GCE AS Level Further Mathematics (8FM0)
– Shadow Paper (Set 1)

8FM0-22 Further Pure Mathematics 2

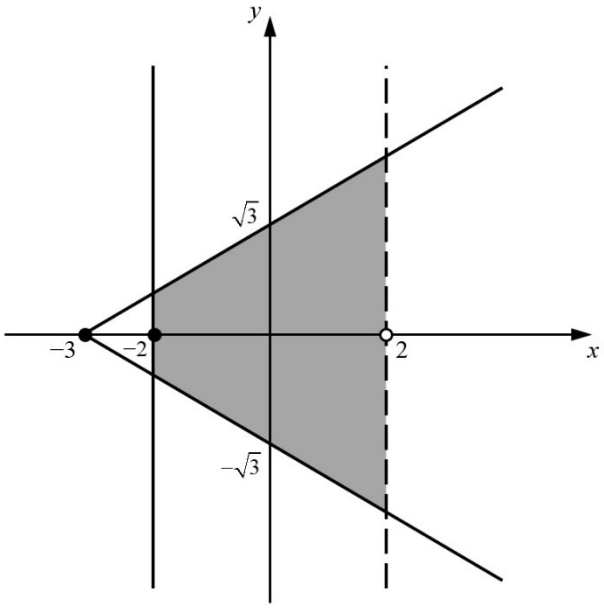
June 2022 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper, which was published in August 2022.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme		Marks	AOs
1		Sector from ± 3 or $\pm\sqrt{3}\mathrm{i}$	**M1**	1.1b
		Strip about either axis	**M1**	1.1b
		Correct strip and sector	**A1**	3.1a
		Fully correct	**A1**	2.5
			(4)	
				(4 marks)

Question	Scheme	Marks	AOs
2	$\begin{vmatrix} 3-\lambda & 7 \\ 2 & -2-\lambda \end{vmatrix} = 0 \Rightarrow (3-\lambda)(-2-\lambda)-14=0$ $\Rightarrow \lambda^2 - \lambda - 20 = 0$	**M1**	3.1a
	$\Rightarrow (\lambda-5)(\lambda+4)=0$ so the eigenvalues are 5 and −4	**A1**	1.1b
	For $\lambda = 5$ eigenvector equations are $\begin{cases} 3x+7y=5x \\ 2x-2y=5y \end{cases} \Rightarrow x=, y=$ OR For $\lambda = -4$ eigenvector equations are $\begin{cases} 3x+7y=-4x \\ 2x-2y=-4y \end{cases} \Rightarrow x=, y=$	**M1**	2.1
	(For $\lambda = 5$, $-2x + 7y = 0$, for $\lambda = -4$, $x + y = 0$ so eigenvectors are) One of $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	**A1**	1.1b
	Both of $\begin{pmatrix} 7 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	**A1**	1.1b
	Hence e.g. $\mathbf{P} = \begin{pmatrix} 7 & -1 \\ 2 & 1 \end{pmatrix}$ or $\mathbf{P} = \begin{pmatrix} -1 & 7 \\ 1 & 2 \end{pmatrix}$	**B1ft**	1.1b
	and $\mathbf{D} = \begin{pmatrix} 5 & 0 \\ 0 & -4 \end{pmatrix}$ or $\mathbf{D} = \begin{pmatrix} -4 & 0 \\ 0 & 5 \end{pmatrix}$ Note: If **P** is given, their **D** must be consistent with it to award this mark.	**B1ft**	2.2a
		(7)	
			(7 marks)

Question	Scheme	Marks	AOs
3(i)	Suppose G has a subgroup of order 11, then (by Lagrange's Theorem) 11 must be a factor of 6 314 757	**M1**	2.1
	But $6 - 3 + 1 - 4 + 7 - 5 + 7 = 9$	**M1**	1.1b
	9 is not divisible by 11, hence 11 is not a factor of $\|G\|$, which contradicts Lagrange's Theorem, so there is no subgroup of order 11.	**A1**	2.4
		(3)	
(ii)(a)	Completes at least one row or column correctly	**M1**	1.1b
	At least 5 rows or columns completed correctly	**A1**	1.1b
	Completely correct	**A1**	1.1b
(b)	As the row and column for 12 repeat the borders, 12 is an identity element for $(X, \times_{22})$	**B1**	2.2a
	Each element has an inverse as follows:	**B1**	1.1b
	Since we know is associative and as there are no new elements in the table, so $(X, \times_{22})$ is closed, hence $(X, \times_{22})$ is a group.	**B1**	2.4
		(6)	
			(9 marks)

