x^3}{256}+\ldots$

ii Valid if $\left|\dfrac{x}{2}\right| < 1 \Rightarrow |x| < 2$

f i $\dfrac{5}{3+2x}$ Write in index form

$= 5(3+2x)^{-1}$ Take out a factor of 3

$= 5\left(3\left(1+\dfrac{2x}{3}\right)\right)^{-1}$

$= 5\times 3^{-1}\left(1+\dfrac{2x}{3}\right)^{-1}$, $3^{-1} = \dfrac{1}{3}$. Use binomial expansion with $n = -1$ and $x = \dfrac{2x}{3}$

$$= \frac{5}{3}\left(1+(-1)\left(\frac{2x}{3}\right)+\frac{(-1)(-2)}{2!}\left(\frac{2x}{3}\right)^2+\frac{(-1)(-2)(-3)}{3!}\left(\frac{2x}{3}\right)^3+\ldots\right)$$

$= \dfrac{5}{3}\left(1-\dfrac{2x}{3}+\dfrac{4x^2}{9}-\dfrac{8x^3}{27}+\ldots\right)$ Multiply by $\dfrac{5}{3}$

$= \dfrac{5}{3}-\dfrac{10x}{9}+\dfrac{20x^2}{27}-\dfrac{40x^3}{81}+\ldots$

ii Valid if $\left|\dfrac{2x}{3}\right| < 1 \Rightarrow |x| < \dfrac{3}{2}$

1 g i

$$\frac{1+x}{2+x}=1-\frac{1}{2+x}$$ Write $\frac{1}{2+x}$ in index form

$$=1-(2+x)^{-1}$$ Take out a factor of 2

$$=1-\left(2\left(1+\frac{x}{2}\right)\right)^{-1}$$

$$=1-\left(2^{-1}\left(1+\frac{x}{2}\right)^{-1}\right)$$ Expand $\left(1+\frac{x}{2}\right)^{-1}$ using the binomial expansion

$$=1-\left(\frac{1}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2!}\left(\frac{x}{2}\right)^2+\frac{(-1)(-2)(-3)}{3!}\left(\frac{x}{2}\right)^3+...\right)\right)$$

$$=1-\left(\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\ldots\right)\right)$$ Multiply $\left(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\ldots\right)$ by $\frac{1}{2}$

$$=1-\left(\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}+\ldots\right)$$

$$=\frac{1}{2}+\frac{1}{4}x-\frac{1}{8}x^2+\frac{1}{16}x^3+\ldots$$

ii Valid for $\left|\frac{x}{2}\right|<1\Rightarrow|x|<2$

1 h i $\sqrt{\dfrac{2+x}{1-x}}$

$= (2+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ Put both in index form

$= 2^{\frac{1}{2}}\left(1+\dfrac{x}{2}\right)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ Expand both using the binomial expansion

$$= \sqrt{2}\left(1+\left(\frac{1}{2}\right)\left(\frac{x}{2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{2}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{x}{2}\right)^3+\dots\right)$$

$$\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-x)^3}{3!}+\dots\right)$$

$= \sqrt{2}\left(1+\dfrac{1}{4}x-\dfrac{1}{32}x^2+\dfrac{1}{128}x^3+\dots\right)\left(1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+\dfrac{5}{16}x^3+\dots\right)$ Multiply out

$$= \sqrt{2}\left(\begin{array}{l}1\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{15}{16}x^3+\dots\right)+\frac{1}{4}x\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\dots\right)\\ -\frac{1}{32}x^2\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\dots\right)+\frac{1}{128}x^3\left(1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\dots\right)+\dots\end{array}\right)$$

$$= \sqrt{2}\left(\begin{array}{l}1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\frac{1}{4}x+\frac{1}{8}x^2\\ +\frac{3}{32}x^3-\frac{1}{32}x^2-\frac{1}{64}x^3+\frac{1}{128}x^3+\dots\end{array}\right)$$

Collect like terms

$= \sqrt{2}\left(1+\dfrac{3}{4}x+\dfrac{15}{32}x^2+\dfrac{51}{128}x^3+\dots\right)$ Multiply by $\sqrt{2}$

$$= \sqrt{2}+\frac{3\sqrt{2}}{4}x+\frac{15\sqrt{2}}{32}x^2+\frac{51\sqrt{2}}{128}x^3+\dots$$

ii Valid if $\left|\dfrac{x}{2}\right|<1$ and $|-x|<1 \Rightarrow |x|<1$ for both to be valid.

