GCE A level Mathematics (9MA0) – Shadow Paper (Set 1)

9MA0-02 AL Pure 2

October 2021 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1(a)	$20 + (31 - 1) \times d = 38 \Rightarrow d = \ldots$	M1	1.1b
	$d = 0.6$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n = \frac{1}{2}n\{2a + (n-1)d\} \Rightarrow S_{300} = \frac{1}{2} \times 300\{2 \times 20 + 299 \times "0.6"\}$	M1	1.1b
	$= 32910$	A1	1.1b
	Answer only scores both marks		
		(2)	

Question	Scheme	Marks	AOs
2(a)	$y \leqslant 9$	B1	2.5
		(1)	
(b)	$f(1.4) = 9 - 4 \times 1.4^2 = 1.16 \Rightarrow \text{gf}(1.4) = g(1.16)$ $= \dfrac{5 \times 1.16}{7 \times 1.16 - 1} = \ldots$	M1	1.1b
	$\text{gf}(1.4) = 0.815 \text{oe e.g.} \dfrac{145}{178}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{5x}{7x-1} \Rightarrow 7xy - y = 5x \Rightarrow x(7y-5) = y$	M1	1.1b
	$(g^{-1}(x) =)\dfrac{x}{7x-5}$	A1	2.2a
		(2)	
			(5 marks)

Question	Scheme	Marks	AOs
3	$log_2(10y+6) - log_2(1-5y) = 3 \Rightarrow log_2 \dfrac{10y+6}{1-5y} = 3$	B1 M1 on EPEN	1.1b
	$log_2 \dfrac{10y+6}{1-5y} = 3 \Rightarrow \dfrac{10y+6}{1-5y} = 2^3 \Rightarrow 8 - 40y = 10y + 6 \Rightarrow y = \ldots$	M1	2.1
	$y = \dfrac{1}{25}$	A1	1.1b
		(3)	
			(3 marks)

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Question	Scheme	Marks	AOs
4	**Examples:** $40\,sin\frac{\theta}{4} \approx 4\left(\frac{\theta}{4}\right), 5\,cos^2\,\theta \approx 5(1-\theta^2)$	M1	1.1a
	Examples: $40\,sin\frac{\theta}{4} + 5\,cos^2\,\theta \approx 40\left(\frac{\theta}{4}\right) + 5(1-\theta^2)$ $40\,sin\frac{\theta}{4} + 5\,cos^2\,\theta = 40\,sin\frac{\theta}{4} + 5\frac{(cos\,2\,\theta + 1)}{2}$ $\approx 40\left(\frac{\theta}{4}\right) + 5(1-\theta^2)$	dM1	1.1b
	$= 10\theta + 5 - 5\theta^2 = 5 + 10\theta - 5\theta^2$	A1	2.1
		(3)	
			(3 marks)

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Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx} = 12x^3 - 84x^2 + 192x - 144$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2} = 36x^2 - 168x + 192$	A1ft	1.1b
		(3)	
(b)(i)	$x = 2 \Rightarrow \frac{dy}{dx} = 36 \times 4 - 168 \times 2 + 192$	M1	1.1b
	$\frac{dy}{dx} = 0$ so there is a stationary point at $x = 2$	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=2}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=1.8} =\ldots \left(\frac{d^2y}{dx^2}\right)_{x=2.2} =\ldots$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1.8} > 0, \left(\frac{d^2y}{dx^2}\right)_{x=2.2} < 0$ Hence point of inflection	A1	2.2a
		(4)	
			(7 marks)

