

MEI STRUCTURED MATHEMATICS

FURTHER CONCEPTS FOR ADVANCED MATHEMATICS, FP1

Practice Paper FP1-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Solve the equation $x^2 + 3x + 4 = 0$. [3]

2 Express $\frac{3+2j}{4-j}$ in the form $x + yj$. [3]

3 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$$

Find

(i) \mathbf{A}^2 , [2]

(ii) \mathbf{A}^{-1} . [1]

4 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$

(i) Use **A** and **B** to prove that matrix multiplication is not commutative. [2]

(ii) Use **A**, **B** and **C** to give an example in which matrix multiplication is associative. [3]

5 (i) Show that $\frac{8}{x+1} - \frac{3}{x-4} \equiv \frac{5(x-7)}{(x+1)(x-4)}$ [2]

(ii) Hence solve the inequality $\frac{8}{x+1} - \frac{3}{x-4} > 5$. [5]

6 The cubic equation $x^3 + px^2 + qx + r = 0$ has roots α , β and γ .

Find

(i) $\alpha^2 + \beta^2 + \gamma^2$. [4]

(ii) The equation which has roots $-\alpha$, $-\beta$ and $-\gamma$. [3]

7 Prove by induction that $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$. [8]

Section B (36 marks)

8 (i) You are given that $\frac{(x+2)(x-3)}{(x-1)} \equiv Ax + B + \frac{C}{(x-1)}$, $x \neq 1$.

Show that $B = 0$ and find A and C .

[6]

(ii) Fig. 8 shows a sketch graph of $y = \frac{(x+2)(x-3)}{(x-1)}$.

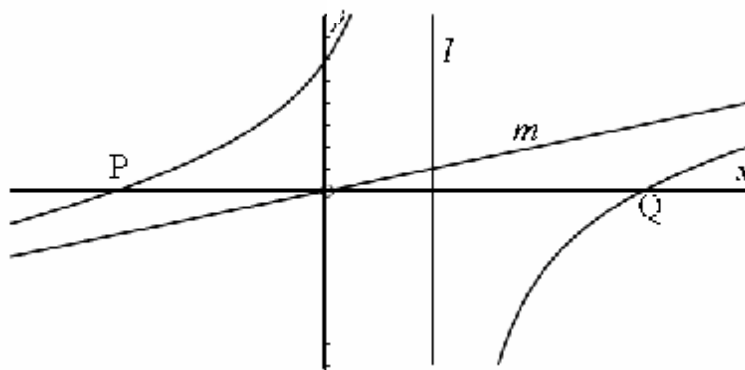


Fig. 8

Write down the coordinates of P and Q , the points where the graph cuts the x axis, and the equation of the line l .

[3]

(iii) The line m has equation $y = x$. Prove that the curve does not cross the line m .

[3]

9 The matrices $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ define transformations in the (x,y) -plane.

(i) (A) Under \mathbf{P} , the point $M(3, 2)$ is transformed to M' . Find the coordinates of M' . [1]

(B) Find \mathbf{P}^2 . [2]

(C) Hence describe the transformation represented by \mathbf{P} . [1]

(ii) (A) Under \mathbf{Q} , the point $M(3, 2)$ is transformed to M'' . Find the coordinates of M'' . [1]

(B) Find \mathbf{Q}^4 . [3]

(C) Hence describe the transformation represented by \mathbf{Q} . [1]

(iii) Find \mathbf{PQ} and describe the transformation that it represents. [3]

- 10** You are given the complex numbers $z_1 = \sqrt{3} + 2j$ and $z_2 = 4\left(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}\right)$.
- (i)** Find the modulus and argument of z_1 . [2]
 - (ii)** Write z_2 in the form $a + bj$ where a and b are to be given exactly. [2]
 - (iii)** Illustrate z_1 and z_2 on an Argand diagram. [2]
 - (iv)** Find $z_2 - z_1$ and indicate this on your Argand diagram. [2]
 - (v)** Describe the locus of the points, z , on the Argand diagram for which $|z - z_1| = \sqrt{3}$.
Sketch the locus on your diagram. [4]