GCE AS Level Mathematics (8MA0) – Shadow Paper (Set 1)

8MA0-01 Pure Mathematics

June 2022 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper, which was published in August 2022.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(4x^3 + \frac{2}{3\sqrt{x}} + 1\right) dx = \frac{4x^4}{4} \ldots + x$	A1	1.1b
	$= \ldots + 2 \times \frac{2}{3} x^{\frac{1}{2}} + \ldots$	A1	1.1b
	$= x^4 - \frac{4}{3} x^{\frac{1}{2}} + x + c$	A1	1.1b
		(4)	
			(4 marks)

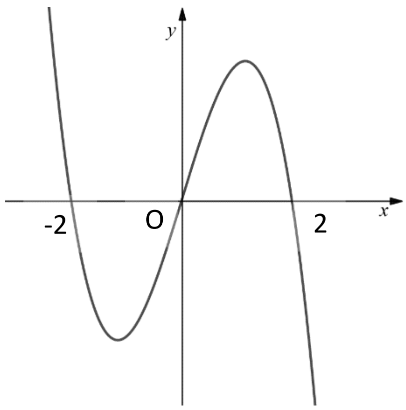
Question	Scheme	Marks	AOs
2(a)	$f(-2) = 3(-2)^3 + 10(-2)^2 + 13(-2) + 10$ $= -24 + 40 - 26 + 10$	M1	1.1b
	$f(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	At least 2 of: $a = 3,\ b = 4,\ c = 5$	M1	1.1b
	All of: $a = 3,\ b = 4,\ c = 5$	A1	1.1b
		(2)	
(c)	$b^2 - 4ac = (4)^2 - 4(3)(5)$	M1	2.1
	$b^2 - 4ac = -44$ which is < 0 so the quadratic has no real roots so $f(x) = 0$ has only 1 real root	A1	2.4
		(2)	
(d)	$(x =)\ 1$	B1	2.2a
		(1)	
			(7 marks)

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{QR}=\overrightarrow{PR}-\overrightarrow{PQ}=10\mathbf{i}-7\mathbf{j}-(2\mathbf{i}+5\mathbf{j})$	M1	1.1a
	$=8\mathbf{i}-12\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\left\|\overrightarrow{QR}\right\|=\sqrt{"8"^2+"(-12)"^2}$	M1	2.5
	$=4\sqrt{13}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS}=\overrightarrow{PQ}+\frac{1}{4}\overrightarrow{QR}=2\mathbf{i}+5\mathbf{j}+\frac{1}{4}("8\mathbf{i}-12\mathbf{j}")=...$ or $\overrightarrow{PS}=\overrightarrow{PR}+\frac{3}{4}\overrightarrow{RQ}=10\mathbf{i}-7\mathbf{j}+\frac{3}{4}("-8\mathbf{i}+12\mathbf{j}")=...$	M1	3.1a
	$=4\mathbf{i}+2\mathbf{j}$	A1	1.1b
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
4(a)(i)	$(2x+1)^2=(x+2)^2+(3x-3)^2-2(x+2)(3x-3)\cos 60°$ oe	M1	3.1a
	Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$3x^2-21x+18=0\Rightarrow x^2-7x+6=0$ *	A1*	2.1
		(3)	
(ii)	$x=6$	B1	3.2a
		(1)	
(b)	$\frac{8}{\sin ACB}=\frac{13}{\sin 60°}\Rightarrow \sin ACB=...\left(\frac{9\sqrt{3}}{13}\right)$ or e.g. $8^2=15^2+13^2-2\times15\times13\cos ACB\Rightarrow \cos ACB=...\left(\frac{11}{13}\right)$	M1	1.1b
	$\theta=$ awrt 32.2	A1	1.1b
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
5(a)	$p = 10^{0.6}$ (or $\log_{10} p = 0.6$) or $q = 10^{0.02}$ (or $\log_{10} q = 0.02$)	M1	1.1b
	$p =$ awrt 3.981 or $q =$ awrt 1.047	A1	1.1b
	$p = 10^{0.6}$ (or $\log_{10} p = 0.6$) and $q = 10^{0.02}$ (or $\log_{10} q = 0.02$)	dM1	3.1a
	$M = 3.981 \times 1.047^t$	A1	3.3
		(4)	
(b)(i)	The initial mass (in g) of bacteria (in the colony).	B1	3.4
(b)(ii)	The ratio of bacteria from one day to the next.	B1	3.4
		(2)	
(c)(i)	5.01 g	B1	2.2a
(c)(ii)	$7 = "3.981" \times "1.047"^t$ or $\log_{10} 7 = 0.02t + 0.6$	M1	3.4
	awrt 12.3 (days)	A1	1.1b
		(3)	
(d)	• The model predicts unlimited growth.	B1	3.5b
		(1)	
			(10 marks)

