OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Practice Paper 1 – Set 1

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \frac{u}{v}, \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

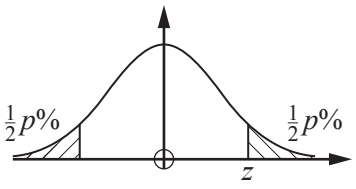
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.575



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 Find $\int 12e^{3x} dx$.

2 Use a counter example to disprove the following statement.

$2^n - 1$ is prime for all $n > 1$

[2]

3 The head of sales of a large company presented Fig. 3 to the board of directors as part of his end-of-year report.

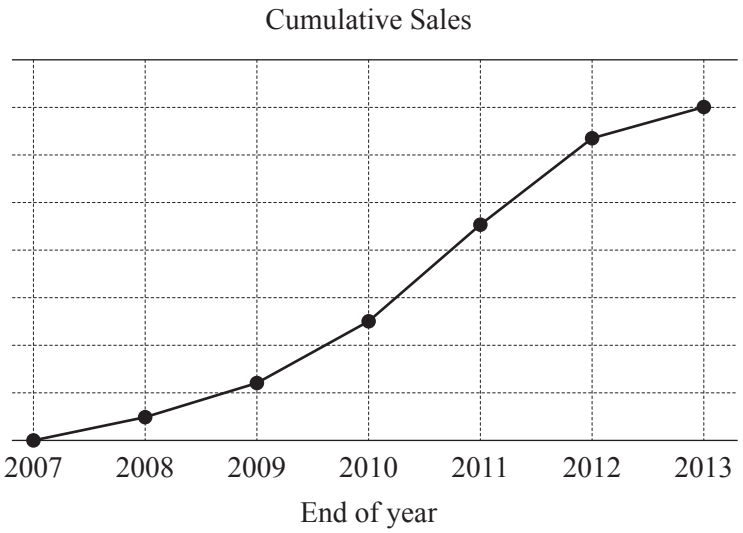


Fig. 3

(i) What key feature is missing from this graph? **[1]**

One director comments that the diagram shows increasing year-on-year sales.

(ii) Explain whether the director is correct. **[1]**

4 Find the equation of the straight line through $(1, 5)$ which is perpendicular to the line with equation $2y = x + 3$. **[3]**

5 Find $\int 18x(3x+1)^7 \,\mathrm{d}x$.

You may wish to use the substitution $u = (3x+1)$.

6 **(i)** Write down the exact values of tan 45° and tan 60°. **[2]**

(ii) **In this question you must show detailed reasoning.**

Show that $\tan 15° = 2 - \sqrt{3}$. **[4]**

7 The sides of a triangle are of length 47, 53 and 94 units. Calculate the size of the largest angle. **[3]**

Answer **all** the questions

Section B (77 marks)

8 The screenshot in Fig. 8 shows the probability distribution for the discrete random variable $X \sim \text{B}(20, 0.9)$.