2 $(5+4x)^{-2} = \left(5\left(1+\frac{4}{5}x\right)\right)^{-2} = 5^{-2}\left(1+\frac{4}{5}x\right)^{-2} = \frac{1}{25}\left(1+\frac{4}{5}x\right)^{-2}$

$$= \frac{1}{25}\left(1+(-2)\left(\frac{4}{5}x\right)+\frac{(-2)(-3)}{2!}\left(\frac{4}{5}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{4}{5}x\right)^3+\ldots\right)$$

$$= \frac{1}{25}\left(1+(-2)\left(\frac{4}{5}x\right)+\frac{(-2)(-3)}{2}\frac{16}{25}x^2+\frac{(-2)(-3)(-4)}{6}\frac{64}{125}x^3+\ldots\right)$$

$$= \frac{1}{25}\left(1-\frac{8}{5}x+\frac{48}{25}x^2-\frac{256}{125}x^3+\ldots\right)$$

$$= \frac{1}{25}-\frac{8}{125}x+\frac{48}{625}x^2-\frac{256}{3125}x^3+\ldots$$

3 a $\sqrt{(4-x)} = (4-x)^{\frac{1}{2}}$

$$= \left[4\left(1-\frac{x}{4}\right)\right]^{\frac{1}{2}}$$

$$= 4^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$$

$$= 2\left[1+\left(\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{4}\right)^3+\ldots\right]$$

$$= 2\left(1-\frac{x}{8}-\frac{x^2}{128}-\frac{x^3}{1024}+\ldots\right)$$

$$= 2-\frac{x}{4}-\frac{x^2}{64}-\frac{x^3}{512}+\ldots$$

Valid for $\left|-\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$

b Substitute $x = \frac{1}{9}$ into both sides of the expansion:

$$\sqrt{\left(4-\frac{1}{9}\right)} \simeq 2-\frac{\frac{1}{9}}{4}-\frac{\left(\frac{1}{9}\right)^2}{64}-\frac{\left(\frac{1}{9}\right)^3}{512}$$

$$\sqrt{\frac{35}{9}} \simeq 2-\frac{1}{36}-\frac{1}{5184}-\frac{1}{373248}$$

$$\frac{\sqrt{35}}{3} \simeq \frac{736055}{373248}$$

3 c $\mathrm{m}(x) \approx 2-\frac{1}{4}x-\frac{1}{64}x^2$

$$\mathrm{m}\left(\frac{1}{9}\right)=\frac{\sqrt{35}}{3}$$

$$\sqrt{35}=3\mathrm{m}\left(\frac{1}{9}\right)$$

$$\approx 3\left(2-\frac{1}{4}\left(\frac{1}{9}\right)-\frac{1}{64}\left(\frac{1}{9}\right)^2\right)$$

$$\approx 3\left(2-\frac{1}{36}-\frac{1}{5184}\right)$$

$$\approx 5.916087963$$

$$\sqrt{35}=5.916079783$$

Percentage error = $\frac{5.916087963-5.916079783}{5.916079783}\times 100=0.000138\%$

4 a $\frac{1}{\sqrt{a+bx}}=(a+bx)^{-\frac{1}{2}}=\left(a\left(1+\frac{b}{a}x\right)\right)^{-\frac{1}{2}}=a^{-\frac{1}{2}}\left(1+\frac{b}{a}x\right)^{-\frac{1}{2}}=\frac{1}{a^{\frac{1}{2}}}\left(1+\frac{b}{a}x\right)^{-\frac{1}{2}}$

$$=\frac{1}{a^{\frac{1}{2}}}\left(1+\left(-\frac{1}{2}\right)\left(\frac{b}{a}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{b}{a}x\right)^2+\ldots\right)$$

$$=\frac{1}{a^{\frac{1}{2}}}\left(1+\left(-\frac{1}{2}\right)\left(\frac{b}{a}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{b^2}{a^2}x^2+\ldots\right)$$

$$=\frac{1}{a^{\frac{1}{2}}}\left(1-\frac{b}{2a}x+\frac{3b^2}{8a^2}x^2+\ldots\right)$$

$$=\frac{1}{a^{\frac{1}{2}}}-\frac{b}{2a^{\frac{3}{2}}}x+\frac{3b^2}{8a^{\frac{5}{2}}}x^2+\ldots=3+\frac{1}{3}x+\frac{1}{18}x^2+\ldots$$

