

MEI

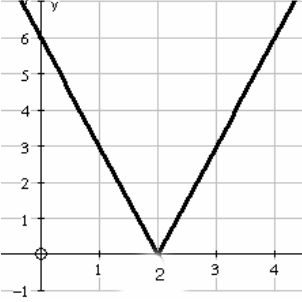
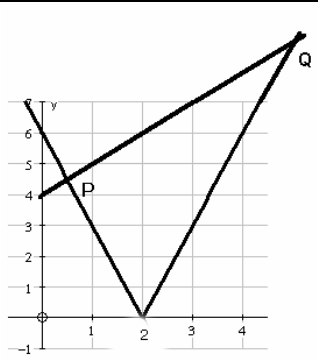
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS OF ADVANCED MATHEMATICS, C3

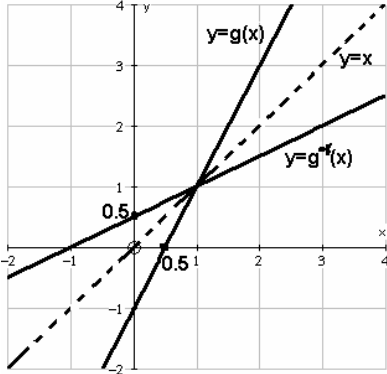
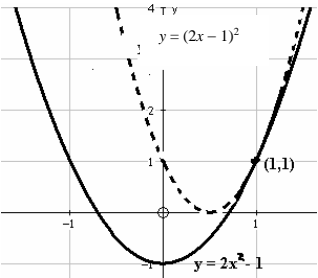
Practice Paper C3-D

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	(i) $y^2 = 4x + 7 \Rightarrow 2y \cdot \frac{dy}{dx} = 4$ $\Rightarrow \frac{dy}{dx} = \frac{2}{y}$	M1 A1 2	
	(ii) $x = \frac{1}{4}(y^2 - 7) \Rightarrow \frac{dx}{dy} = \frac{1}{4} \cdot 2y = \frac{y}{2} = \frac{1}{\frac{2}{y}}$	B1 M1 A1 3	
2	(i) $e^{2x} + 2 + e^{-2x}$	B1 1	
	(ii) $= \int (e^{2x} + 2 + e^{-2x}) dx$ $= \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$	B1 B1 B1 3	One for each exponential term, one for both 2x and constant.
3	(i) 	B1 B1 2	One for two half lines; one for correct orientation and meeting at (2, 0).
	(ii) 	At P $-3x + 6 = x + 4$ $\Rightarrow x = \frac{1}{2}$ At Q $3x - 6 = x + 4$ $\Rightarrow x = 5$ The solution is $x = \frac{1}{2}, 5$. As shown on graph	M1 A1 A1 E1 4
4	$\int x \sin 3x dx; \quad u = x \Rightarrow \frac{du}{dx} = 1, \frac{dv}{dx} = \sin 3x \Rightarrow v = -\frac{1}{3} \cos 3x$ $= -\frac{1}{3} x \cos 3x + \int \frac{1}{3} \cos 3x dx$ $= -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + c$	M1 A1 M1 A1 4	Choice of u $-\frac{1}{3} \cos 3x$ Correct form c must be seen

5		$t = \ln \sqrt{\frac{5}{x-3}}$ $= -\frac{1}{2} \ln \frac{(x-3)}{5}$ $\Rightarrow -2t = \ln \frac{(x-3)}{5}$ $\Rightarrow e^{-2t} = \frac{(x-3)}{5} \Rightarrow x = 5e^{-2t} + 3$	M1 M1 A1 A1	Rules of logs Change to exponentials 4
6	(i)	P(1,0)	B1	1
	(ii)	$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - 1 \cdot \ln x}{x^2}$ $= \frac{1 - \ln x}{x^2}$ <p>At Q, gradient is zero, so $x = e$. Q is $(e, \frac{1}{e})$.</p> $\frac{d^2y}{dx^2} = \frac{x^2(-\frac{1}{x}) - (1 - \ln x) \cdot 2x}{x^4}$ $= \frac{-3 + 2 \ln x}{x^3}$ <p>When $x = e$, this is -ve, so Q is a maximum.</p>	M1 A1 M1 A1 M1 A1	quotient rule = 0 Or equivalent methods 6 For $1/e$.
7	(i)	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$ $\Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = \frac{1}{2} \times 2\pi r = \pi r$	M1 A1 M1 A1	4
	(ii)	<p>So when $r = 6$,</p> $\frac{dA}{dt} = 6\pi (= 15.707\dots)$ <p>The area increases at $15.7 \text{ km}^2\text{h}^{-1}$, to 3sf.</p>	M1 A1	Substituting 2

