

Exercise 4A

$$1 \text{ a } 5 \text{ rev min}^{-1} = 5 \times 2\pi \text{ rad min}^{-1} = \frac{5 \times 2\pi}{60} \text{ rad s}^{-1} \approx 0.524 \text{ rad s}^{-1}$$

$$\text{b } 120 \text{ rev min}^{-1} = 120 \times 2\pi \text{ rad min}^{-1} = \frac{120 \times 2\pi}{60} \text{ rad s}^{-1} \approx 12.6 \text{ rad s}^{-1}$$

$$\text{c } 4 \text{ rad s}^{-1} = 4 \times 60 \text{ rad min}^{-1} = \frac{4 \times 60}{2\pi} \text{ rev min}^{-1} \approx 38.2 \text{ rev min}^{-1}$$

$$\text{d } 3 \text{ rad s}^{-1} = 3 \times 60 \times 60 \text{ rad h}^{-1} = \frac{3 \times 60 \times 60}{2\pi} \text{ rev h}^{-1} \approx 1720 \text{ rev h}^{-1}$$

$$2 \text{ a } v = r\omega : v = 20 \times 4 = 80 \text{ ms}^{-1}$$

$$\text{b } \text{Distance per minute} = 40 \times 2\pi \times 20 = 1600\pi \text{ m}$$

$$\text{Distance per second} = \frac{1600\pi}{60} \approx 83.8 \text{ m}, v = 83.8 \text{ ms}^{-1}$$

$$3 \text{ a } \omega = \frac{v}{r} = \frac{2}{0.25} = 8 \text{ rad s}^{-1}$$

← Need to convert cm to m.

$$\text{b } 8 \text{ rad s}^{-1} = 8 \times 60 \text{ rad min}^{-1} = \frac{8 \times 60}{2\pi} \approx 76.4 \text{ rev min}^{-1}$$

$$4 \text{ a } v = r\omega : v = 0.8 \times 2.5 = 2 \text{ ms}^{-1}$$

$$\text{b } 25 \text{ rev min}^{-1} = 25 \times 2\pi \text{ rad min}^{-1} = \frac{25 \times 2\pi}{60} \text{ rad s}^{-1} = 2.617... \text{ rad s}^{-1}$$

$$v = r\omega : v = 0.8 \times 2.617... \approx 2.09 \text{ ms}^{-1}$$

$$5 \text{ a } \text{time} = \frac{\text{distance}}{\text{speed}} = \frac{2\pi \times 50}{7} \approx 44.9 \text{ s}$$

$$\text{b } \omega = \frac{v}{r} = \frac{7}{50} = 0.14 \text{ rad s}^{-1}$$

$$6 \text{ a } 1 \text{ rev in } 10 \text{ s} = 0.1 \text{ rev s}^{-1} = 0.1 \times 2\pi \text{ rad s}^{-1} \approx 0.628 \text{ rad s}^{-1}$$

$$\text{b } v = r\omega : v = 0.12 \times 0.628... \approx 0.0754 \text{ ms}^{-1}$$

$$\text{c } v = r\omega : v = 0.08 \times 0.628... \approx 0.0503 \text{ ms}^{-1}$$

$$7 \text{ a } 2 \text{ circuits} = 2 \times 2\pi \text{ radians in } 45 \text{ seconds} = \frac{4\pi}{45} \text{ rad s}^{-1} \approx 0.279 \text{ rad s}^{-1}$$

$$\text{b } 40 \text{ km h}^{-1} = \frac{40 \times 1000}{3600} = 11.1... \text{ ms}^{-1}$$

$$r = \frac{v}{\omega} = \frac{11.111}{0.279} \approx 39.8 \text{ m}$$

$$8 \quad 10 \text{ rev min}^{-1} = 10 \times 2\pi \text{ rad min}^{-1} = \frac{10 \times 2\pi}{60} \text{ rad s}^{-1} = 1.047... \text{ rad s}^{-1}$$

$$v = r\omega: \text{Aalia's speed} = 3 \times 1.047... \approx 3.14 \text{ m s}^{-1},$$

$$\text{Bethany's speed} = 5 \times 1.047... \approx 5.24 \text{ m s}^{-1}$$

$$9 \quad \text{a} \quad 1 \text{ circuit in 26 seconds} = \frac{2\pi}{26} \text{ rad s}^{-1} = 0.2416... \text{ rad s}^{-1}$$

$$\text{b} \quad v = r\omega: v = 1.5 \times 0.2416... = 0.362 \text{ m s}^{-1}$$

$$10 \quad 150 \text{ km h}^{-1} = \frac{150 \times 1000}{3600} \text{ m s}^{-1} \approx 41.666... \text{ m s}^{-1}$$

$$\omega = \frac{v}{r} = \frac{41.666...}{750} \approx 0.056 \text{ rad s}^{-1}$$

$$11 \quad \text{a} \quad \text{Hour hand: } 2\pi \text{ radians in 12 hours} = \frac{2\pi}{12 \times 3600} \text{ rad s}^{-1} \approx 0.000145 \text{ rad s}^{-1}$$

$$\text{Minute hand: } 2\pi \text{ radians in 1 hour} = \frac{2\pi}{3600} \text{ rad s}^{-1} \approx 0.00175 \text{ rad s}^{-1}$$

$$\text{b} \quad v = r\omega: \text{End of hour hand moves at } 0.1 \times 0.00145 = 1.45 \times 10^{-5} \text{ m s}^{-1}$$

$$\text{End of minute hand moves at } 0.15 \times 0.00175 = 2.62 \times 10^{-4} \text{ m s}^{-1}$$

$$12 \quad 1200 \text{ rev min}^{-1} = 1200 \times 2\pi \text{ rad min}^{-1} = \frac{1200 \times 2\pi}{60} \text{ rad s}^{-1} = 125.66... \text{ rad s}^{-1}$$

$$v = r\omega: v = 0.5 \times 125.66... = 62.8 \text{ m s}^{-1}$$

$$13 \quad \text{a} \quad 45 \text{ rev min}^{-1} = 45 \times 2\pi \text{ rad min}^{-1} = \frac{45 \times 2\pi}{60} \text{ rad s}^{-1} = 4.712... \text{ rad s}^{-1} = 4.71 \text{ rad s}^{-1}$$

$$\text{b} \quad r = \frac{v}{\omega} = \frac{12}{4.712} \approx 2.55 \text{ cm}$$

Working is in cm, not m here.

$$14 \quad \text{Distance travelled in one year} = 2\pi \times 1.5 \times 10^{11} \text{ m},$$

$$\text{So speed} = \frac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 3600} \approx 29900 \text{ m s}^{-1}$$

We cannot achieve more than 2 s.f. accuracy in the answer because one of the original values only contained 2 s.f.

$$15 \quad v > 5 \text{ m s}^{-1}$$

$$r = \frac{v}{\omega} > \frac{5}{\omega}$$

$$r > 5 \text{ m}$$

Challenge

The dots are diametrically opposite each other when they are at their maximum distance.

$$\text{Angular speed} = \frac{2\pi}{190}$$

$$\omega = \left(\frac{2\pi}{190} \right) \div 0.2$$

$$= \frac{\pi}{19} \text{ rads}^{-1}$$