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \frac{\pi}{2} - \frac{\theta}{6}$	B1	2.2a
		(1)	
(b)	Area = $2 \times \frac{1}{2}r^2\left(\frac{\pi}{2} - \frac{\theta}{6}\right) + \frac{1}{2}(3r)^2\frac{\theta}{3}$	M1	2.1
	$= \frac{1}{2}r^2\pi - \frac{1}{6}r^2\theta + \frac{3}{2}r^2\theta = \frac{8}{6}r^2\theta + \frac{1}{2}r^2\pi = r^2(\frac{4}{3}\theta + \frac{1}{2}\pi)$ *	A1*	1.1b
		(2)	
(c)	Perimeter = $6r + 2r\left(\frac{\pi}{2} - \frac{\theta}{6}\right) + 3r\theta$	M1	3.1a
	$= 6r + r\pi + 3r\theta - \frac{2r\theta}{6}$ or e.g. $r\left(6 + \pi + \frac{8\theta}{3}\right)$	A1	1.1b
		(2)	
			(5 marks)

Question	Scheme	Marks	AOs

7(a)	$y = x^3 - 12x^2 + 38x - 25 \Rightarrow \frac{dy}{dx} = 3x^2 - 24x + 38$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=6} = 3 \times 6^2 - 24 \times 6 + 38 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 6)$	M1	2.1
	$y = 2x - 25$	A1	1.1b
		(4)	
(b)	Both *C* and *l* pass through (0, –25) and so *C* meets *l* again on the *y*-axis	B1	2.2a
		(1)	
(c)	$\pm\int\left(x^3 - 12x^2 + 38x - 25 - (2x - 25)\right)dx$ $= \pm\left(\frac{x^4}{4} - \frac{12}{3}x^3 + \frac{36}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{12}{3}x^3 + \frac{36}{2}x^2\right]_0^6$ $= \left(\frac{1296}{4} - \frac{2592}{3} + \frac{1296}{2}\right)(-0)$	dM1	2.1
	=108	A1	1.1b
		(4)	

Question	Scheme	Marks	AOs
8(a)	$\frac{d}{dx}(2y^2) = 4y\frac{dy}{dx}$ or $\frac{d}{dx}(qxy) = qx\frac{dy}{dx} + qy$	M1	2.1
	$3px^2 + qx\frac{dy}{dx} + qy + 4y\frac{dy}{dx} = 0$	A1	1.1b
	$(qx + 4y)\frac{dy}{dx} = -3px^2 - qy \Rightarrow \frac{dy}{dx} = \ldots$	dM1	2.1
	$\frac{dy}{dx} = \frac{-3px^2 - qy}{qx + 4y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3 + q(-1)(-5) + 2(-5)^2 = 28$	M1	1.1b
	$8x + 13y + 73 = 0 \Rightarrow m = -\frac{8}{13}$	B1	2.2a
	$\frac{-3p(-1)^2 - q(-5)}{q(-1) + 4(-5)} = \frac{13}{8}$ or $\frac{q(-1) + 4(-5)}{3p(-1)^2 + q(-5)} = -\frac{8}{13}$	M1	3.1a
	$-p + 5q = -22, -24p + 53q = -260 \Rightarrow p = \ldots, q = \ldots$	dM1	1.1b
	$p = 2, q = -4$	A1	1.1b
		(5)	
		(9 marks)	

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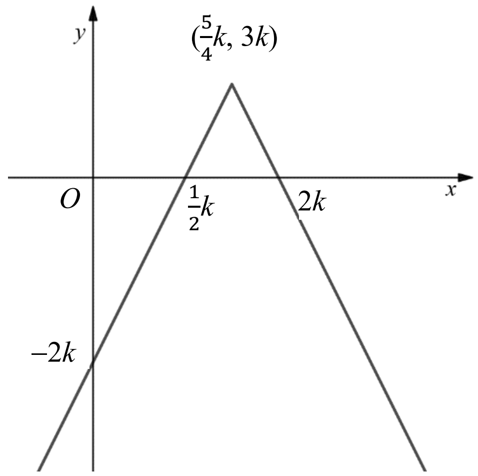
Question	Scheme	Marks	AOs
9	$a = -\frac{16}{25}$ or $r = -\frac{4}{5}$	B1	2.2a
	$\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n sin(180n - 90)^\circ = \frac{-\frac{16}{25}}{1-(-\frac{4}{5})}$	M1	3.1a
	$= -\frac{16}{25}$ *	A1*	1.1b
		(3)	

