GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8\ \text{m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. The mechanism for releasing the ball on a pinball machine contains a light elastic spring of natural length 15 cm and modulus of elasticity λ.

The spring is held compressed to a length of 9 cm by a force of 4.5 N.

(a) Find λ. **(3 marks)**

(b) Find the work done in compressing the spring from a length of 9 cm to a length of 5 cm.

(4 marks)

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2. A small bead P is threaded onto a smooth circular wire of radius 0.8 m and centre O which is fixed in a vertical plane.

The bead is projected from the point vertically below O with speed u m s^{-1} and moves in complete circles about O.

(a) Suggest a suitable model for the bead. **(1 mark)**

(b) Given that the minimum speed of P is 60% of its maximum speed, use the principle of conservation of energy to show that $u = 7$.

(6 marks)

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3. At time t seconds the acceleration, a m s^{-2}, of a particle is given by

$$a = \frac{4}{(1+t)^3}.$$

When $t = 0$, the particle has velocity 1 m s^{-1} and displacement 3 m from a fixed origin O.

(a) Find an expression for the velocity of the particle in terms of t. **(4 marks)**

(b) Show that when $t = 3$ the particle is 10.5 m from O. **(5 marks)**

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4. A particle of mass 0.5 kg is moving on a straight line with simple harmonic motion.

At time $t = 0$ the particle is instantaneously at rest at the point A. It next comes instantaneously to rest 3 seconds later at the point B where $AB = 4$ m.

(a) For the motion of the particle write down

(i) the period,

(ii) the amplitude. **(2 marks)**

(b) Find the maximum kinetic energy of the particle in terms of π. **(4 marks)**

The point C lies on AB at a distance of 1.2 m from B.

(c) Find the time it takes the particle to travel directly from A to C, giving your answer in seconds correct to 2 decimal places.

(4 marks)

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5. When a particle of mass M is at a distance of x metres from the centre of the moon, the gravitational force, F N, acting on it and directed towards the centre of the moon is given by

$$F = \frac{(4.90\times10^{12})M}{x^2}.$$

A rocket is projected vertically into space from a point on the surface of the moon with initial speed u m s^{-1}. Given that the radius of the moon is (1.74×10^6) m,

(a) show that the speed of the rocket, v m s^{-1}, when it is x metres from the centre of the moon is given by

$$v^2 = u^2 + \frac{a}{x} - b,$$

where a and b are constants which should be found correct to 3 significant figures.

(7 marks)

(b) Find, correct to 2 significant figures, the minimum value of u needed for the rocket to escape the moon's gravitational attraction.

(4 marks)

Turn over

6.

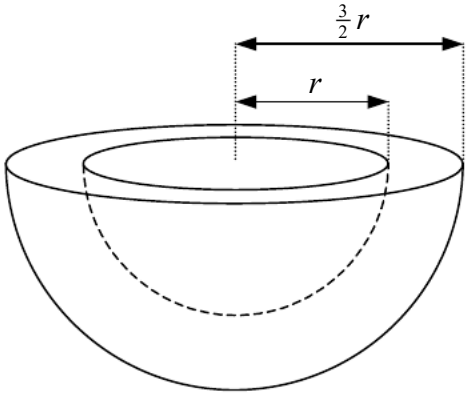

Fig. 1

Figure 1 shows a bowl formed by removing from a solid hemisphere of radius $\frac{3}{2}r$ a smaller hemisphere of radius r having the same axis of symmetry and the same plane face.

(a) Show that the centre of mass of the bowl is a distance of $\frac{195}{304}r$ from its plane face.

(7 marks)

The bowl has mass M and is placed with its curved surface on a smooth horizontal plane. A stud of mass $\frac{1}{2}M$ is attached to the outer rim of the bowl.

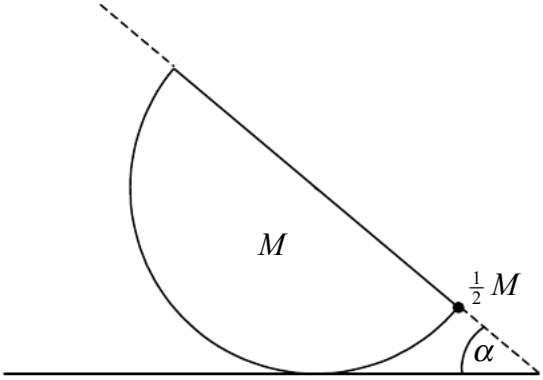

Fig. 2

When the bowl is in equilibrium its plane surface is inclined at an angle α to the horizontal as shown in Figure 2.

(b) Find tan α. **(6 marks)**

7. A cyclist is travelling round a circular bend of radius 25 m on a track which is banked at an angle of 35° to the horizontal.

In a model of the situation, the cyclist and her bicycle are represented by a particle of mass 60 kg and air resistance and friction are ignored.

Using this model and assuming that the cyclist is not slipping,

(a) find, correct to 3 significant figures, the speed at which she is travelling. **(5 marks)**

In tests it is found that the cyclist must travel at a minimum speed of 10 m s^{-1} to prevent the bicycle from slipping down the slope. A more refined model is now used with a coefficient of friction between the bicycle and the track of μ.

Using this model,

(b) show that $\mu = 0.227$, correct to 3 significant figures, **(8 marks)**

(c) find, correct to 2 significant figures, the maximum speed at which the cyclist can travel without slipping up the slope.

(5 marks)

END