MEI

Mathematics in Education and Industry

# MEI STRUCTURED MATHEMATICS

## FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

# Practice Paper FP1-B

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS**

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

**INFORMATION**

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

## Section A (36 marks)

**1** Solve the inequality $x^3 - 4x > 0$. [2]

**2** You are given that $A(x-1)(x-2) + Bx(x-1) + Cx(x-2) \equiv x^2 + x + 1$.

Find the values of $A$, $B$ and $C$. [4]

**3** You are given that the equation $x^3 + px^2 + qx + r = 0$ has roots $\alpha$, $-\alpha$ and $\beta$.

**(i)** Show that $q = -\alpha^2$. [1]

**(ii)** Show that $r = pq$.
Show that this is true for the equation $x^3 + 7x^2 + 19x + 133 = 0$ but that it has only one real root. [5]

**4** **(i)** Express $2 + 3j$ in the modulus-argument form $(r, \theta)$ where $r$ and $\theta$ are given to two decimal places. [3]

**(ii)** Sketch on an Argand diagram the locus $|z - z_1| = 2$ where $z_1 = 2 + 3j$. [3]

**5** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$

**(i)** Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$. [2]

**(ii)** Show that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. [4]

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**6** **(i)** Show that $(r+1)^2 \times r^2 - r^2 \times (r-1)^2 = 4r^3$. [2]

**(ii)** Hence find $\sum_{r=1}^{n} r^3$. [4]

**7** You are given the equation $x^3 - 3x^2 + 7x - 5 = 0$.

**(i)** Show by substitution that $x = 1 + 2j$ satisfies this equation. [3]

**(ii)** Write down a second root of the equation. [1]

**(iii)** Find the third root of the equation. [2]

## Section B (36 marks)

**8** A curve has equation $y = \dfrac{4(2x-1)}{(x+1)^2}$.

**(i)** Write down the equation of the asymptote that is parallel to the $y$-axis. [1]

**(ii)** Find the second asymptote of the curve. Describe clearly the behaviour of the curve for large positive and negative values of $x$. [5]

**(iii)** Find the values of $x$ for which $y = 1$. [3]

**(iv)** Sketch the curve, showing clearly where it cuts the $x$ axis. [3]

**9** A reflection in a line $l$ on the coordinate plane is represented by the matrix **A** where

$$\mathbf{A} = \begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}.$$

**(i)** Find the image of the point (3, 6).
Hence write down the equation of the mirror line, $l$. [4]

**(ii)** The matrix $\mathbf{T} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ represents a rotation. By considering the image of the point (3, 2), find the centre and the angle of the rotation. [3]

**(iii)** Find **TA**. [2]

**(iv)** Show that under the transformation **TA** the point (1, -3) is invariant. Hence state the equation of the line of invariant points under the transformation **TA**. [3]

**10** The quadratic equation $z^2 + 6z + 25 = 0$ has complex roots $\alpha$ and $\beta$.

**(i)** Find the roots in the form $p + q\mathrm{j}$. [3]

**(ii)** Find the modulus and argument of each root.
Illustrate both roots on an argand diagram. [5]

**(iii)** Find the value of $\alpha^2 + \beta^2$.
Hence find the equation with roots $\alpha^2$ and $\beta^2$. [4]