GCE Examinations

Advanced Subsidiary / Advanced Level

Mechanics
Module M3

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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M3 Paper F – Marking Guide

1. *(a)* impulse = $\int_1^4 F \; dt = \int_1^4 (4t + 3) \; dt$ M1

$= [2t^2 + 3t]_1^4 = (32 + 12) - (2 + 3) = 39$ Ns M1 A1

(b) $\int_0^T F \; dt = m(v - u)$ M1

$\therefore [2t^2 + 3t]_0^T = 1.5(22 - 0)$ A1

giving $2T^2 + 3T - 33 = 0$ A1

quad. form. gives $T = {}^-4.88, 3.38$; $T > 0 \;\; \therefore \; T = 3.38$ s (3sf) M1 A1 **(8)**

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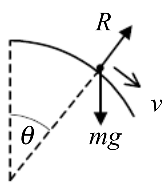
2. *(a)* con. of ME: $\frac{1}{2}mv^2 = mgh$ M1

giving $v^2 = 2g(r - r\cos\theta) = 2 \times 9.8 \times 1.25(1 - \cos\theta) = 24.5(1 - \cos\theta)$ M1 A1

(b)



resolve ↙: $mg\cos\theta - R = \frac{mv^2}{r} = \frac{mv^2}{1.25}$ M1 A1

leaves surface when $R = 0 \;\; \therefore \; v^2 = 1.25g\cos\theta$ M1

combining, $24.5(1 - \cos\theta) = 1.25g\cos\theta$ M1

$\therefore 24.5 = \cos\theta\,(1.25 \times 9.8 + 24.5)$

giving $\cos\theta = \frac{24.5}{36.75} = \frac{2}{3}$ A1 **(8)**

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3. *(a)* $a = v\frac{dv}{dx} = \frac{500 - kx}{150}$ M1

$\therefore \int 150v \; dv = \int 500 - kx \; dx$ M1

$75v^2 = 500x - \frac{1}{2}kx^2 + c$ A1

$x = 0, v = 0 \;\; \therefore \; c = 0$ M1

$x = 40, v = 16 \;\; \therefore \; 19200 = 20000 - 800k$ giving $k = 1$ M1 A1

(b) v_{max} when $a = 0 \;\; \therefore \; \frac{500 - x}{150} = 0$ so $x = 500$ M1

$75{v_{max}}^2 = 250000 - 125000$ giving $v_{max} = 41$ ms^{-1} (2sf) M1 A1

(c) when $v = 0$, $500x - \frac{1}{2}x^2 = 0$ M1

$\therefore \frac{1}{2}x(1000 - x) = 0$ so $x = 0$ (at O) or $1000 \;\; \therefore \; OA = 1000$ m M1 A1 **(12)**

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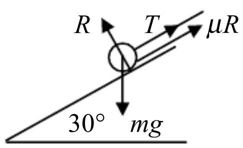
4. *(a)*



resolve ↖: $R - 2g\cos 30 = 0 \;\; \therefore \; R = \sqrt{3}\,g$ M1 A1

friction = $\mu R = \frac{1}{6}\sqrt{3} \times \sqrt{3}\,g = \frac{1}{2}g$ A1

work done = $Fs = \frac{1}{2} \times 9.8 \times 2.2 = 10.78$ J M1 A1

(b) change in GPE = $mgh = 2 \times 9.8 \times 2.2\sin 30 = 21.56$ J (loss) M1 A1

(c) work done = loss of GPE – gain of EPE M1

$\therefore 10.78 = 21.56 - \frac{\lambda x^2}{2l}$ M1 A1

giving $\frac{\lambda \times 0.7^2}{2 \times 1.5} = 10.78$ so $\lambda = 66$ N M1 A1 **(12)**

