GCE Examinations

Mechanics
Module M3

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

When a numerical value of g is required, use $g = 9.8\ \text{m s}^{-2}$.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is attached to a fixed point A and a particle of mass m is attached to the other end.

The particle is released from rest at A and falls vertically until it comes to rest instantaneously at the point B.

Find the distance AB in terms of a. **(7 marks)**

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2. A particle P of mass 0.25 kg is moving on a horizontal plane.

At time t seconds the velocity, $\mathbf{v}$ m s^{-1}, of P relative to a fixed origin O is given by

$$\mathbf{v} = \ln(t + 1)\mathbf{i} - e^{-2t}\mathbf{j}, \quad t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in the horizontal plane.

(a) Find the acceleration of P in terms of t. **(3 marks)**

(b) Find, correct to 3 significant figures, the magnitude of the resultant force acting on P when $t = 1$. **(4 marks)**

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3. A coin of mass 5 grams is placed on a vinyl disc rotating on a record player. The distance between the centre of the coin and the centre of the disc is 0.1 m and the coefficient of friction between the coin and the disc is μ. The disc rotates at 45 revolutions per minute around a vertical axis at its centre and the coin moves with it and does not slide.

By modelling the coin as a particle and giving your answers correct to an appropriate degree of accuracy, find

(a) the speed of the coin, **(2 marks)**

(b) the horizontal and vertical components of the force exerted on the coin by the disc. **(4 marks)**

Given that the coin is on the point of moving,

(c) show that, correct to 2 significant figures, $\mu = 0.23$. **(3 marks)**

4.

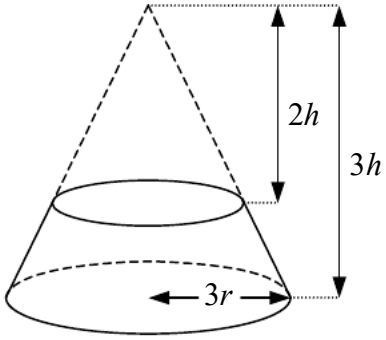

Fig. 1

A stand used to reach high shelves in a storeroom is in the shape of a frustum of a cone. It is modelled as a uniform solid formed by removing a right circular cone of height $2h$ from a similar cone of height $3h$ and base radius $3r$ as shown in Figure 1.

(a) Show that the centre of mass of the stand is a distance of $\frac{33}{76}h$ from its larger plane face.

(7 marks)

The stand is stored hanging in equilibrium from a point on the circumference of the larger plane face. Given that $h = 2r$,

(b) find, correct to the nearest degree, the acute angle which the plane faces of the stand make with the vertical.

(4 marks)

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5. A particle of mass 0.8 kg is moving along the positive x-axis at a speed of 5 m s^{-1} away from the origin O. When the particle is 2 metres from O it becomes subject to a single force directed towards O. The magnitude of the force is $\frac{k}{x^2}$ N when the particle is x metres from O.

Given that when the particle is 4 m from O its speed has been reduced to 3 m s^{-1},

(a) show that $k = \frac{128}{5}$, **(8 marks)**

(b) find the distance of the particle from O when it comes to instantaneous rest. **(4 marks)**

Turn over

6.

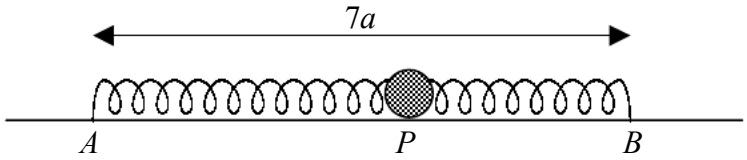

Fig. 2

Figure 2 shows a particle P of mass m which lies on a smooth horizontal table. It is attached to a point A on the table by a light elastic spring of natural length $3a$ and modulus of elasticity λ, and to a point B on the table by a light elastic spring of natural length $2a$ and modulus of elasticity 2λ. The distance between the points A and B is $7a$.

(a) Show that in equilibrium $AP = \frac{9}{2}a$. **(5 marks)**

The particle is released from rest at a point Q where Q lies on the line AB and $AQ = 5a$.

(b) Prove that the subsequent motion of the particle is simple harmonic with a period of $\pi\sqrt{\frac{3ma}{\lambda}}$. **(9 marks)**

7.

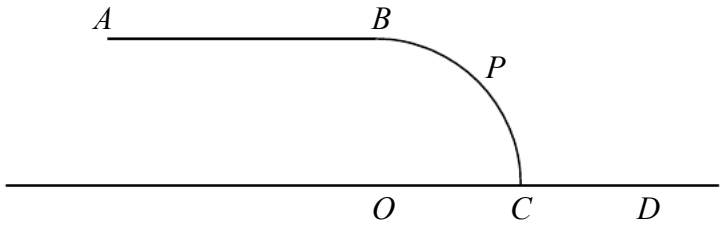

Fig. 3

Figure 3 shows a vertical cross-section through part of a ski slope consisting of a horizontal section AB followed by a downhill section BC. The point O is on the same horizontal level as C and BC is a circular arc of radius 30 m and centre O, such that $\angle BOC = 90°$.

A skier of mass 60 kg is skiing at $12\,\text{m}\,\text{s}^{-1}$ along AB.

(a) Assuming that friction and air resistance may be neglected, find the magnitude of the loss in reaction between the skier and the surface at B. **(4 marks)**

The skier subsequently leaves the slope at the point P.

(b) Find, correct to 3 significant figures, the speed at which the skier leaves the slope. **(8 marks)**

(c) Find, correct to 3 significant figures, the speed of the skier immediately before hitting the ground again at the point D which is on the same horizontal level as C. **(3 marks)**

END