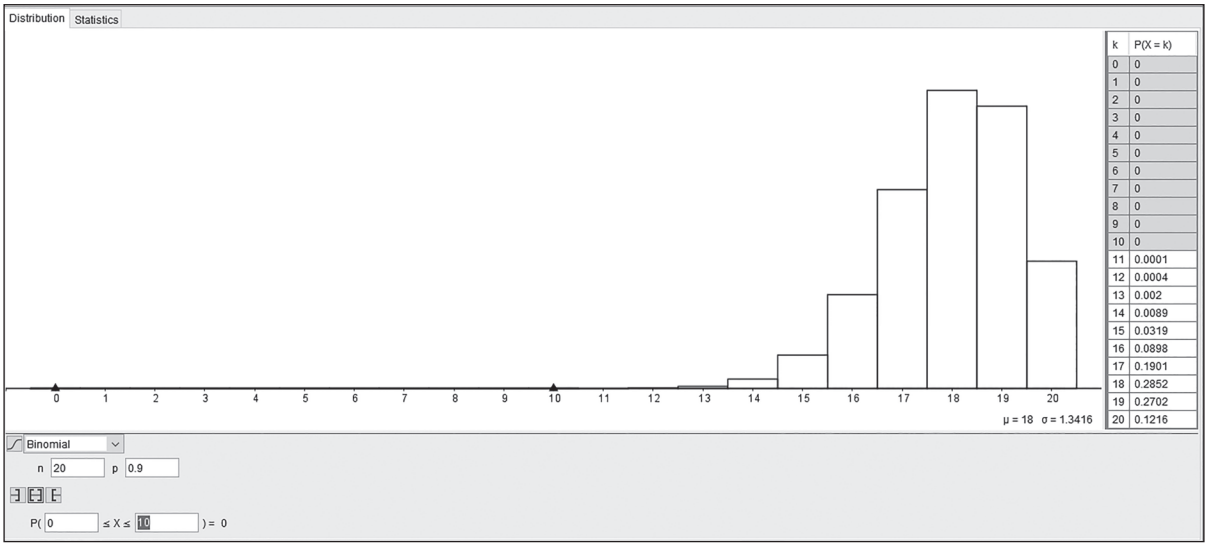


Fig. 8

(i) Describe the shape of the distribution. **[1]**

(ii) Explain why the values of $\text{P}(X = k)$ for $k = 0$ to 10 inclusive are recorded as 0 in the table in the screenshot. **[1]**

(iii) State which of the values from 0 to 20 is

(A) the most likely value of X, **[1]**

(B) the least likely value of X. **[1]**

9 **(a)** **(i)** The pre-release data shows that the total population of the 239 countries in th
7 174 654 290. The populations of a sample of 10 countries are given in Fig. 9

Country	Population
Tuvalu	10 782
Equatorial Guinea	722 254
Somalia	10 428 043
Denmark	5 569 077
Burma	55 746 253
Norway	5 147 792
Botswana	2 155 784
Rwanda	12 337 138
Sint Maarten	39 689
Swaziland	1 419 623

Fig. 9.1

Show that the mean population per country for the whole world is much larger than the mean population per country for this sample. **[3]**

(ii) Rebecca takes a large number of different samples of 10 countries. She finds that the mean population per country is usually smaller for the sample than it is for the whole world. Explain whether this suggests that the sampling was not random. **[2]**

(b) Fig. 9.2 shows data for Norway.

Country	Population	GDP per capita (US$)	Health expenditure (% of GDP)
Norway	5 147 792	55 400	9.1

Fig. 9.2

Calculate Norway's health expenditure per person in US$. **[2]**

(c) As part of an investigation into the factors which might be associated with life ex
diagrams in Fig. 9.3 are drawn. The line of best fit and the corresponding value
correlation coefficient are also shown for each scatter diagram.