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5. *(a)*

portion	mass	y	my
large cylinder	$\rho\times\pi(\frac{4}{3}r)^2\times\frac{3}{2}h = \frac{8}{3}\rho\pi r^2h$	$\frac{3}{4}h$	$2\rho\pi r^2h^2$
small cylinder	$\rho\pi r^2h$	$\frac{1}{2}h$	$\frac{1}{2}\rho\pi r^2h^2$
flask	$\frac{5}{3}\rho\pi r^2h$	$\bar{y}$	$\frac{3}{2}\rho\pi r^2h^2$

ρ = mass per unit volume y coords. taken vert. from open face M2 A3

$\frac{5}{3}\rho\pi r^2h\times\bar{y} = \frac{3}{2}\rho\pi r^2h^2 \therefore \bar{y} = \frac{3}{2}h\div\frac{5}{3} = \frac{9}{10}h$ M1 A1

(b)

portion	mass	y	my
flask	$\frac{5}{3}\rho\pi r^2h$	$\frac{9}{10}h$	$\frac{3}{2}\rho\pi r^2h^2$
liquid	$k\rho\pi r^2h$	$\frac{1}{2}h$	$\frac{1}{2}k\rho\pi r^2h^2$
full flask	$(\frac{5}{3}+k)\rho\pi r^2h$	$\frac{15}{22}h$	$(\frac{3}{2}+\frac{1}{2}k)\rho\pi r^2h^2$

ρ = mass per unit volume y coords. taken vert. from open face M2 A2

$(\frac{5}{3}+k)\rho\pi r^2h\times\frac{15}{22}h = (\frac{3}{2}+\frac{1}{2}k)\rho\pi r^2h^2$ M1

$\therefore 15(\frac{5}{3}+k) = 22(\frac{3}{2}+\frac{1}{2}k),\ 25+15k = 33+11k,\ k = 2$ M1 A1

(c) e.g. when full the centre of mass is in top half so easily knocked over B2 **(16)**

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6. *(a)* $v^2 = \omega^2(a^2 - x^2)$ B1

$x = \pm 3,\ v = \pm 6 \ \therefore 36 = \omega^2(a^2 - 9)$ giving $\omega^2 = \frac{36}{a^2-9}$ M1 A1

$x = \pm 2.25,\ v = \pm 8 \ \therefore 64 = \omega^2(a^2 - \frac{81}{16})$ M1

combining, $64 = \frac{36}{a^2-9}(a^2 - \frac{81}{16})$ M1

giving $16(a^2 - 9) = 9(a^2 - \frac{81}{16})$ A1

$256a^2 - 2304 = 144a^2 - 729$

so $a^2 = \frac{1575}{112} = \frac{225}{16},\ a = \frac{15}{4} \ \therefore AB = 7.5$ m M1 A1

(b) $a^2 = \frac{225}{16} \ \therefore 64 = \omega^2(\frac{225}{16} - \frac{81}{16})$ so $\omega^2 = \frac{64}{9},\ \omega = \frac{8}{3}$ M1 A1

period $= \frac{2\pi}{\omega} = 2\pi\div\frac{8}{3} = \frac{3}{4}\pi$ s M1 A1

(c) KE $= \frac{1}{2}mv^2 = \frac{1}{2}\times 2.5\times\frac{64}{9}(\frac{225}{16} - 1.05^2) = 115.2$ J M1 A2

(d) $x = a\cos\omega t \ \therefore -\frac{3.75}{2} = 3.75\cos\omega t$ M1 A1

$\cos\omega t = -\frac{1}{2} \ \therefore \omega t = \frac{2\pi}{3}$ M1

$\therefore t = \frac{2\pi}{3}\div\frac{8}{3} = \frac{\pi}{4}$ s A1 **(19)**

Total **(75)**

Performance Record – M3 Paper F

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	Total
Topic(s)	variable force, impulse	motion in a vertical circle	variable accel.	elastic string, EPE	centre of mass	SHM	
Marks	8	8	12	12	16	19	75
Student							