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MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

CONCEPTS OF ADVANCED MATHEMATICS, C2

Practice Paper C2-A

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Qu		Answer	Mark	Comment
Section A				
1		$\frac{dy}{dx} = 2 - 5x$ $\Rightarrow y = 2x - \frac{5}{2}x^2 + c$ When $x = 0, y = 1$ $\Rightarrow 1 = c \Rightarrow y = 2x - \frac{5}{2}x^2 + 1$	M1 A1 M1 A1 **4**	
2	**(i)**	$\log_2 5 + \log_2 1.6 = \log_2 5 \times 1.6$ $= \log_2 8 = 3$	M1 A1 **2**	
	(ii)	$2^x = 3 \Rightarrow x \ln 2 = \ln 3$ $\Rightarrow x = \frac{\ln 3}{\ln 2}$ ≈ 1.5850	M1 A1 A1 **3**	
3	**(i)**	A.P.: 5 + 8 + 11… $\Rightarrow a_8 = 5 + 7\times 3 = £26$	M1 A1 **2**	
	(ii)	We require $S_n > 200$ so solve $S_n = 200$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right) = \frac{n}{2}\left(10 + (n-1)3\right) = 200$ $\Rightarrow 10n + 3n(n-1) = 400 \Rightarrow 3n^2 + 7n - 400 = 0$ $\Rightarrow n = \frac{-7 \pm \sqrt{49 + 4800}}{6} = \frac{-7 \pm 69.6}{6} = 10.4$ So minimum time is 11 years.	M1 A1 M1 A1 **4**	
4		$\frac{dy}{dx} = 3x^2 + 2$ When $x = 0, \frac{dy}{dx} = 2 \Rightarrow y - 7 = 2(x - 0)$ $\Rightarrow y = 2x + 7$	B1 B1 M1 A1 **4**	
5	**(i)**	. $2\sin^2\theta + 3\cos\theta = 2 - 2\cos^2\theta + 3\cos\theta$	B1 **1**	
		$2\sin^2\theta + 3\cos\theta = 3 \Rightarrow 2 - 2\cos^2\theta + 3\cos\theta = 3$ $\Rightarrow 2\cos^2\theta - 3\cos\theta + 1 = 0$ $\Rightarrow (2\cos\theta - 1)(\cos\theta - 1) = 0 \Rightarrow \cos\theta = 1, \frac{1}{2}$ $\Rightarrow \theta = 0^\circ, 60^\circ, 300^\circ, 360^\circ$	B1 M1 A1 A1 **5**	For 0,60,360 For 300

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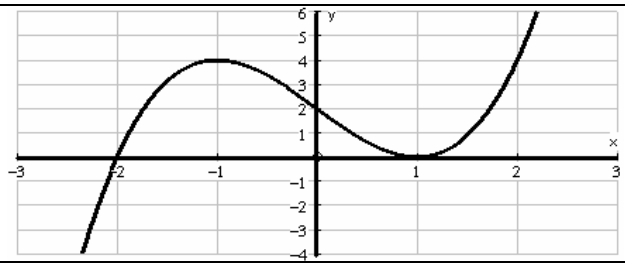
6	**(i)**	$s = r\theta \Rightarrow r = \frac{45}{2} = 22.5\,(\text{cm})$	M1 A1 **2**	
	(ii)	$A = \frac{1}{2}r^2\theta = 22.5^2 = 506.25\,(\text{cm}^2)$	M1 A1 **2**	
7	**(i)**	$S_{10} = a\frac{1-r^{10}}{1-r}; \quad r = 0.8, a = 5$ $\Rightarrow S_{10} = 5\frac{1-0.8^{10}}{0.2} = 22.32$	M1 A1 **2**	
	(ii)	$S_\infty = a\frac{1}{1-r} = 25$	B1 **1**	
8		$BC^2 = 5^2 + 6^2 - 2\times5\times6\times\cos110$ $= 61 + 20.52 = 81.52$ $\Rightarrow BC = 9.03\,\text{cm}$	M1 A1 B1 A1 A1 **5**	For getting neg for cos110
Section B				
9	**(i)**	Multiply out to get $x^3 - 3x + 2$	M1 A1 **2**	
	(ii)	$y = x^3 - 3x + 2 \Rightarrow \frac{dy}{dx} = 3x^2 - 3$ $\frac{dy}{dx} = 0$ when $x = \pm1$ $\frac{d^2y}{dx^2} = 6x;$ when x = -1, $\frac{d^2y}{dx^2} < 0$ so maximum when x = 1 $\frac{d^2y}{dx^2} > 0$ so minimum. when x = 1, y = 0	M1 A1 M1 M1 E1 M1 B1 **7**	
	(iii)		B1 **1**	
	(iv)	For values of k within max and min values of y there are three roots. When $x = -1$, $y = 4$ i.e. $k < 0$ and $k > 4$	M1 B1 A1 **3**	