$\times_{22}$	2	4	6	8	10	12	14	16	18	20
2	4	8	12	16	20	2	6	10	14	18
4	8	16	2	10	18	4	12	20	6	14
6	12	2	14	4	16	6	18	8	20	10
8	16	10	4	20	14	8	2	18	12	6
10	20	18	16	14	12	10	8	6	4	2
12	2	4	6	8	10	12	14	16	18	20
14	6	12	18	2	8	14	20	4	10	16
16	10	20	8	18	6	16	4	14	2	12
18	14	6	20	12	4	18	10	2	16	8
20	18	14	10	6	2	20	16	12	8	4

x	2	4	6	8	10	12	14	16	18	20
x^{-1}	6	14	2	18	10	12	4	20	8	16

Question	Scheme	Marks	AOs
4(i)(a)	$468 = 5 \times 84 + 48$	**M1**	1.1b
	$84 = 1 \times 48 + 36$; $48 = 1 \times 36 + 12$; $36 = 3 \times 12$ (+ 0)	**M1**	1.1b
	Hence $h = 8$	**A1**	2.2a
		(3)	
(b)	Using back substitution $12 = 48 - 1 \times 36$	**M1**	1.1b
	$= 48 - (84 - 1 \times 48) = 2 \times 48 - 84$ $= 2(468 - 5 \times 84) - 84$	**M1**	1.1b
	$= 2 \times 468 - 11 \times 84$ (so $a = 2$ and $b = -11$)	**A1**	1.1b
		(3)	
(c)	$11 \times 84 = 2 \times 468 - 12 \equiv -12 \pmod{468}$ or $11 \times 84 = 924$ so $925 - 468k =$	**M1**	3.1a
	$c = 468 - 12 = 456$	**A1**	1.1b
		(2)	
(ii)	eg $3^{15} \equiv \left(3^3\right)^5 \equiv 27^5 \equiv \left(-1\right)^5 \pmod{14}$	**M1**	1.1b
	$\equiv -1 \pmod{14}$	**dM1**	1.1b
	$\equiv 13 \pmod{14}$	**A1**	2.2a
		(3)	
			(11 marks)

Question	Scheme	Marks	AOs
5(a)	• Immediately after the first tablets are taken there is 30 mg in the person, so $u_0 = 30$ • Reduction by 70% through day means just before next tablet is taken there is $0.3u_n$ mg of the vitamin left in the person • The next tablet taken adds 15 mg to the amount just before the tablet is taken, giving $u_{n+1} = 0.3u_n + 15$	**B1** **B1**	2.4 3.3
		(2)	
(b)	$u_1 = 0.3\times 30 + 15 = 24$	**B1**	3.1a
	So $\begin{cases} 30 = a + b \\ 24 = \frac{3}{10}a + b \end{cases} \Rightarrow 30 - 24 = a\left(1 - \frac{3}{10}\right) \Rightarrow a = ...$	**M1**	1.1b
	$a = \frac{60}{7}$ and $b = \frac{150}{7}$	**A1** **A1**	1.1b 1.1b
		(4)	
(c)	In long term $u_n = \frac{60}{7}\times 0.3^n + \frac{150}{7} \to \frac{150}{7}$ as $0.3^n \to 0$	**M1**	3.4
	Minimum amount of vitamin occurs just before a tablet is taken, so is $\frac{150}{7} - 15 = \frac{45}{7} = 6\frac{3}{7}$ mg	**M1**	3.1b
	This is greater than 6 mg and so there is always at least 6 mg of the vitamin in the person. The course of the vitamin will be effective.	**A1**	3.2a
		(3)	
			(9 marks)