Section B			
8	(i)	P(-1,0)	B1 1
	(ii)	$y = x\sqrt{1+x}$ $= x(1+x)^{\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = 1 \cdot (1+x)^{\frac{1}{2}} + x \cdot \frac{1}{2}(1+x)^{-\frac{1}{2}}$ $= \frac{2(1+x) + x}{2(1+x)^{\frac{1}{2}}}$ $= \frac{3x+2}{2\sqrt{1+x}}$	M1 A1 M1 E1 4
	(iii)	<p>At a turning point, gradient is zero. $x = -\frac{2}{3}$ there. Then</p> $y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ $= -\frac{2\sqrt{3}}{9}$ <p>These are the coordinates of the TP.</p> <p>At P the gradient is undefined.</p>	M1 A1 A1 B1 4
	(iv)	$\int_{-1}^0 x\sqrt{1+x} dx$ $= \int_0^1 (u-1)u^{\frac{1}{2}} du, \quad u = 1+x$ $\Rightarrow \frac{du}{dx} = 1$ $= \int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$ $= \left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} \right]_0^1$ $= -\frac{4}{15}$ <p>The magnitude represents the area bounded by the curve and axis between P and O. It is negative because the curve is below the axis (except at the end points).</p>	M1 E1 M1A1 A1 B1 B1 7
	(v)	$y = x\sqrt{1+x} \sin 2x$ $\Rightarrow \frac{dy}{dx} = \frac{3x+2}{2\sqrt{1+x}} \sin 2x + x\sqrt{1+x} \cdot 2 \cos 2x$	M1A1 2

9	<p>(i) Range of f is $[0, \infty)$, of g $(-\infty, \infty)$. f has no inverse because (say) for any value of $f > 0$ there are 2 corresponding values of x</p>	<p>B1 B1 E1</p>	<p>3</p>
	<p>(ii)</p> $y = 2x - 1$ $\Rightarrow x = \frac{1}{2}y + \frac{1}{2}$ $\Rightarrow g^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$ 	<p>M1 A1 B1 B1</p>	<p>One mark for one line, and one mark for second correctly related</p> <p>4</p>
	<p>(iii)</p> $gf(x) = 2x^2 - 1$ $fg(x) = (2x - 1)^2$	<p>B1 B1</p>	<p>2</p>
	<p>(iv)</p> $2x^2 - 1 = (2x - 1)^2$ $\Rightarrow 0 = 2x^2 - 4x + 2$ $\Rightarrow 0 = 2(x - 1)^2$ $\Rightarrow x = 1$ 	<p>M1 A1 B1 B1</p>	<p>$y = (2x - 1)^2$ $y = (2x - 1)^2$</p> <p>4</p>
	<p>(v)</p> $f(x+a) = (x+a)^2$ $g^2(x) = 2(2x-1) - 1 = 4x - 3$ $f(x+a) = g^2(x) \Rightarrow (x+a)^2 = 4x - 3$ $\Rightarrow x^2 + (2a-4)x + a^2 + 3 = 0$ <p>\Rightarrow There are two roots to this equation if</p> $(2a-4)^2 > 4(a^2 + 3)$ <p>i.e. $4a^2 - 16a + 16 > 4a^2 + 12$</p> $\Rightarrow 16a < 4 \Rightarrow a < \frac{1}{4}$	<p>B1 M1 M1 A1 A1</p>	<p>Both fns correct Equating Using $b^2 - 4ac$ Correct inequality Result</p> <p>5</p>

