OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Practice Paper – Set 4

Time allowed: 2 hours

You must have:
• Printed Answer Booklet

You may use:
• a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

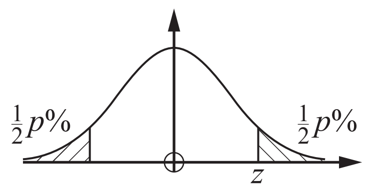
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (25 marks)

1 Express $\dfrac{14+6x}{(1-x)(3+2x)}$ in partial fractions.

2 Find the quotient and the remainder when x^3-2x+3 is divided by $x+2$. **[3]**

3 The 5th term of an arithmetic progression is 11 and the 11th term of the progression is 20. Find

- the common difference,
- the first term,
- the sum of the first 40 terms of the progression. **[5]**

4 **(a)** Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1+3x)^{-1}$. **[3]**

(b) State the range of values of x for which this expansion is valid. **[1]**

5 A is the event "Tom forgets to bring his calculator to his mathematics class".
B is the event "Tom forgets to bring his textbook to his mathematics class".

You are given that $\mathrm{P}(A) = 0.5$ and $\mathrm{P}(B) = 0.6$.

The probability that Tom forgets to bring both his textbook and his calculator to his mathematics class is 0.2.

(a) Calculate $\mathrm{P}(A|B)$. **[1]**

(b) Calculate $\mathrm{P}(A|B')$. **[3]**

(c) State, with a reason, whether or not A and B are independent events. **[1]**

6 **In this question you must show detailed reasoning.**

The equation of a curve is $y = \dfrac{\ln(x+2)}{\cos x}$.

Find the equation of the tangent to the curve at the point where $x = 0$. **[5]**

Answer **all** the questions

Section B (75 marks)

7 Qasim has just opened a café in Burnton. He decides to conduct some mark customers.

One Monday morning at 11 am he asked every customer in the café to fill out a questionnaire which included asking for the customer's age. The results he collected are shown in the stem-and-leaf diagram in Fig. 7.1.

1	8	9					
2	0	1	2	3	5	7	9
3	0	2	5	6	8		
4	3	7					
5	1	1					
6	8						

Key 2 | 0 represents an age of 20.

Fig. 7.1

(a) What name is given to the sampling method used by Qasim? **[1]**

(b) Explain why the sample does not represent a simple random sample of all the customers who use Qasim's café. **[1]**

(c) Describe the shape of the distribution of the ages in the sample. **[1]**

(d) For the data in Fig. 7.1 find

- the median,
- the interquartile range. **[3]**

Kai works at Qasim's café. He believes that Qasim's sample data may not be representative of all Qasim's customers. One week he asks every customer to fill in the questionnaire. Summary statistics for the customers who filled in the questionnaire are shown in Fig. 7.2.

Number of respondents	237
Age of youngest person	7
Lower quartile	27
Median	31
Upper quartile	39
Age of oldest person	81

Fig. 7.2

(e) Comment on whether the statistics in Fig. 7.2 provide any evidence to support Kai's belief. **[2]**

8 Two siblings, Layla and Wilson, have identical smart phones with an app cal which provides the user with statistics concerning the amount of time spen per day. The statistics for several months of use for Layla's phone are shown measured in minutes.

Number of days	120
Shortest time	12.2
Lower Quartile	46.4
Median	56.1
Upper Quartile	66.1
Mean	56.2
Standard deviation	14.5
Longest time	99.8

Fig. 8

Layla believes that X, the amount of screen time she uses per day on her smart phone, may be modelled by the Normal distribution.

(a) **Without** doing any calculation, state one feature of the data in Fig. 8 which suggests that Layla may be correct. **[1]**

(b) Verify that the values for the quartiles are consistent with the model

$X \sim \mathrm{N}(56.2, 14.5^2)$. **[2]**

(c) Calculate $\mathrm{P}(X > 90)$. **[1]**

Layla believes that Y, the amount of screen time used per day by Wilson on his smart phone, may be modelled by $Y = 2X$.

(d) Use this model to calculate $\mathrm{P}(Y > 90)$. **[2]**

Each night Layla and Wilson leave their phones on the kitchen table before going to bed. On one occasion their father picked up one of the phones and said:

"This phone has been used for more than an hour and a half today. There is a 99% chance that this is Wilson's phone."

(e) Use Layla's models for the distributions of X and Y to determine whether Layla and Wilson's father's statement is correct. **[2]**

9 Roxanne is employed by an employment agency called Supertemps. In any give required to work up to a maximum of 5 days.

Both Roxanne and her partner, Alex, suggest ways in which the number of d required to work in a week may be modelled.

