

MEI

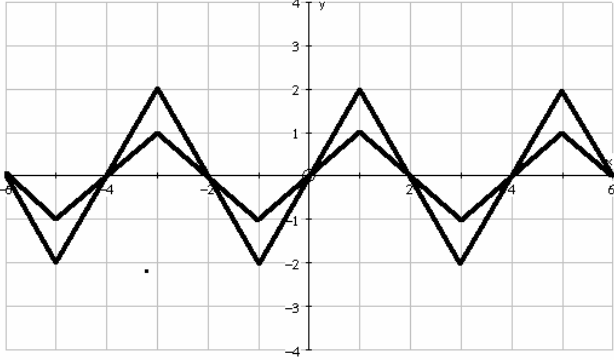
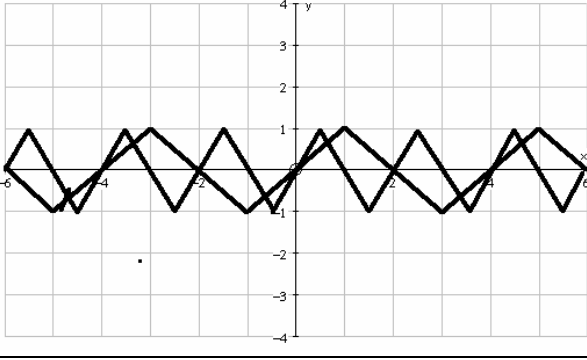
Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

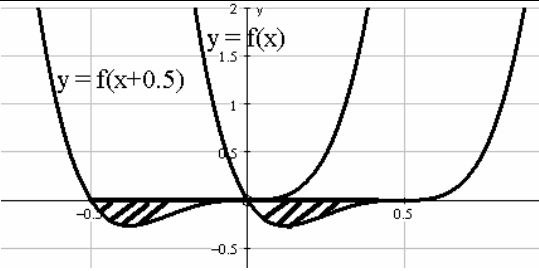
METHODS OF ADVANCED MATHEMATICS, C3

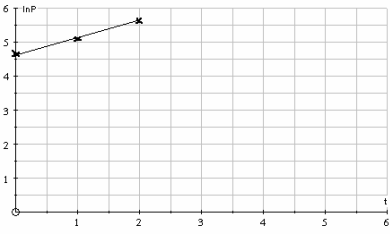
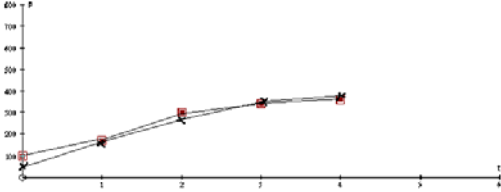
Practice Paper C3-C

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	Take $n = 11$; the result is divisible by 11. ($n = 10$ is the smallest number)	M1A1 2	
2	(i) $f(-x)$ $= (-x)^3 $ $= -x^3 $ $= x^3 $ $= f(x)$ This argument holds regardless of whether x is positive, negative or zero.	M1 A1 2	Using $-x$
	(ii) $g(-x)$ $= (-x-1)^3$ $g(1) \neq g(-1)$ (say), so the function is not odd.	M1 A1 2	
3	(i) 	B1 B1 2	One for the correct amplitude and one for a function of the right period correctly placed.
	(ii) 	B1 B1 2	One for a function of the right period and one for the correct amplitude correctly placed.
4	$V = \frac{4}{3}\pi r^3$ $\Rightarrow \frac{dV}{dr} = 4\pi r^2$ and $\frac{dr}{dt} = 2$ $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $= 4\pi \times 10^2 \times 2$ $= 800\pi$ $(= 2513.27\dots)$ The rate of increase of V is $2500 \text{ cm}^3/\text{sec}$, to 2sf (say).	M1 B1 B1 A1 A1 5	B1 for number, B1 for units.

5	(i)	$x^2 + y^2 = 25$ $\Rightarrow 2x + 2y \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$	M1A1 E1 3	
	(ii)	<p>Gradient of normal = $\frac{4}{3}$</p> $\Rightarrow y - 4 = \frac{4}{3}(x - 3)$ $\Rightarrow 3y = 4x$ <p>Or equivalent.</p>	B1 M1 A1 3	
6	(a)	$\int x \cos 2x dx$ $= \frac{x}{2} \sin 2x - \int \frac{1}{2} \sin 2x dx$ $= \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x + c$	M1 A1 A1 3	
	(b)	$\int_2^3 \frac{x}{x^2 + 1} dx$ $u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x$ $x = 2 \Rightarrow u = 5, x = 3 \Rightarrow u = 10$ $= \frac{1}{2} \int_5^{10} \frac{1}{u} du = \frac{1}{2} [\ln u]_5^{10} = \frac{1}{2} \ln 2$	M1 A1 A1 A1 A1 5	Substitution Integral in u Limits Integral Answer
7		<p>For turning point $y = e^{-x} \sin x$</p> $\Rightarrow \frac{dy}{dx} = -e^{-x} \sin x + e^{-x} \cos x$ $= 0 \text{ when } \sin x = \cos x \Rightarrow x = \frac{\pi}{4} \text{ (in this range)}$ $\Rightarrow OA' = \frac{\pi}{4}$ <p>For intersection: $e^{-x} \sin x = e^{-x} \Rightarrow \sin x = 1$</p> $\Rightarrow x = \frac{\pi}{2} \text{ (in this range)}$ $\Rightarrow OB' = \frac{\pi}{2} \Rightarrow A'B' = \frac{\pi}{4}$	M1 A1 M1 A1 M1 A1 A1 7	Product = 0 Solving