Equating coefficients gives $\frac{1}{a^{\frac{1}{2}}}=3$, so $a=\frac{1}{9}$

and $-\frac{b}{2\left(\frac{1}{9}\right)^{\frac{3}{2}}}=\frac{1}{3}$

$$-\frac{b}{\frac{2}{27}}=\frac{1}{3}$$

$$b=-\frac{2}{81}$$

$$a=\frac{1}{9},\ b=-\frac{2}{81}$$

4 b x^3 term of $3\left(1-\frac{2}{9}x\right)^{-\frac{1}{2}} = 3\left(\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{2}{9}x\right)^3\right)$

$$= -3\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6}\frac{8}{729}x^3$$

$$= \frac{5}{486}x^3$$

Coefficient $= \frac{5}{486}$

5 $\frac{3+2x-x^2}{4-x} \equiv (3+2x-x^2)(4-x)^{-1}$ Write $\frac{1}{4-x}$ as $(4-x)^{-1}$

$= (3+2x-x^2)\left(4\left(1-\frac{x}{4}\right)\right)^{-1}$ Take out a factor of 4

$= (3+2x-x^2)\frac{1}{4}\left(1-\frac{x}{4}\right)^{-1}$ Expand $\left(1-\frac{x}{4}\right)^{-1}$ using the binomial expansion

$= (3+2x-x^2)\frac{1}{4}\left(1+(-1)\left(-\frac{x}{4}\right)+\frac{(-1)(-2)}{2!}\left(-\frac{x}{4}\right)^2+\ldots\right)$ Ignore terms higher than x^2

$= (3+2x-x^2)\frac{1}{4}\left(1+\frac{x}{4}+\frac{x^2}{16}+\ldots\right)$ Multiply expansion by $\frac{1}{4}$

$= (3+2x-x^2)\left(\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}+\ldots\right)$ Multiply result by $(3+2x-x^2)$

$$= 3\left(\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}+\ldots\right)+2x\left(\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}+\ldots\right)-x^2\left(\frac{1}{4}+\frac{x}{16}+\frac{x^2}{64}+\ldots\right)$$

$= \frac{3}{4}+\frac{3}{16}x+\frac{3}{64}x^2+\frac{1}{2}x+\frac{1}{8}x^2-\frac{1}{4}x^2+\ldots$ Ignore any terms bigger than x^2

$$= \frac{3}{4}+\frac{11}{16}x-\frac{5}{64}x^2$$

Expansion is valid if $\left|\frac{-x}{4}\right| < 1 \Rightarrow |x| < 4$

6 a $\dfrac{1}{\sqrt{5+2x}}=(5+2x)^{-\frac{1}{2}}=5^{-\frac{1}{2}}\left(1+\dfrac{2}{5}x\right)^{-\frac{1}{2}}=\dfrac{1}{\sqrt{5}}\left(1+\dfrac{2}{5}x\right)^{-\frac{1}{2}}$

$$=\frac{1}{\sqrt{5}}\left(1+\left(-\frac{1}{2}\right)\left(\frac{2}{5}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{2}{5}x\right)^2+\ldots\right)$$

$$=\frac{1}{\sqrt{5}}\left(1+\left(-\frac{1}{2}\right)\left(\frac{2}{5}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\frac{4}{25}x^2+\ldots\right)$$

$$=\frac{1}{\sqrt{5}}\left(1-\frac{1}{5}x+\frac{3}{50}x^2+\ldots\right)$$

$$=\frac{1}{\sqrt{5}}-\frac{1}{5\sqrt{5}}x+\frac{3}{50\sqrt{5}}x^2+\ldots$$

b $\dfrac{2x-1}{\sqrt{5+2x}}=\dfrac{2x-1}{\sqrt{5}}\left(1+\dfrac{2}{5}x\right)^{-\frac{1}{2}}$