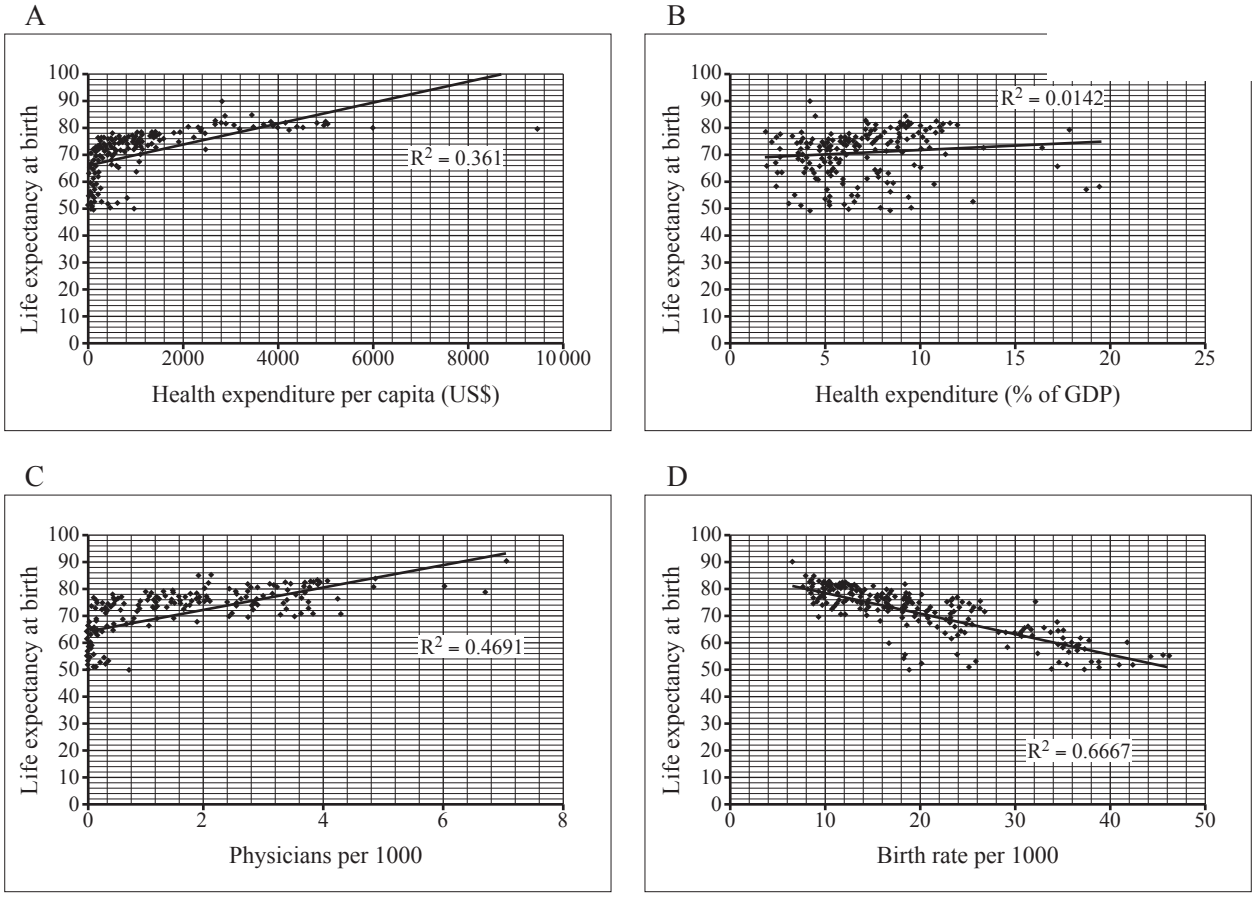


Fig. 9.3

(i) Which of the four factors appears to have the strongest positive association with life expectancy at birth? Give a reason for your answer. **[2]**

(ii) Explain why the line of best fit in scatter diagram B is not a good model for the relationship between the two variables. **[1]**

10 Every evening at bedtime my cat Arthur decides whether to spend the night inside or outside the house.

If Arthur spends a night inside, the probability that he will spend the next night outside is 0.6.

If Arthur spends a night outside, the probability that he will spend the next night inside is 0.9.

Arthur spends a Saturday night inside.

(i) Calculate the probability that Arthur will decide to spend the next Monday night outside. **[3]**

(ii) Show that it is extremely likely that Arthur will spend at least one out of the next seven nights (Sunday to Saturday) inside. **[3]**

11 **In this question you must show detailed reasoning.**

A curve has parametric equations

$$x = \cos t - 3t \text{ and } y = 3t - 4\cos t - \sin 2t, \text{ for } 0 \leqslant t \leqslant \pi.$$

Show that the gradient of the curve is always negative. **[7]**

12 **(i)** The response time to a roadside callout from a breakdown recovery firm may be modelled by a Normal distribution. Over a long period it has been noted that 8% of the response times are less than 58 minutes and 19% of the response times are greater than 86 minutes. Determine estimates of the mean and standard deviation of the response time. **[7]**

(ii) The recovery firm claim that 95% of their response times are less than 90 minutes. Investigate this claim. **[2]**

13 When a particular type of glass is filled to a depth of x cm with champagne, the volume of champagne in the glass is modelled by the formula

$$V = kx^2 - \frac{x^3}{3},$$

where $V = 36$ when $x = 3$.

(i) Find the value of k. **[1]**

When full, a glass contains 140 cm^3 of champagne and the depth is 7.5 cm.