Question	Scheme	Marks	AOs
6(a)	2^7 or 128 as the constant term	B1	1.1b
	$\left(2 - \frac{5x}{6}\right)^7 = ... + {}^7C_1(2)^6\left(-\frac{5x}{6}\right) + {}^7C_2(2)^5\left(-\frac{5x}{6}\right)^2 + {}^7C_3(2)^4\left(-\frac{5x}{6}\right)^3 + ...$ $= ... + 7 \times (2)^6\left(-\frac{5x}{6}\right) + 21 \times (2)^5\left(-\frac{5x}{6}\right)^2 + 35 \times (2)^4\left(-\frac{5x}{6}\right)^3 + ...$	M1 A1	1.1b 1.1b
	$= 128 - \frac{1120}{3}x + \frac{1400}{3}x^2 - \frac{8750}{27}x^3 + ...$	A1	1.1b
		(4)	
(b)	Coefficient of x^2 is $\frac{1}{3} \times "\frac{1400}{3}" + \frac{1}{3} \times "-\frac{8750}{27}"$	M1	3.1a
	$= \frac{3850}{81}$	A1	1.1b
		(2)	
			(6 marks)

Question	Scheme		Marks	AOs
7(a)	$4x-x^3=x\left(4-x^2\right)$		M1	1.1b
	$4x-x^3=x(2-x)(2+x)$ oe		A1	1.1b
			(2)	
(b)		A cubic with correct orientation	B1	1.1b
		Passes though origin, (2, 0) and (–2, 0)	B1	1.1b
			(2)	
(c)	$y=4x-x^3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=4-3x^2=0 \Rightarrow x=(\pm)\frac{2\sqrt{3}}{3} \Rightarrow y=\ldots$		M1	3.1a
	$y=(\pm)\frac{16\sqrt{3}}{9}$		A1	1.1b
	$\left\{k \in \mathbb{R}:-\frac{16\sqrt{3}}{9}<k<\frac{16\sqrt{3}}{9}\right\}$ oe		A1ft	2.5
			(3)	
				(7 marks)

Question	Scheme	Marks	AOs
8(a)	$(k=)\ 100$	B1	1.1b
		(1)	
(b)	$500=100+1900\text{e}^{-0.1t} \Rightarrow 1900\text{e}^{-0.1t}=400$	M1	3.1b
	$-0.1t=\ln\left(\frac{4}{19}\right) \Rightarrow t=\ldots$	M1	1.1b
	awrt 15.6 minutes	A1	1.1b
		(3)	
(c)	$\left(\frac{\text{d}V}{\text{d}t}=\right)\ -190\text{e}^{-0.1t}$ $\left(\frac{\text{d}V}{\text{d}t}\right)_{t=20}=-190\text{e}^{-0.1\times 20}$	M1	3.1b
	= awrt 25.7 (litres per minute)	A1	1.1b
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
9(a)(i)	$\log_5\left(\frac{x}{25}\right)=\log_5 x-\log_5 25=p-2$	B1	1.2
(ii)	$\log_5\left(\sqrt[3]{x}\right)=\frac{1}{3}p$	B1	1.1b
		(2)	
(b)	$5\log_5\left(\frac{x}{25}\right)+3\log_5\left(\sqrt[3]{x}\right)=-28\Rightarrow 5p-10+p=-28\Rightarrow p=...$	M1	1.1b
	$p=-3$	A1	1.1b
	$\log_5 x=-3\Rightarrow x=5^{-3}$	M1	1.1b
	$x=\frac{1}{125}$	A1	1.1b
		(4)	
	Alternative for (b) not using (a):		
	$5\log_5\left(\frac{x}{25}\right)+3\log_5\left(\sqrt[3]{x}\right)=-28\Rightarrow\log_5\left(\frac{x}{25}\right)^5+\log_5\left(\sqrt[3]{x}\right)^3=-28$ $\Rightarrow\log_5\frac{x^6}{9765625}=-28$	M1	1.1b
	$\Rightarrow\frac{x^6}{9765625}=5^{-28}$ or equivalent eg $x^6=5^{-18}$	A1	1.1b
	$x^6=9765625\times5^{-28}\Rightarrow x^6=5^{10}\times5^{-28}=5^{-18}\Rightarrow x=\left(5^{-18}\right)^{\frac{1}{6}}=5^{-3}$	M1	1.1b
	$x=\frac{1}{125}$	A1	1.1b
			(6 marks)

Question	Scheme	Marks	AOs
10(a)	$y=\frac{1}{4}x^2-\sqrt{x}+1 \Rightarrow \frac{dy}{dx}=\frac{1}{2}x-\frac{1}{2}x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$x=4 \Rightarrow y=3$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=4}=\frac{1}{2}\times 4-\frac{1}{2}\times 4^{-\frac{1}{2}}\left(=\frac{7}{4}\right) \quad \therefore y-3=\frac{7}{4}(x-4)$	M1	2.1
	$7x-4y-16=0$*	A1*	1.1b
		(5)	
(b)	$\int\left(\frac{x^2}{4}-\sqrt{x}+1\right)dx=\frac{x^3}{12}-\frac{2}{3}x^{\frac{3}{2}}+x(+c)$	M1 A1	1.1b 1.1b
	$y=0 \Rightarrow x=\frac{16}{7}$	B1	2.2a
	Area of R is $\left[\frac{x^3}{12}-\frac{2}{3}x^{\frac{3}{2}}+x\right]_0^4-\frac{1}{2}\times\left(4-"\frac{16}{7}"\right)\times"3"=4-\frac{18}{17}$	M1	3.1a
	$=\frac{10}{7}$	A1	1.1b
		(5)	
			(10 marks)

