

STATISTICS (C) UNIT 1 TEST PAPER 3

1. A die is biased so that the probability of scoring a 6 is $\frac{1}{5}$.
- (i) Find the probability that the die is thrown at least 10 times without scoring a 6 before the first 6 is scored. [2]
- (ii) Write down an expression for the probability that the first 6 is scored on the n th throw. [2]
2. The mean mark obtained by a class of 20 pupils in a test was μ . The marks, x , were such that $\Sigma (x - \mu)^2 = 7578 \cdot 2$.
- (i) Find the standard deviation of the pupils' marks. [2]
- (ii) Given further that $\mu = 58 \cdot 7$, find the value of Σx^2 . [3]
3. The discrete random variable X has the probability function given by the following table:
- | | | | | | | | |
|----------|------|------|------|------|-----|------|-----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(X=x)$ | 0.09 | 0.12 | 0.22 | 0.16 | p | $2p$ | 0.2 |
- (i) Show that $p = 0.07$ [2]
- (ii) Find the values of $E(X)$ and $\text{Var}(X)$. [5]
4. Of the students who enrol on a university course, 15% drop out within the first year and a further 5% of the original total drop out before the end of the course. One year, 25 students enrol. Find the probability that there are still more than 20 students on the course
- (i) after one year, [2]
- (ii) at the end of the course. [2]
- State an assumption that you have made. Is this assumption likely to be valid? [2]
- 10 of the students are mature students. The probability that at least 8 of these will still be on the course after a year is calculated to be 0.3, to 1 decimal place.
- Find the proportion of mature students who drop out during the first year. [4]
5. Sixteen cards have been lost from a pack, which therefore contains only 36 cards. Two cards are drawn at random from the pack. The probability that **both** cards are red is $\frac{1}{3}$.
- (i) Show that r , the number of red cards in the pack, satisfies the equation
- $$r(r - 1) = 420. \quad [3]$$
- (ii) Hence or otherwise find the value of r . [3]
- (iii) Find the conditional probability that, when two cards are drawn at random from the pack, the first one is red given that at least one is red. [4]

6. Twenty pairs of observations are made of two variables x and y , which are believed to be related.

It is found that

$$\Sigma x = 200, \quad \Sigma y = 174, \quad \Sigma x^2 = 6201, \quad \Sigma y^2 = 5102, \quad \Sigma xy = 5200.$$

Find

(i) the product-moment correlation coefficient between x and y , [2]

(ii) the equation of the regression line of y on x . [3]

Given that $p = x + 30$ and $q = y + 50$,

(iii) write down the product-moment correlation coefficient between p and q . [1]

(iv) Find the equation of the regression line of q on p , in the form $q = mp + c$. [3]

(v) Estimate the value of q when $p = 46$, stating any assumptions you make. [3]

7. In a survey of natural habitats, the numbers of trees in sixty equal areas of land were recorded,

as follows:

17	12	9	23	40	32	11	5	34	22	31	8
15	45	10	52	14	13	29	43	69	30	15	47
35	6	24	13	19	26	9	31	27	18	6	20
22	18	30	51	49	35	50	25	8	10	26	31
33	29	40	37	38	44	24	34	42	38	11	23

(i) Construct a stem-and-leaf diagram to illustrate this data, using the groupings 5 - 9, 10 - 14, 15 - 19, 20 - 24, etc. [6]

(ii) Estimate the median of the distribution. [2]

The first and third quartiles of the distribution are 14 and 35 respectively.

(iii) On graph paper construct a box-and-whisker plot for the data, showing your scale and clearly indicating any outliers. [4]

STATISTICS 1 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. (i) $(0.8)^{10} = 0.107$ (ii) $(0.8)^{n-1}(0.2)$ M1 A1; M1 A1 4
2. (i) s.d. = $\sqrt{(7578.2 \div 20)} = 19.5$ M1 A1
 (ii) $\Sigma x^2 \div 20 - 58.7^2 = 19.466^2$ $\Sigma x^2 = 76\,492$ M1 A1 A1 5
3. (i) $3p + 0.79 = 1$ $3p = 0.21$ $p = 0.07$ M1 A1
 (ii) $E(X) = 3.22$ M1 A1
 $\text{Var}(X) = E(X^2) - 3 \cdot 22^2 = 14.26 - 3 \cdot 22^2 = 3.892$ M1 A1 A1 7
4. (i) $X \sim B(25, 0.85)$: $P(X > 20) = 1 - P(X \leq 20) = 1 - 0.3179 = 0.682$ M1 A1
 (ii) $X \sim B(25, 0.8)$: $P(X > 20) = 1 - P(X \leq 20) = 1 - 0.5793 = 0.421$ M1 A1
 Assumed each student's leaving is independent of all others B1
 May not be true - some may influence others B1
 $B(10, p)$: $P(X \geq 8) = 0.3$ so $P(X \leq 7) = 0.7$ M1 A1 A1
 From tables, $p = \frac{2}{3}$ so $\frac{1}{3}$ drop out in first year A1 10
5. (i) $\frac{r}{36} \times \frac{r-1}{35} = \frac{1}{3}$ $r(r-1) = 12 \times 35 = 420$ M1 A1 A1
 (ii) $r^2 - r - 420 = 0$ $(r-21)(r+20) = 0$ $r = 21$ M1 M1 A1
 (iii) $P(\text{at least one red}) = 1 - \frac{5}{12} \times \frac{14}{35} = \frac{5}{6}$ M1 A1
 $P(\text{1st red} \mid \text{at least one red}) = \frac{\frac{7}{12}}{\frac{5}{6}} = \frac{7}{10}$ M1 A1 10
6. (i) $S_{xx} = 4201$, $S_{yy} = 3588.2$, $S_{xy} = 3460$ $r = 0.891$ M1 A1 A1
 (ii) $y - 8.7 = \frac{3460}{4201}(x - 10)$ $y = 0.824x + 0.464$ M1 M1 A1
 (iii) Same as between x and y : 0.891 B1
 (iv) $q - 50 = 0.824(p - 30) + 0.464$ $q = 0.824p + 25.8$ M1 A1 A1
 (v) When $p = 46$, $q \approx 63.6$ Assumed these values of p and q are within or close to the range from which the data was collected M1 A1 B1 12
7. (i) Stem-and-leaf diagram drawn M2 A1 any one group
 Totals in groups: 7, 8, 6, 7, 6, 9, 5, 5, 3, 3, 0, 0, 1 A1 A1 A1
 (ii) Median = 30th - 31st value = 26 M1 A1
 (iii) Box plot drawn Outlier at 69 B3 B1 12