Section B				
8	(i)	$\frac{dy}{dx} = 5x \cdot 6(2x-1)^2 + 5(2x-1)^3$ $= 5(2x-1)^2(8x-1)$ <p>Where the gradient is zero, $x = \frac{1}{2}$ (double root, where the curve touches the x-axis) or $x = \frac{1}{8}$ (at S).</p>	<p>M1A1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">4</p>	
	(ii)	<p>Area has magnitude</p> $\left \int_0^{\frac{1}{2}} 5x(2x-1)^3 dx \right $ $= \left \int_0^{\frac{1}{2}} (40x^4 - 60x^3 + 30x^2 - 5x) dx \right $ $= \left \left[8x^5 - 15x^4 + 10x^3 - \frac{5x^2}{2} \right]_0^{\frac{1}{2}} \right $ $= \left \frac{1}{4} - \frac{15}{16} + \frac{10}{8} - \frac{5}{8} \right $ $= \left -\frac{1}{16} \right $ <p>So area is $\frac{1}{16}$ units²..</p>	<p>B1</p> <p>M1A1</p> <p>A2</p> <p>A1</p> <p style="text-align: right;">6</p>	<p>(or at end if the -ve sign is explained)</p> <p>Expand or integrate by parts</p> <p>A1 for some terms right</p> <p><i>Alternatively by substitution</i></p>
	(iii)	$f(x+0.5)$ $= 5(x+0.5)(2[x+0.5]-1)^2$ $= 40x^3(x+0.5)$	<p>M1</p> <p>A1</p> <p style="text-align: right;">2</p>	
	(iv)	$\int_{-\frac{1}{2}}^0 40x^3(x+0.5) dx$ $= \int_{-\frac{1}{2}}^0 (40x^4 + 20x^3) dx$ $= \left[8x^5 + 5x^4 \right]_{-\frac{1}{2}}^0 = 0 - \left(-\frac{8}{32} + \frac{5}{16} \right) = -\frac{1}{16}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p style="text-align: right;">4</p>	<p>Multiply out Terms</p> <p>Both terms</p>
	(v)	 <p>The area representing the one integral is a translation of that representing the other, so their values are equal.</p>	<p>B1</p> <p>E1</p> <p style="text-align: right;">2</p>	

9	(i)	$P = P_0 e^{kt}$ $\Rightarrow \ln P = \ln P_0 + kt$ <p>So if y is identified with $\ln P$, m with k and x with t, we have $y = mx + c$; gradient k, intercept $\ln P_0$.</p>	M1A1 E1 3													
	(ii)	<table border="1" data-bbox="300 342 667 488"> <thead> <tr> <th>T</th> <th>P</th> <th>$\ln P$ (to 3dp)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>100</td> <td>4.61</td> </tr> <tr> <td>1</td> <td>170</td> <td>5.14</td> </tr> <tr> <td>2</td> <td>300</td> <td>5.70</td> </tr> </tbody> </table> <p>Gradient estimate (about) $k = 0.55$; Intercept (about) 4.61; So $P_0 = \exp(4.61) = 100$, to 2 sf.</p> 	T	P	$\ln P$ (to 3dp)	0	100	4.61	1	170	5.14	2	300	5.70	B1 B1 B1 M1 A1 B1 6	for table of values points plotted Straight line Gradient P_0
T	P	$\ln P$ (to 3dp)														
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	(iii)	<table border="1" data-bbox="300 712 523 969"> <thead> <tr> <th>t</th> <th>P (to 1 dp)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>52.2</td> </tr> <tr> <td>1</td> <td>170</td> </tr> <tr> <td>2</td> <td>287.8</td> </tr> <tr> <td>3</td> <td>336.1</td> </tr> <tr> <td>4</td> <td>357.4</td> </tr> </tbody> </table> 	t	P (to 1 dp)	0	52.2	1	170	2	287.8	3	336.1	4	357.4	M1 A1 M1 A1 5	For 1 st curve For 2 nd curve
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	(iv)	$375 = 150 \arctan(t - 1) + 170$ $\Rightarrow t = 1 + \tan \frac{205}{150}$ $= 5.83\dots$ <p>So the population will exceed 375 in 6 years.</p>	B1 M1 A1 B1 4													