(ii) Determine whether the model works well for this value. **[2]**

When Leonie pours champagne into a glass of this type, the volume in the glass after t seconds is modelled by the formula

$$V = 140\left(1 - \text{e}^{-0.2t}\right).$$

(iii) (*A*) Verify that after 5 seconds the depth of champagne is between 5.20 cm and 5.21 cm. **[2]**

(*B*) Find the rate at which the depth of champagne is increasing 5 seconds after she starts pouring. **[4]**

(*C*) According to the model, would it be faster for Leonie to pour two half glasses of 70 cm^3 or one full glass of 140 cm^3 of champagne? **[2]**

14 Between the islands of Tenerife and La Gomera there is a resident population of pilot w
past showed that 20% of the adult population were male. Due to a change in environm
thought that the proportion of males has decreased.

In order to investigate this, scientists caught and released a random sample of 43 differe
Exactly 3 of these whales were found to be male.

(i) Carry out a hypothesis test at the 5% level to investigate whether there is any evidence that the proportion of males has decreased. **[6]**

Previous studies also showed that the mean length of adult females was 2.98 metres. On another occasion the scientists caught a random sample of 39 different adult female pilot whales; the length, x metres, of each whale was measured before it was released. The data are summarized below.

$$\Sigma x = 120.7 \text{ and } \Sigma x^2 = 384.75$$

(ii) Carry out a hypothesis test at the 5% level to investigate whether there is any evidence that the mean length of adult females has changed. **[8]**

15 Fig. 15 shows the graph of $\mathrm{f}(x) = 2x + \frac{1}{x} + \ln x - 4$.

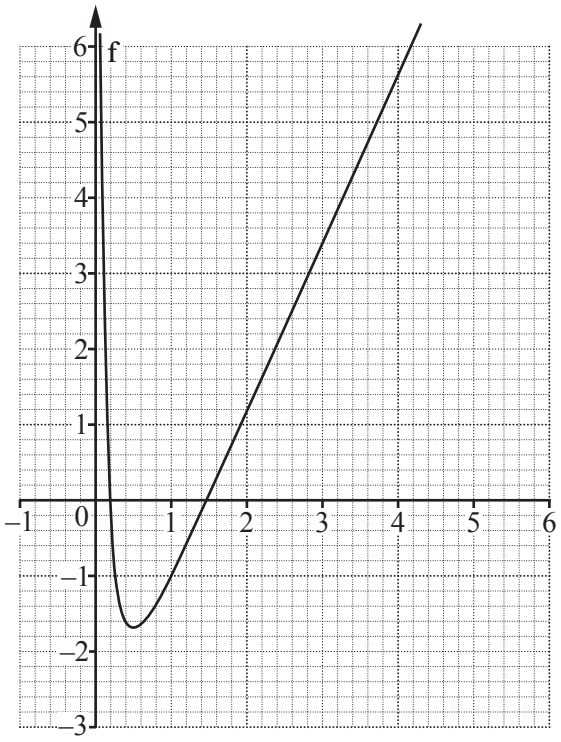


Fig. 15

(i) Show that the equation

$$2x + \frac{1}{x} + \ln x - 4 = 0$$

has a root, α, such that $0.1 < \alpha < 0.9$.

(ii) Obtain the following Newton-Raphson iteration for the equation in part **(i)**.

$$x_{r+1} = x_r - \frac{2x_r^3 + x_r + x_r^2(\ln x_r - 4)}{2x_r^2 - 1 + x_r}$$

[3]

(iii) Explain why this iteration fails to find α using each of the following starting values.

(*A*) $x_0 = 0.4$ **[2]**

(*B*) $x_0 = 0.5$ **[2]**

(*C*) $x_0 = 0.6$ **[2]**

16 A particular condition affects 0.8% of the population. 90.1% of the population as a whole carry a certain gene. 9.85% of the population neither carry the gene nor are affected by the condition. Paul discovers that he carries the gene. He believes that it is very likely that he will be affected by the condition. Determine whether or not he is correct. **[5]**

END OF QUESTION PAPER