Question	Scheme	Marks	AOs
11(a)	$(x \pm 3)^2 + (y \pm 4)^2$	M1	1.1b
	(i) Centre is (3, 4)	A1	1.1b
	(ii) Radius is 2	A1	1.1b
		(3)	
(b)	$3y + 2x + 6 = 0 \Rightarrow y = -\frac{2}{3}x + ... \Rightarrow -\frac{2}{3} \to \frac{3}{2}$	B1	2.2a
	$m_N = \frac{3}{2} \Rightarrow y - 4 = \frac{3}{2}(x-3)$ $y - 4 = \frac{3}{2}(x-3),\ 3y + 2x + 6 = 0 \Rightarrow x = ...,\ y = ...$	M1	3.1a
	Intersection is at $\left(-\frac{9}{13}, -\frac{20}{13}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(3+\frac{9}{13}\right)^2 + \left(4+\frac{20}{13}\right)^2}$ So distance required is $\sqrt{\left("3"+"\frac{9}{13}"\right)^2 + \left("4"+"\frac{20}{13}"\right)^2} - "2"$	dM1	3.1a
	$= \frac{24\sqrt{13}}{13} - 2$ (or awrt 4.66)	A1	1.1b
		(5)	
			(8 marks)

Question	Scheme	Marks	AOs
12(a)	$V=\pi r^2h=7800\Rightarrow h=\dfrac{7800}{\pi r^2}$ $\left(\text{or } rh=\dfrac{7800}{\pi r} \text{ or } \pi rh=\dfrac{7800}{r}\right)$	B1	1.1b
	$C=0.05\left(\pi r^2+2\pi rh\right)+0.08\left(\pi r^2\right)$	M1	3.4
	$C=0.13\pi r^2+0.1\pi rh=0.13\pi r^2+0.1\pi r\left(\dfrac{7800}{\pi r^2}\right)$	dM1	2.1
	$C=0.13\pi r^2+\dfrac{780}{r}$ *	A1*	1.1b
		(4)	
(b)	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0.26\pi r-\dfrac{780}{r^2}$	M1 A1	3.4 1.1b
	$\dfrac{\mathrm{d}C}{\mathrm{d}r}=0\Rightarrow r^3=\dfrac{780}{0.26\pi}\Rightarrow r=...$	M1	1.1b
	$r=\sqrt[3]{\dfrac{3000}{\pi}}=9.85...$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ 0.26\pi+\dfrac{1560}{r^3}=0.26\pi+\dfrac{1560}{\text{"9.85"}^3}$	M1	1.1b
	$\left(\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right)\ (2.45..)>0$ Hence minimum (cost)	A1	2.4
		(2)	
(d)	$C=0.13\pi(\text{"9.85"})^2+\dfrac{780}{\text{"9.85"}}$	M1	3.4
	$(C=)119$	A1	1.1b
		(2)	
			(12 marks)

Question	Scheme	Marks	AOs
13(a)	$\dfrac{1}{\sin\theta}+\dfrac{1}{\tan\theta}=\dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}$ or $\dfrac{1+\cos\theta}{\sin\theta}$	M1	1.1b
	$=\dfrac{1+\cos\theta}{\sin\theta}\times\dfrac{1-\cos\theta}{1-\cos\theta}=\dfrac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}=\dfrac{\sin^2\theta}{\sin\theta\left(1-\cos\theta\right)}$	dM1	2.1
	$=\dfrac{\sin\theta}{1-\cos\theta}$ *	A1*	1.1b
		(3)	
(b)	$\dfrac{\sin 2x}{1-\cos 2x}=4\sin 2x,\ \sin 2x\neq 0$	M1	2.1
	$1=4-4\cos 2x$	A1	1.1b
	$\cos 2x=\dfrac{3}{4}\Rightarrow 2x=\ldots\Rightarrow x=\ldots$	M1	1.1b
	$x=20.7°, 159.3°$	A1 A1	1.1b 1.1b
		(5)	
			(8 marks)

Question	Scheme	Marks	AOs
14(i)	The statement is **not true** because e.g. when x = 4, y = -1, $\dfrac{x}{y}=\dfrac{4}{-1}=-4$ (which is < 1)	B1	2.3
		(1)	
(ii)	$n^3-3n^2+2n=n\left(n^2-3n+2\right)=n\left(n-1\right)\left(n-2\right)$	M1	2.1
	$n(n-1)(n-2)$ is the product of 3 consecutive integers	A1	2.2a
	As $n(n-1)(n-2)$ is a multiple of 2 **and** a multiple of 3 it must be a multiple of 6 and so n^3-3n^2+2n is divisible by 6 for all integers n	A1	2.4
		(3)	
			(4 marks)