$$=(2x-1)\left(\frac{1}{\sqrt{5}}-\frac{1}{5\sqrt{5}}x+\frac{3}{50\sqrt{5}}x^2+\ldots\right)$$

$$=\frac{2}{\sqrt{5}}x-\frac{2}{5\sqrt{5}}x^2-\frac{1}{\sqrt{5}}+\frac{1}{5\sqrt{5}}x-\frac{3}{50\sqrt{5}}x^2+\ldots$$

$$=-\frac{1}{\sqrt{5}}+\frac{11}{5\sqrt{5}}x-\frac{23}{50\sqrt{5}}x^2+\ldots$$

7 a $(16-3x)^{\frac{1}{4}}=\left(16\left(1-\dfrac{3}{16}x\right)\right)^{\frac{1}{4}}=2\left(1-\dfrac{3}{16}x\right)^{\frac{1}{4}}$

$$=2\left(1+\left(\frac{1}{4}\right)\left(-\frac{3}{16}x\right)+\frac{\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)}{2!}\left(-\frac{3}{16}x\right)^2+\ldots\right)$$

$$=2\left(1+\left(\frac{1}{4}\right)\left(-\frac{3}{16}x\right)+\frac{\left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)}{2}\frac{9}{256}x^2+\ldots\right)$$

$$=2\left(1-\frac{3}{64}x-\frac{27}{8192}x^2+\ldots\right)$$

$$=2-\frac{3}{32}x-\frac{27}{4096}x^2+\ldots$$

7 b Let $x = 0.1$

$$\sqrt[4]{15.7} \approx 2 - \frac{3}{32}(0.1) - \frac{27}{4096}(0.1)^2$$
$$= 1.991$$

8 a
$$\frac{3}{4-2x} = 3(4-2x)^{-1} = 3\left(4\left(1-\frac{1}{2}x\right)\right)^{-1} = \frac{3}{4}\left(1-\frac{1}{2}x\right)^{-1}$$
$$= \frac{3}{4}\left(1+(-1)\left(-\frac{1}{2}x\right)+\frac{(-1)(-2)}{2!}\left(-\frac{1}{2}x\right)^2+\ldots\right)$$
$$= \frac{3}{4}\left(1-\left(-\frac{1}{2}x\right)+\frac{1}{4}x^2+\ldots\right)$$
$$= \frac{3}{4}+\frac{3}{8}x+\frac{3}{16}x^2+\ldots$$
$$\frac{2}{3+5x} = 2(3+5x)^{-1} = 2\left(3\left(1+\frac{5}{3}x\right)\right)^{-1} = \frac{2}{3}\left(1+\frac{5}{3}x\right)^{-1}$$
$$= \frac{2}{3}\left(1+(-1)\left(\frac{5}{3}x\right)+\frac{(-1)(-2)}{2!}\left(\frac{5}{3}x\right)^2+\ldots\right)$$
$$= \frac{2}{3}\left(1-\frac{5}{3}x+\frac{25}{9}x^2+\ldots\right)$$
$$= \frac{2}{3}-\frac{10}{9}x+\frac{50}{27}x^2+\ldots$$
$$\frac{3}{4-2x}-\frac{2}{3+5x}$$
$$= \frac{3}{4}+\frac{3}{8}x+\frac{3}{16}x^2+\ldots-\left(\frac{2}{3}-\frac{10}{9}x+\frac{50}{27}x^2+\ldots\right)$$
$$= \frac{1}{12}+\frac{107}{72}x-\frac{719}{432}x^2+\ldots$$

b $\mathrm{g}(0.01) = \dfrac{3}{4-2(0.01)} - \dfrac{2}{3+5(0.01)} = 0.0980311$

c Using the series expansion:

$$\mathrm{g}(0.01) \approx \frac{1}{12}+\frac{107}{72}(0.01)-\frac{719}{432}(0.01)^2 = 0.098028009$$

Percentage error = $\dfrac{0.0980311-0.098028009}{0.098028009}\times 100 = 0.0032\%$