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.511$ or $b = \frac{47}{92}$	B1	2.2a
	$0 = \text{"0.511"} \times -0.6 + log_{10}\, a \Rightarrow a = 10^{0.306...}$	M1	3.1a
	$T = 2.03l^{0.511}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Question	Scheme	Marks	AOs
11(a)			
	$\wedge$ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a $\wedge$ shape	B1	1.1b
		(4)	
(b)	$x = \frac{1}{2}k$	B1	2.2a
	$8k - 4x = 2x - k \Rightarrow x = \ldots$	M1	3.1a
	$x = \frac{3k}{2}$	A1	1.1b
	Set notation is required here for this mark $\left\{x: \frac{k}{2} < x < \frac{3k}{2}\right\}$	A1	2.5
		(4)	
(c)	$x = 5k$ **or** $y = 5 - 6k$	B1ft	2.2a
	$x = 5k$ **and** $y = 5 - 6k$	B1ft	2.2a
		(2)	
			(10 marks)

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Question	Scheme	Marks	AOs
12(a)	$u=5+\sqrt{4x} \Rightarrow x=\frac{(u-5)^2}{4} \Rightarrow \frac{dx}{du}=\frac{u-5}{2}$ or $u=5+2\sqrt{x} \Rightarrow \frac{du}{dx}=10(4x)^{-\frac{1}{2}}$	B1	1.1b
	$\int \frac{8x}{5+\sqrt{4x}}\,dx=\int \frac{8(u-5)^2}{4u}\times\frac{u-5}{2}\,du=\int \frac{(u-5)^3}{u}\,du$	M1	2.1
	$\int_0^{100} \frac{8x}{5+2\sqrt{x}}\,dx=\int_5^{25} \frac{(u-5)^3}{u}\,du$	A1	1.1b
		(3)	
(b)	$\int \frac{u^3-15u^2+75u-125}{u}\,du$	M1	3.1a
	$=\left[\frac{u^3}{3}-\frac{15u^2}{2}+75u-125\ln u\right]$	A1	1.1b
	$=\left[\frac{15625}{3}-\frac{9375}{2}+1875-125\ln 5^2 -\left(\frac{125}{3}-\frac{375}{2}-375+125\ln 5\right)\right]$	dM1	2.1
	$=\frac{6500}{3}-125\ln 5$	A1	1.1b
		(4)	
(7 marks)			

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Question	Scheme	Marks	AOs
13(a)	$y = \sec^3\theta \Rightarrow \dfrac{dy}{d\theta} = 3\sec^2\theta\sec\theta\, tan\,\theta$	B1	1.1b
	$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	M1	1.1b
	$\dfrac{dy}{dx} = \dfrac{-3\sec^3\theta\, tan\,\theta}{2\, sin\, 2\,\theta}$	A1	1.1b
		(3)	
(b)	$y = 8 \Rightarrow \sec^3\theta = 8 \Rightarrow cos^3\,\theta = \dfrac{1}{8} \Rightarrow cos\,\theta = \dfrac{1}{2}$	M1	3.1a
	$\theta = \dfrac{\pi}{3} \Rightarrow \dfrac{dy}{dx} = \dfrac{-3\sec^3\left(\frac{\pi}{3}\right) tan\left(\frac{\pi}{3}\right)}{2\, sin\left(\frac{2\pi}{3}\right)} = \ldots$	M1	2.1
	$= -24$	A1	2.2a
		(3)	

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	(6 marks)

