

## MEI STRUCTURED MATHEMATICS

### METHODS FOR ADVANCED MATHEMATICS, C3

## Practice Paper C3-D

Additional materials: Answer booklet/paper  
Graph paper  
List of formulae (MF2)

**TIME** 1 hour 30 minutes

### INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

### INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.
- **You are reminded of the need for clear presentation in your answers.**

**Section A (36 marks)**

**1** You are given that  $y^2 = 4x + 7$ .  
**(i)** Use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $y$ . [2]

**(ii)** Make  $x$  the subject of the equation.

Find  $\frac{dx}{dy}$  and hence show that in this case  $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ . [3]

**2** **(i)** Expand  $(e^x + e^{-x})^2$ . [1]

**(ii)** Hence find  $\int (e^x + e^{-x})^2 dx$ . [3]

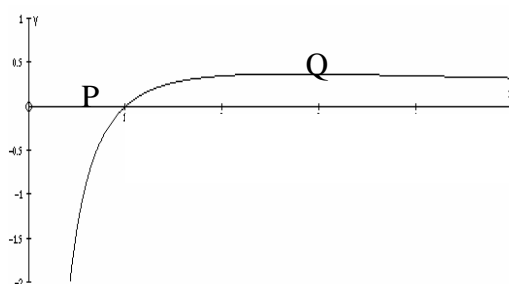
**3** **(i)** Sketch the graph of  $y = |3x - 6|$ . [2]

**(ii)** Solve the equation  $|3x - 6| = x + 4$  and illustrate your answer on your graph. [4]

**4** Find  $\int x \sin 3x dx$ . [4]

**5** Make  $x$  the subject of  $t = \ln \sqrt{\frac{5}{(x-3)}}$ . [4]

**6** The function  $f(x)$  is defined as  $f(x) = \frac{\ln x}{x}$ . The graph of the function is shown in Fig. 6.



**Fig. 6**

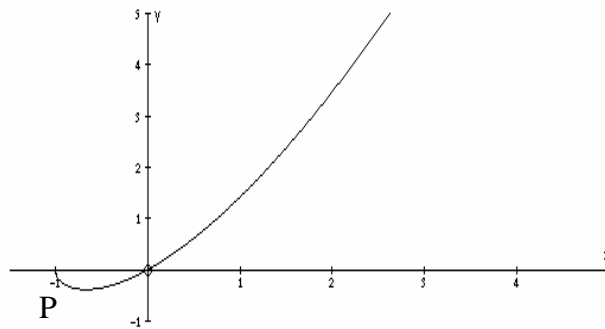
**(i)** Give the coordinates of the point, P, where the curve crosses the  $x$ -axis. [1]

**(ii)** Use calculus to find the coordinates of the stationary point, Q, and show that it is a maximum. [6]

- 7 An oil slick is circular with radius  $r$  km and area  $A$  km<sup>2</sup>. The radius increases with time at a rate given by  $\frac{dr}{dt} = 0.5$ , in kilometres per hour.
- (i) Show that  $\frac{dA}{dt} = \pi r$ . [4]
- (ii) Find the rate of increase of the area of the slick at a time when the radius is 6 km. [2]

**Section B (36 marks)**

- 8 Fig. 8 shows the graph of  $y = x\sqrt{1+x}$ . The point P on the curve is on the  $x$ -axis.



**Fig. 8**

- (i) Write down the coordinates of P. [1]
- (ii) Show that  $\frac{dy}{dx} = \frac{3x+2}{2\sqrt{1+x}}$ . [4]
- (iii) Hence find the coordinates of the turning point on the curve. What can you say about the gradient of the curve at P? [4]
- (iv) By using a suitable substitution, show that  $\int_{-1}^0 x\sqrt{1+x} \, dx = \int_0^1 \left( u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$ . Evaluate this integral, giving your answer in an exact form. What does this value represent? [7]
- (v) Use your answer to part (ii) to differentiate  $y = x\sqrt{1+x} \sin 2x$  with respect to  $x$ . (You need not simplify your result.) [2]

9 The functions  $f(x)$  and  $g(x)$  are defined by

$$f(x) = x^2, \quad g(x) = 2x - 1,$$

for all real values of  $x$ .

- (i) State the ranges of  $f(x)$  and  $g(x)$ .  
Explain why  $f(x)$  has no inverse. [3]
- (ii) Find an expression for the inverse function  $g^{-1}(x)$  in terms of  $x$ .  
Sketch the graphs of  $y = g(x)$  and  $y = g^{-1}(x)$  on the same axes. [4]
- (iii) Find expressions for  $gf(x)$  and  $fg(x)$ . [2]
- (iv) Solve the equation  $gf(x) = fg(x)$ .  
  
Sketch the graphs of  $y = gf(x)$  and  $y = fg(x)$  on the same axes to illustrate your answer. [4]
- (v) Show that the equation  $f(x + a) = g^2(x)$  has no solution if  $a > \frac{1}{4}$ . [5]