MEI

Mathematics in Education and Industry

# MEI STRUCTURED MATHEMATICS

## CONCEPTS FOR ADVANCED MATHEMATICS, C2

# Practice Paper C2-B

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

**TIME** 1 hour 30 minutes

*INSTRUCTIONS*

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

*INFORMATION*

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- There is an **INSERT SHEET** for question 11.
- The total number of marks for this paper is **72**.

**Section A ( 36 marks)**

**1** Find all the angles in the range $0^0 \le x \le 360^0$ satisfying the equation $\sin x + \frac{1}{2}\sqrt{3} = 0$. [3]

**2** Solve the equation $3^x = 15$, giving your answer correct to 4 decimal places. [3]

**3** The sum to infinity of a geometric series is 5 and the first term is 2.
Find the common ratio of the series. [3]

**4** The first 3 terms of an arithmetical progression are 7, 5.9 and 4.8.

Find

**(i)** the common difference, [1]

**(ii)** the smallest value of $n$ for which the sum to $n$ terms is negative. [4]

**5** The gradient of a curve is given by the function $\frac{dy}{dx} = 2 - x$.
The curve passes through the point (1, 2).

Find the equation of the curve. [4]

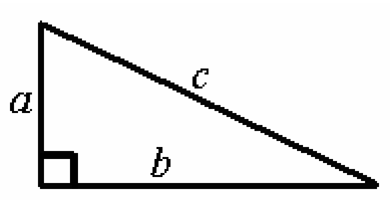
**6** Evaluate $\int_1^2 \left( x^2 + \frac{1}{x^2} \right) dx$. [5]

**7** **(i)** Using the triangle, show that [3]

$\sin^2 x + \cos^2 x = 1$.

**(ii)** Hence prove that [2]

$1 + \tan^2 x = \frac{1}{\cos^2 x}$.




**8** Draw two sketches of the graph of $y = \sin x$ in the range $0^o \le x \le 360^o$.

**(i)** On the first sketch, draw also a sketch of $y = \sin(2x)$. [2]

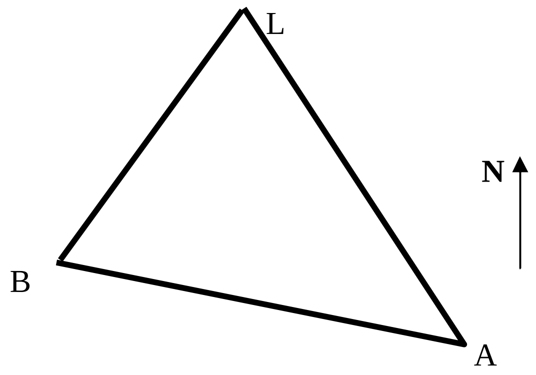
**(ii)** On the second sketch, draw also a sketch of $y = 2\sin x$. [2]

**9** A sector of a circle has an angle of 0.8 radians. The arc length is 5 cm.
Calculate the radius of the circle and the area of the sector. [4]

**Section B (36 marks)**

**10** At 1200 the captain of a ship observes that the bearing of a lighthouse is $340^o$. His position is at A.
At 1230 he takes another bearing of the lighthouse and finds it to be $030^o$. During this time the ship moves on a constant course of $280^o$ to the point B.

His plot on the chart is as shown in Fig. 11 below.




**Not to scale**

**Fig. 11**

**(i)** Write down the size of the angles LAB and LBA. [2]

**(ii)** The captain believes that at A he is 5 km from L. Assuming that LA is exactly 5 km, show that LB is 4.61 km , correct to 2 decimal places, and find AB.
Hence calculate the speed of the ship. [8]

**(iii)** The speed of the ship is actually 10 kilometres per hour. Given that the bearings of $340^o$ and $030^o$ and the ship's course of $280^o$ are all accurate, calculate the true value of the distance LA. [2]

**11** ***You should use the insert sheet for this question*.**

John records the speed of a car in metres per second over a period of 10 seconds.
His results are shown in the table below.

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $v$ | 0 | 9 | 15 | 18 | 15 | 10 |

**(i)** The speed-time graph on the insert sheet provides the axes and the first two points plotted. Plot the remainder of these points and join them with a smooth curve. [2]

The area between this curve and the $t$-axis represents the distance travelled by the car in this time.

**(ii)** Using the trapezium rule with 6 values of $t$ estimate the area under the curve to give the distance travelled. Illustrate on your graph the area found. [3]

**(iii)** John's teacher suggests that the equation of the curve could be $v = 6t - \frac{1}{2}t^2$.

Find, by calculus, the area between this curve and the $t$ axis. [5]

**(iv)** Plot this curve on your graph. Comment on whether the estimates obtained in parts (ii) and (iii) are overestimates or underestimates. [2]

**12** Fig. 12 shows a window. The base and sides are parts of a rectangle with dimensions $2x$ metres horizontally by $y$ metres vertically. The top is a semicircle of radius $x$ metres. The perimeter of the window is 10 metres.

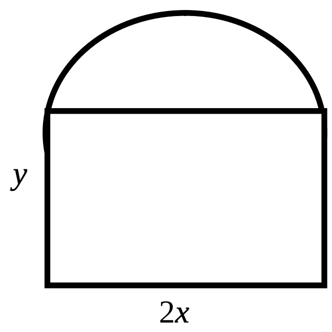



**Fig.12**

**(i)** Express $y$ as a function of $x$. [2]

**(ii)** Find the total area, $A$ m$^2$, in terms of $x$ and $y$. Use your answer to part (i) to show that this simplifies to

$$A = 10x - 2x^2 - \frac{1}{2}\pi x^2 .$$ [4]

**(iii)** Prove that for the maximum value of $A$, $y = x$ exactly. [6]

NAME OF CANDIDATE:……………………………….

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**11** Speed-time graph with the first two points plotted.