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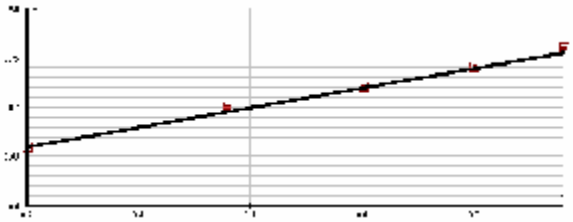
10	**(i)**	$y = ax^b \Rightarrow \log y = \log ax^b$ $\Rightarrow \log y = \log a + \log x^b$ i.e. $\log y = \log a + b\log x$ This is of the form of $y = mx + c$ So plotting the points will give a straight line where intercept is $\log a$ and gradient is b.	M1 A1 B1 **3**	 Explanation
	(ii)	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$\log x$</td><td>0.3</td><td>0.48</td><td>0.60</td><td>0.70</td><td>0.78</td></tr><tr><td>y</td><td>4.6</td><td>5.0</td><td>5.3</td><td>5.5</td><td>5.7</td></tr><tr><td>$\log y$</td><td>0.66</td><td>0.70</td><td>0.72</td><td>0.74</td><td>0.76</td></tr></table>  Straight line so model is appropriate. Gradient of line $\approx \frac{1}{5} \Rightarrow b = \frac{1}{5}$ Gives intercept $\approx 0.6 \Rightarrow a = 4$ i.e. $y = 4x^{0.2}$	B1 B1 B1 B1 B1 **7**	Correct $\log x$ values Correct $\log y$ values Correct plot Straight line drawn Or substitute if the origin not on graph
	(iii)	$y = 4 \times (2.8)^{0.2} \approx 4.91$	M1 A1 **2**	
11	**(i)**	$\frac{dy}{dx} = -x + 3$ $= 0$ when $x = 3$ $\Rightarrow y = 1.5$ (Alt: Highest point is where $x = 3$ $\Rightarrow y = -\frac{9}{2} + 9 - 3 = 1.5$)	M1 M1 A1 A1 **4**	Differentiate Set = 0 For x For y Alt. B2 M1 A1
	(ii)	$\text{Area} = 1 \times 2 + \int_2^4 \left(-\frac{1}{2}x^2 + 3x - 3\right) dx + 1 \times 2$ $= 4 + \left[-\frac{x^3}{6} + \frac{3x^2}{2} - 3x\right]_2^4$ $= 4 + \left(-\frac{64}{6} + \frac{48}{2} - 12\right) - \left(-\frac{8}{6} + \frac{12}{2} - 6\right)$ $= 4 + 1\frac{1}{3} + 1\frac{1}{3} = 6\frac{2}{3}$	M1 A1 B1 M1 A1 A1 A1 **7**	 Adding on the 4 Definite integral $2^2/_3$ c.a.o.