

**MEI**

Mathematics in Education and Industry

# **MEI STRUCTURED MATHEMATICS**

## **METHODS OF ADVANCED MATHEMATICS, C3**

### **Practice Paper C3-B**

### **MARK SCHEME**

Qu	Answer	Mark	Comment
<b>Section A</b>			
1	<p>Call the numbers <math>n, n + 1</math> and <math>n + 2</math>                      At least one of the numbers is even, and so the product is a multiple of 2.                      If <math>n</math> is a multiple of 3 then so is the product.                      If <math>n = 3k + 1</math> then <math>n + 2</math> is a multiple of 3                      If <math>n = 3k + 2</math> then <math>n + 1</math> is a multiple of 3.</p> <p><math>n</math> must have one of the forms <math>3k, 3k + 1</math> or <math>3k + 2</math>.                      Therefore whichever it is one of the three numbers is a multiple of 3 and so the product is a multiple of 3.                      Since it is also a multiple of 2 it is a multiple of 6.</p>	B1 M1  M1  E1  4	Algebra Divisibility by 2  Divisibility by 3  conclusion
2	(i)	B1  B1  2	Right part  Left part
	(ii) Line $y = 5$ to be shown on graph. $-1 < x < 4$	M1 A1  2	
3	(i) $y = (x^2 + 3)^5$ Let $u = x^2 + 3 \Rightarrow \frac{du}{dx} = 2x$  $y = u^5 \Rightarrow \frac{dy}{du} = 5u^4$  $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5u^4 \times 2x = 10x(x^2 + 3)^4$	M1 A1   A1  3	Chain rule $\frac{dy}{du}$
	(ii) $y = \frac{\sin 2x}{x}$ Let $u = \sin 2x \Rightarrow \frac{du}{dx} = 2 \cos 2x$  $v = x \Rightarrow \frac{dv}{dx} = 1$  $\Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{2x \cos 2x - \sin 2x}{x^2}$	M1  A1   A1  3	Quotient rule
4	$y^2 = 5x - 4 \Rightarrow 2y \frac{dy}{dx} = 5 \Rightarrow \frac{dy}{dx} = \frac{5}{2y}$  When $x = 8, y^2 = 36 \Rightarrow y = \pm 6$  $\Rightarrow$ gradients = $\frac{5}{12}$ and $-\frac{5}{12}$	M1 A1  A1 A1 A1  5	

5		$x = x_0 e^{-3t} \Rightarrow e^{3t} = \frac{x_0}{x}$ $\Rightarrow 3t = \ln\left(\frac{x_0}{x}\right) \Rightarrow t = \frac{1}{3} \ln\left(\frac{x_0}{x}\right)$ $\Rightarrow t = \ln\left(\frac{x_0}{x}\right)^{\frac{1}{3}}$ <p>i.e. <math>a = x_0</math>, <math>b = \frac{1}{3}</math></p>	<p>M1 A1</p> <p>A1 A1</p> <p><b>4</b></p>	<p>Take logs</p> <p>or any equivalent method</p>
6	(i)	$\int (2x-3)^7 dx. \quad \text{Let } u = 2x-3, \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2} du$ $= \int \frac{1}{2} u^7 du = \frac{u^8}{2 \times 8} = \frac{1}{16} (2x-3)^8 + c$	<p>M1 A1</p> <p>A1</p> <p><b>3</b></p>	<p>or B3 cao</p>
	(ii)	<p>The substitution <math>u = x^2 + 1</math> gives <math>\frac{du}{dx} = 2x</math></p> $\Rightarrow \int_1^2 x(x^2 + 1)^3 dx = \int_2^5 \frac{1}{2} u^3 du$ $= \left[ \frac{u^4}{8} \right]_2^5$ $= \frac{609}{8} (= 76\frac{1}{8})$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p><b>5</b></p>	<p>Using sub</p> <p>Correct int</p> <p>Correct limits</p> <p>Int</p> <p>Ans</p>
7	(i)	$f^2(x) = 4x$	<p>B1</p> <p><b>1</b></p>	
	(ii)	$fgh(x) = fg(x+2)$ $= f(x+2)^2$ $= 2(x+2)^2$	<p>M1</p> <p>A1</p> <p>A1</p> <p><b>3</b></p>	<p>correct order of functions</p>
	(iii)	$y = h(x)$ $= x+2$ $\Rightarrow x = y-2$ $h^{-1}(x) = x-2$	<p>B1</p> <p><b>1</b></p>	

Section B				
8	(i)	$0 = (x+2)e^{-x}$ $\Rightarrow x = -2$ so $(-2,0)$ and $(0,2)$	B1 B1	2
	(ii)	$y = (x+2)e^{-x}$ $\Rightarrow \frac{dy}{dx} = -e^{-x}(x+1) = 0 \Rightarrow x = -1$ SP is $(-1,e)$	M1 A1 M1 A1	Product rule  = 0  4
	(iii)	$\Rightarrow \frac{d^2y}{dx^2} = xe^{-x}$ At $(-1,e)$ this is negative, so SP is a maximum.	M1 A1 A1	3
	(iv)		B1	1
	(v)	At $(0,2)$ gradient is $-1$ so gradient of normal is 1 Normal is $y = x + 2$ . $y = x + 2, y = (x+2)e^{-x}$ $\Rightarrow 0 = (x+2)(1 - e^{-x})$ $\Rightarrow x = -2$ (or 0) New intersection point is $(-2,0)$ .	B1 M1 A1	3
	(vi)	Required area is $\int_1^3 (x+2)e^{-x} dx$ $= [-e^{-x}(x+2)]_1^3 + \int_1^3 e^{-x} dx$ $= [-e^{-x}(x+2)]_1^3 + [-e^{-x}]_1^3$ $= \frac{-6}{e^3} + \frac{4}{e}$	B1 M1 A1 A1 A1	or equivalent  5

9	(i) (A)		The transformation is a stretch with the $x$ -axis invariant and of scale factor 2.	B1  B1  <b>2</b>	Same orientation  y values doubled
	(i) (B)		The transformation is a reflection in the $y$ -axis.	B1  B2  <b>3</b>	same shape  Inversion
	(i) (C)		The transformation is a translation of 2 units parallel to the $x$ -axis, ie $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	B1  B2  <b>3</b>	Same shape  Moved 2 to the right
	(ii)	There is a set of values of $y$ (for example, $y = 1$ ) for which there are three corresponding values of $x$ (so an inverse would be multivalued).		B1  B1  <b>2</b>	
	(iii)			B1  <b>1</b>	
	(iv)	$f(x) = x^2(x+2)$ $\Rightarrow f'(x) = 3x^2 + 4x$ So the gradient at (1,3) is 7. The gradient on the inverse (which is a reflection of the original in $y = x$ ) is therefore $^{-1}/7$ .		M1 A1 M1 A1  <b>4</b>	
	(v)	The graph and its reflection must intersect on the axis of reflection, ie $y = x$ , so solve $y = x, y = x^2(x+2)$ $\Rightarrow x = x^2(x+2)$ $\Rightarrow 0 = x(x^2 + 2x - 1)$ $\Rightarrow x = 0, -1 \pm \sqrt{2}$ The positive non-zero root is as given.		M1  M1  E1  <b>3</b>	