GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 6 questions.

When a numerical value of g is required, use $g = 9.8\ \text{m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. A particle P of mass 1.5 kg moves from rest at the origin such that at time t seconds it is subject to a single force of magnitude $(4t + 3)$ N in the direction of the positive x-axis.

(a) Find the magnitude of the impulse exerted by the force during the interval $1 \leq t \leq 4$.

(3 marks)

Given that at time T seconds, P has a speed of 22 m s^{-1},

(b) find the value of T correct to 3 significant figures. **(5 marks)**

2.

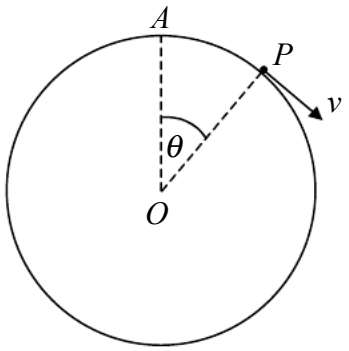

Fig. 1

A particle P of mass 0.5 kg is at rest at the highest point A of a smooth sphere, centre O, of radius 1.25 m which is fixed to a horizontal surface.

When P is slightly disturbed it slides along the surface of the sphere. Whilst P is in contact with the sphere it has speed v m s^{-1} when $\angle AOP = \theta$ as shown in Figure 1.

(a) Show that $v^2 = 24.5(1 - \cos\theta)$. **(3 marks)**

(b) Find the value of $\cos\theta$ when P leaves the surface of the sphere. **(5 marks)**

---

3. A car starts from rest at the point O and moves along a straight line. The car accelerates to a maximum velocity, V m s^{-1}, before decelerating and coming to rest again at the point A.

The acceleration of the car during this journey, a m s^{-2}, is modelled by the formula

$$a = \frac{500 - kx}{150},$$

where x is the distance in metres of the car from O.

Using this model and given that the car is travelling at 16 m s^{-1} when it is 40 m from O,

(a) find k, **(6 marks)**

(b) show that $V = 41$, correct to 2 significant figures, **(3 marks)**

(c) find the distance OA. **(3 marks)**

4.

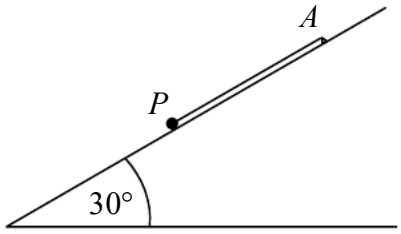

Fig. 2

A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.5 m and modulus of elasticity λ. The other end of the string is fixed to a point A on a rough plane inclined at an angle of 30° to the horizontal as shown in Figure 2. The coefficient of friction between P and the plane is $\frac{1}{6}\sqrt{3}$.

P is held at rest at A and then released. It first comes to instantaneous rest at the point B, 2.2 m from A. For the motion of P from A to B,

(a) show that the work done against friction is 10.78 J, **(5 marks)**

(b) find the change in the gravitational potential energy of P. **(2 marks)**

By using the work-energy principle, or otherwise,

(c) find λ. **(5 marks)**

Turn over

5.

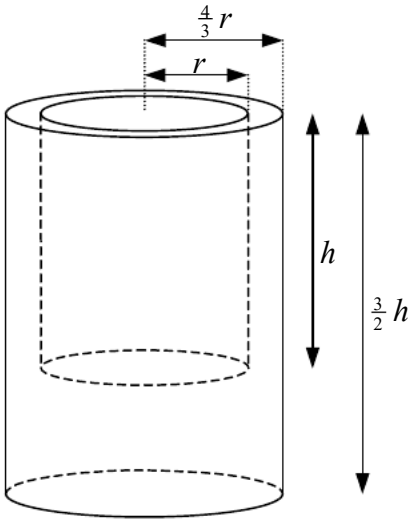

Fig. 3

A flask is modelled as a uniform solid formed by removing a cylinder of radius r and height h from a cylinder of radius $\frac{4}{3}r$ and height $\frac{3}{2}h$ with the same axis of symmetry and a common plane as shown in Figure 3.

(a) Show that the centre of mass of the flask is a distance of $\frac{9}{10}h$ from the open end of the flask.

(7 marks)

The flask is made from a material of density ρ and is filled to the level of the open plane face with a liquid of density $k\rho$. Given that the centre of mass of the flask and liquid together is a distance of $\frac{15}{22}h$ from the open end of the flask,

(b) find the value of k. **(7 marks)**

(c) Explain why it may be advantageous to make the base of the flask from a more dense material.

(2 marks)

---

6. A particle P of mass 2.5 kg is moving with simple harmonic motion in a straight line between two points A and B on a smooth horizontal table. When P is 3 m from O, the centre of the oscillations, its speed is 6 m s^{-1}. When P is 2.25 m from O, its speed is 8 m s^{-1}.

(a) Show that $AB = 7.5$ m. **(8 marks)**

(b) Find the period of the motion. **(4 marks)**

(c) Find the kinetic energy of P when it is 2.7 m from A. **(3 marks)**

(d) Show that the time taken by P to travel directly from A to the midpoint of OB is $\frac{\pi}{4}$ seconds. **(4 marks)**

END