Question	Scheme	Marks	AOs
14(a)	$\frac{dV}{dt} = 0.9 - 0.2h$	B1	3.1b
	$V = 48h \Rightarrow \frac{dV}{dh} = 48$ or $\frac{dh}{dV} = \frac{1}{48}$	B1	3.1b
	$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{0.9 - 0.2h}{48}$ or e.g. $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} \Rightarrow 0.9 - 0.2h = 48\frac{dh}{dt}$	M1	2.1
	$480\frac{dh}{dt} = 9 - 2h\ *$	A1*	1.1b
		(4)	
(b)	$480\frac{dh}{dt} = 9 - 2h \Rightarrow \int \frac{480}{9 - 2h} dh = \int dt$ $\Rightarrow e.g.\ \alpha\ ln(9 - 2h) = t(+c)$ oe	M1	3.1a
	$t = -240\ ln(9 - 2h)\ (+c)$ oe	A1	1.1b
	$t = 0, h = 3 \Rightarrow 0 = -240\ ln(9 - 6) + c \Rightarrow c = \ldots (240\ ln\ 3)$	M1	3.4
	$t = 240\ ln(3) - 240\ ln(9 - 2h)$	A1	1.1b
	$t = 240\ ln\frac{3}{9 - 2h} \Rightarrow \frac{t}{240} = ln\frac{3}{9 - 2h} \Rightarrow e^{\frac{t}{240}} = \frac{3}{9 - 2h}$ $\Rightarrow 3e^{-\frac{t}{240}} = 9 - 2h \Rightarrow h = \ldots$	ddM1	2.1
	$h = \frac{9}{2} - \frac{3}{2}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t \to \infty, \mathrm{e}^{-\frac{t}{240}} \to 0$ • When $h > 4.5,\ \frac{\mathrm{d}V}{\mathrm{d}t} < 0$ • Flow in = flow out at max h so $0.2h = 4.5 \rightarrow h = 4.5$ • As $e^{-\frac{t}{240}} > 0, h < 4.5$ • $h = 7 \Rightarrow \frac{dV}{dt} = -0.5$ or $\frac{dh}{dt} = -\frac{1}{960}$ • $\frac{dh}{dt} = 0 \Rightarrow h = 4.5$ • $h = 7 \Rightarrow 4.5 - 1.25e^{-\frac{t}{240}} = 7 \Rightarrow e^{-\frac{t}{240}} < 0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 7m high so the tank will never become full • If $h = 7$ the tank would be emptying so can never be full • The equation can't be solved when $h = 7$	A1	3.2a

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		(2)	
			(12 marks)

Question	Scheme	Marks	AOs
15(a)	$R = \sqrt{13}$	B1	1.1b
	$tan\,\alpha = \frac{2}{3}$	M1	1.1b
	$\alpha = 0.588$	A1	1.1b
		(3)	
(b)(i)	$5 + 3\sqrt{13}$	B1ft	3.4
(ii)	$cos(0.2t + 0.588) = 1 \Rightarrow 0.2t + 0.588 = 2\pi$ $\Rightarrow t = \ldots$	M1	3.4
	$t = 28.5$	A1	1.1b
		(3)	
(c)	$5 + 3\sqrt{13}\,cos(0.2t + 0.588) = 0$ $cos(0.2t + 0.588) = -\frac{5}{3\sqrt{13}}$	M1	3.4
	$cos(0.2t + 0.588) = -\frac{5}{3\sqrt{13}} \Rightarrow 0.2t + 0.588 = cos^{-1}\left(-\frac{5}{3\sqrt{13}}\right)$ $\Rightarrow t = \frac{1}{0.2}\left(cos^{-1}\left(-\frac{5}{3\sqrt{13}}\right) - 0.588\right)$	dM1	1.1b
	So the time required is e.g.: $\frac{1}{0.2}(4.231\ldots - 0.588) - \frac{1}{0.2}(2.051\ldots - 0.588)$	dM1	3.1b
	$=10.9$	A1	1.1b
		(4)	
(d)	e.g. the "5" would need to vary	B1	3.5c
		(1)	
			(11 marks)

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