- Roxanne thinks that X may be modelled using the discrete uniform distribution.

- Alex suggests the following formula to model the probability distribution of X.

$$\mathrm{P}(X = x) = k(2x - 5)^2 \text{ for } x = 0, 1, 2, 3, 4, 5.$$

(a) Calculate the value of k for Alex's model. **[2]**

Alex and Roxanne record the number of days Roxanne is required to work in each of the first 42 weeks of her employment. The results are summarised in Fig. 9.

Number of days	0	1	2	3	4	5
Frequency	13	7	1	0	7	14

Fig. 9

(b) Investigate how well each model fits the data. **[6]**

10 For a period of 20 years until the end of 2017 the parish council of the village of Springmoor have met every fortnight to discuss local affairs. Throughout this period Mr. Loparts acted as chairperson.

The parish council secretary has noted that the mean length of time of meetings throughout this period was 142 minutes, and the standard deviation of the length of time of meetings was 17.4 minutes.

At the beginning of 2018 Miss Patel took over as chairperson. At the end of 2018 the secretary stated that he believed that the mean length of time of meetings is lower since Miss Patel became chairperson.

The secretary generated a random sample of lengths of times of meetings in 2018. The times, in minutes, are as follows.

123, 137, 119, 135, 136, 143, 126, 126, 125.

Assume that the length of time of meetings may be modelled by the Normal distribution and that the variance of the length of time of meetings is unaltered.

Conduct a hypothesis test at the 5% level to determine whether there is any evidence to support the secretary's belief that the mean length of meeting has decreased. **[7]**

11 **(a)** Write $\cos^2 x$ in terms of $\cos 2x$.

(b) Express $6\sin 2x + 8\cos 2x$ in the form $R\cos(2x - \theta)$, where $0 < \theta < \frac{\pi}{2}$.

In this question you must show detailed reasoning.

(c) Hence solve the equation $6\sin 2x + 16\cos^2 x = 13$ for $0 \leqslant x \leqslant 2\pi$, giving your answers correct to 3 significant figures. **[5]**

12 The day length, Y hours, is defined as the difference between the time the sun rises and the time the sun sets on a particular day. For Manchester, England, the following model is proposed for years which are not leap years.

$$Y = a\sin\left(\frac{2\pi}{365}t + b\right) + c,$$

where t is the time in days since the start of the year and a, b and c are constants.

The maximum value of Y, which is 17.03, occurs on June 21st, when $t = 172$. The minimum value of Y, which is 7.47, occurs on December 21st, when $t = 355$.

(a) Show that $a = 4.78$ and $c = 12.25$. **[2]**

(b) Determine the value of b correct to 3 significant figures. **[2]**

On September 1st, when $t = 244$, the day length is recorded as 13.76 hours.

(c) Show that the model is a good fit for this value. **[2]**

In Reykjavik, Iceland, on June 21st the maximum day length was 21.14 hours and on December 21st the minimum day length was 4.12 hours.

(d) Use this information to refine the model for Manchester to produce a possible model for the day length in Reykjavik. **[1]**

On September 1st the day length in Reykjavik is recorded as 14.56 hours.

(e) Determine whether your possible model for Reykjavik is a good fit for this value. **[1]**

13 The large data set (LDS1) provides information on average life expectancy at bi the world. Fig. 13.1 shows the entry for South Sudan, in Africa.

Country	life expectancy at birth
South Sudan	#N/A

Fig. 13.1

No data concerning average life expectancy at birth is available for South Sudan.

(a) Explain why the spreadsheet entry is #N/A instead of simply being left blank. **[1]**

Summary statistics for the values given for average life expectancy at birth for all the countries in Africa apart from South Sudan were generated using software. These are shown in Fig. 13.2.

Mean	61.21418
Standard Deviation (s)	7.83837
Lowest Score	49.81
Highest Score	79.36
Distribution Range	29.55
Total Number of Scores	55
Number of Distinct Scores	54
Lowest Class Value	45
Highest Class Value	79.99
Number of Classes	7
Class Range	5

Fig. 13.2

(b) Explain why the mean of 61.21418 may not represent a good estimate of the average life expectancy at birth of all people in Africa. **[1]**

Fig. 13.3 shows a frequency diagram of average life expectancies of countries in South Sudan.

