

GCE AS Level Further Mathematics (8FM0) – Shadow Paper (Set 1)
8FM0-26 Further Mechanics 1

June 2022 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper, which was published in August 2022.

Guidance on the use of codes within this document

M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.

A1 – accuracy mark. This mark is generally given for a correct answer following correct working.

B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.

Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs	
1(a)	<i>ABE</i> <i>BCDE</i> lamina			
	$\frac{3}{2}a^2$ $9a^2$ $\frac{21}{2}a^2$	B1	1.2	Any equivalent ratios e.g. 3 : 18 : 21
	a $\frac{3}{2}a$ \bar{y}	B1	1.2	Or correct distances from a parallel axis
	Moments about <i>AD</i>	M1	2.1	Or moments about a parallel axis
	$(\frac{3}{2}a^2 \times a) + (9a^2 \times \frac{3}{2}a) + = \frac{21}{2}a^2\bar{y}$	A1	1.1b	Correct unsimplified equation for their axis
	$\bar{y} = \frac{10a}{7}$ *	A1*	2.2a	Correct given answer correctly obtained If they have centre of mass at (xa, ya) then the a might not be seen in the working. Otherwise, with no a in the working the maximum score is B1B0M1A0A0
	(5)			
1(b)	Moments about <i>F</i> , $Mg \times (3a - \frac{10a}{7}) = 3aT$	M1	3.1a	A complete method to obtain an equation in T only
	$T = \frac{11Mg}{21}$ (0.523809... Mg)	A1	1.1b	0.52 Mg or better
	(2)			
(7 marks)				

Question	Scheme	Marks	AOs		
2.(a)	$OABC$ $ODEF$	CD framework			
	$8a$ $8a$	$\frac{\pi a}{16a + \pi a}$	B1	1.2	Any equivalent ratios
	$(-)\frac{1}{2}a$ $\frac{1}{2}a$	$\frac{4a}{\pi}$ \bar{x}	B1	1.2	Or correct distances from a parallel axis
	Moments about BC		M1	2.1	Or moments about a parallel axis Must be using framework. If BC included twice mark as a misread.
	$-8a \times \frac{1}{2}a + \pi a \times \frac{4a}{\pi} + 8a \times \frac{1}{2}a = (16a + \pi a)\bar{x}$		A1	1.1b	Correct unsimplified equation for their axis. Allow within a vector equation
	$\bar{x} = \frac{4a}{16 + \pi} *$		A1*	2.2a	Correct given answer correctly obtained
			(5)		
2(b)	Angle $ABD = \tan^{-1}\left(\frac{2a}{a}\right)$		M1	1.1b	Correct relevant angle (or side if they use the cosine rule) Do not need to evaluate: accept $\tan \alpha = \dots$ or $\alpha = \tan^{-1} \dots$ (e.g. $63.4\dots^\circ$ or $90^\circ - 63.4\dots^\circ$)
	Angle $ABG =$ $\tan^{-1}\left(\frac{a + \frac{4a}{16 + \pi}}{a - \frac{4a}{16 + \pi}}\right) = \tan^{-1}\left(\frac{20 + \pi}{12 + \pi}\right)$		M1	1.1b	Another correct relevant angle (or side if they use the cosine rule) Do not need to evaluate: accept $\tan \beta = \dots$ or $\beta = \tan^{-1} \dots$ (e.g. $56.8\dots^\circ$ or $90^\circ - 56.8\dots^\circ$)
	Angle = $ABD - ABG$		M1	3.1a	Correct method for finding the required angle
	6.6317568...		A1	1.1b	6.6° or better
			(4)		

2(c)	Moments about OB	M1	2.1	Complete method to give an equation in k only
	$kM2a \sin 45^\circ = \sqrt{2}M\bar{x}$	A1	1.1b	Correct equation in k only
	$k = \frac{4}{16 + \pi}$ (= 0.208969...)	A1	1.1b	0.21 or better
		(3)		
2(c) alt	Moments about O	M1	2.1	
	$kM \begin{pmatrix} 0 \\ 2a \end{pmatrix} - M \begin{pmatrix} \frac{4a}{16+\pi} \\ \frac{4a}{16+\pi} \end{pmatrix} = (k+1)M \begin{pmatrix} -\lambda \\ \lambda \end{pmatrix}$	A1	1.1b	
	$k = \frac{4}{16 + \pi}$ (= 0.208969...)	A1	1.1b	
		(3)		
(12 marks)				

Question	Scheme	Marks	AOs	
3(a)	Resolving vertically	M1	3.4	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	$R \cos \alpha - F \sin \alpha = mg$	A1	1.1b	Correct equation
	Equation of motion horizontally	M1	3.4	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	$R \sin \alpha + F \cos \alpha = \frac{mV^2}{r}$	A1	1.1b	Correct equation
	Use of $F = \mu R$	M1	3.4	Independent but must be used in an equation
	Solve for V	M1	3.1b	Substitute for trig and solve for V . Dependent on preceding M marks.
	$V = \sqrt{\frac{(5+12\mu)rg}{12-5\mu}}$ *	A1*	1.1b	Correct given answer correctly obtained
		(7)		
3(b)	Use of $\mu = 0$ oe	M1	2.1	If they don't use $\mu = 0$, we need to see the first 6 marks from (a), without friction
	$U = \sqrt{\frac{5rg}{12}}$	A1	1.1b	cao
		(2)		
3(c)	Since $5+12\mu > 5$ and $12-5\mu < 12$ oe	M1	2.1	Any convincing argument
	$\frac{5}{12} < \frac{5+12\mu}{12-5\mu}$ and hence $U < V$ *	A1*	2.2a	Given answer correctly obtained
				SC: Allow M1A0 if they work in reverse to show that if $U < V$ then $\mu > 0$ and make an appropriate comment
		(2)		
(11 marks)				

Question	Scheme	Marks	AOs	
4(a)	$a = \frac{dv}{dt} = \frac{1}{3} \times 6e^{3t}$	M1	1.1b	Need to see evidence of attempt to differentiate v wrt t , not just a statement of intent.
	$= 3v - 1$ *	A1*	1.1b	Given answer correctly obtained
4(b)		(2)		
	$2 \text{ (ms}^{-2}\text{)}$	B1	1.1b	cao
		(1)		
4(c)	$\frac{dx}{dt} = \frac{1}{3}(2e^{3t} + 1)$ and integrate	M1	3.3	Set up differential equation and attempt to solve
	$x = \frac{1}{3}(\frac{2}{3}e^{3t} + t)(+C)$	A1	1.1b	Condone missing C
	Put either $\frac{1}{3}(2e^{3t} + 1) = 1$ or 3 and solve for t	M1	2.1	Use at least one of the given speeds to find a t value
	$t = 0$	A1	1.1b	cao
	$t = \frac{1}{3}\ln 4$ (0.1540327...)	A1	1.1b	0.15 or better
	Substitute their t values into their x expression and subtract	M1	3.1a	Substitute their t values to find x values and showing subtracting. Need to see evidence. M0 if using 1 and 3.
	$\frac{2}{3} + \frac{1}{9}\ln 4$ (m)	A1	1.1b	oe. cao
		(7)		
(10 marks)				