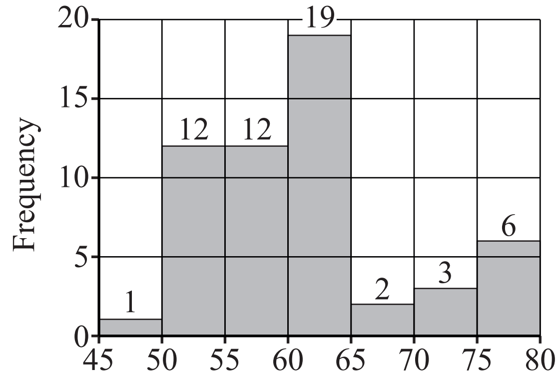


Fig. 13.3

(c) Explain why it would be incorrect to call the diagram in Fig. 13.3 a histogram. **[1]**

(d) Draw a histogram to represent the data, using class boundaries at 45, 55, 60, 65 and 80. **[3]**

Fig. 13.4 shows a scatter diagram of median age in years against average life expectancy for countries in Africa excluding South Sudan. This was generated using a spreadsheet.

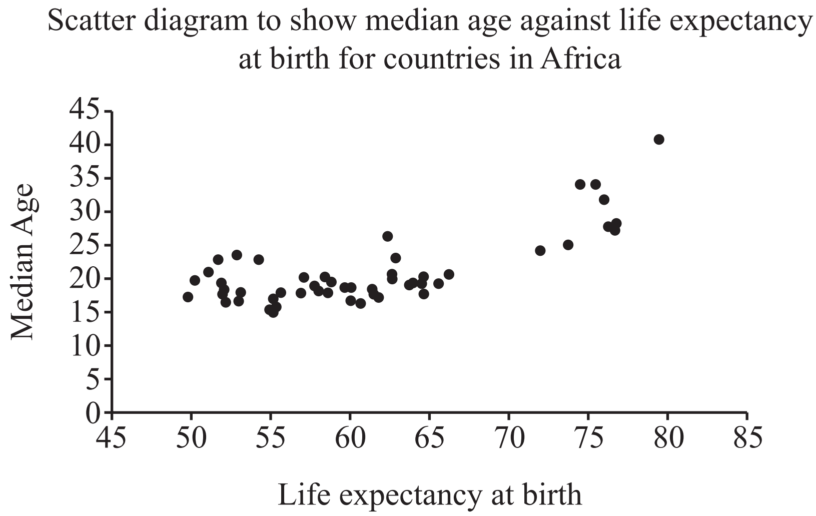


Fig. 13.4

The median age for the population of South Sudan is given as 17.

(e) With reference to Fig. 13.4, comment on whether it might be possible to obtain a reliable estimate of the average life expectancy at birth for South Sudan. **[2]**

14 The 24 members of staff at the Blackley branch of the Midshires Bank decid
They will each pay £5 per week to enter the lottery.

Each week, three of the numbers 1, 2, 3 and 4 will be placed in a random order b
member of staff will make 3 different attempts, entered into their computer wc
what the computer has chosen.

(a) Show that the probability that a particular member of staff enters the correct numbers in the correct order in any week is $\frac{1}{8}$. **[3]**

If exactly one person chooses the right numbers in the right order, that person will win a prize of all £120 paid by staff to enter the lottery. If **no-one** chooses the right numbers, **or if more than one person** chooses the right numbers, the prize will be 'rolled over' to the next week.

(b) Calculate the probability that someone will win the prize in the first week. **[2]**

(c) Calculate the probability that the customer services manager, Marak, will win the prize in each of the first two weeks. **[2]**

In fact, when the lottery started, nobody won until the prize reached £6000. At that point Milena, the chief cashier, won. Marak commented that there was a one in a million chance of this happening to Milena.

(d) Determine whether Marak's comment is correct. **[4]**

15 In this question you must show detailed reasoning.

The curve $y = \ln x$ passes through the point (a, b), where $a > 1$.

The area A is bounded by the x-axis, the line $x = a$ and the curve $y = \ln x$.

The area B is bounded by the x-axis, the y-axis, the line $y = b$ and the curve $y =$

The area A is equal in magnitude to the area B.

(a) Show that a satisfies the equation $pa\ln a + qa + r = 0$, where p, q and r are constants to be determined. **[7]**

The value of a is found using the Newton-Raphson method on a spreadsheet. The output is shown in Fig. 15.

r	x_r
0	4
1	5.177399
2	4.931531
3	4.921571
4	4.921554

Fig. 15

Heidi states that the value of a is 4.921554 correct to 6 decimal places.

(b) Determine whether she is correct. **[2]**

END OF QUESTION PAPER