GCE A level Mathematics (9MA0) – Shadow Paper (Set 1)

9MA0-01 AL Pure 1

October 2021 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1	$f(2) = a(2)^3 + 3(2)^2 - 5a(2) + 8 = 0$	M1	3.1a
	$20 - 2a = 0 \Rightarrow a = ...$	M1	1.1b
	$a = 10$	A1	1.1b
		(3)	
			(3 marks)
	Notes		

Question	Scheme	Marks	AOs
2(a)	$f(x) = (x-3)^2 \pm ...$	M1	1.2
	$f(x) = (x-3)^2 - 2$	A1	1.1b
		(2)	
(b)(i)	$P = (0,7)$	B1	1.1b
(b)(ii)	$Q = (3,-2)$	B1ft	1.1b
		(2)	
			(4 marks)
	Notes		

Question	Scheme	Marks	AOs
3(a)	$u_2 = k - 9, u_3 = k - 3 - \frac{42}{k-9}$	M1	1.1b
	$(6u_1 - 2) + 5u_2 - u_3 = 0 \Rightarrow 42 - 2 + 5k - 45 - k + 3 + \frac{42}{k-9} = 0$	dM1	1.1b
	$4k - 2 + \frac{42}{k-9} = 0 \quad \Rightarrow \quad (4k-2)(k-9) + 42 = 0$ $\Rightarrow 4k^2 - 36k - 2k + 18 + 42 = 0$ $\Rightarrow 4k^2 - 38k + 60 = 0 *$	A1*	2.1
		(3)	
(b)	$k = 2, (7.5)$	M1	1.1b
	$k = 2$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)5$	B1	2.2a
		(1)	
			(6 marks)
Notes			

Question	Scheme	Marks	AOs
4(a)	$f'(x) = 2x + \frac{6x-6}{3x^2 - 6x + 7}$	M1 A1	1.1b 1.1b
	$2x + \frac{6x-6}{3x^2-6x+7} = 0 \Rightarrow 2x(3x^2 - 6x + 7) + 6x - 6 = 0$	dM1	1.1b
	$3x^3 - 6x^2 + 10x - 3 = 0 *$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \frac{1}{10}(3 + 6(0.3)^2 - 3(0.3)^3)$	M1	1.1b
	$x_2 = 0.3459$	A1	1.1b
	(ii) $x_4 = 0.3636$	A1	1.1b
		(3)	
(c)	$h(x) = 3x^3 - 6x^2 + 10x - 3$ $h(0.3665) = 0.544\ldots h(0.3655) = -0.0000598\ldots$	M1	3.1a
	States: • there is a change of sign • $f'(x)$ is continuous • $\alpha = 0.366$ to 3dp	A1	2.4
		(2)	
			(9 marks)
Notes			

Question	Scheme	Marks	AOs
5(a)	$u_3 = £30000 \times 1.09^2 = (£)35643 *$	B1*	1.1b
		(1)	
(b)	$30000 \times 1.09^{n-1} > 180000$	M1	1.1b
	$1.09^{n-1} > 6 \Rightarrow n - 1 > \frac{ln\,6}{ln(1.09)}$ or e.g. $1.09^{n-1} > 6 \Rightarrow n - 1 > log_{1.09}(6)$	M1	3.1b
	Year 22	A1	3.2a
		(3)	
(c)	$S_{30} = \frac{30000(1 - 1.09^{30})}{1 - 1.09}$	M1	3.4
	Awrt (£) 4 090 000	A1	1.1b
		(2)	
			(6 marks)
Notes			

Question	Scheme	Marks	AOs
6(a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = -2\boldsymbol{i} - 6\boldsymbol{j} - 3\boldsymbol{k} + 6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k} = ...$	M1	1.1b
	$= 4\boldsymbol{i} - 9\boldsymbol{j} - \boldsymbol{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $(AC^2) = "4^2 + 9^2 + 1^2", (AB^2) = 2^2 + 6^2 + 3^2, (BC^2) = 6^2 + 3^2 + 2^2$	M1	1.1b
	$4^2 + 9^2 + 1^2 = 2^2 + 6^2 + 3^2 + 6^2 + 3^2 + 2^2 - 2\sqrt{2^2 + 6^2 + 3^2}\sqrt{6^2 + 3^2 + 2^2}\cos ABC$	M1	3.1a
	$98 = 98 - 2\sqrt{7}\sqrt{7}\cos ABC$ $\Rightarrow \cos ABC = \frac{98 - 98}{2\sqrt{7}\sqrt{7}} = 0 *$	A1*	2.1
		(3)	

Question	Scheme	Marks	AOs
7(a)(i)	$(x-6)^2 + (y+4)^2 = ...$	M1	1.1b
	$(6, -4)$	A1	1.1b
(ii)	$r = \sqrt{"6"^2 + "-4"^2 - 7}$	M1	1.1b
	$r = 3\sqrt{5}$	A1	1.1b
		(4)	
(b)	$y = 3x + k \Rightarrow x^2 + (2x+k)^2 - 12x + 8(2x+k) + 7 = 0$ $\Rightarrow x^2 + 4x^2 + 4kx + k^2 - 12x + 16x + 8k + 7 = 0$	M1	2.1
	$\Rightarrow 5x^2 + (4k+4)x + k^2 + 8k + 7 = 0$	A1	1.1b
	$b^2 - 4ac = 0 \Rightarrow (4k+4)^2 - 4 \times 5 \times (k^2 + 8k + 7) = 0$	M1	3.1a
	$\Rightarrow 4k^2 + 128k + 124 = 0 \Rightarrow k = ...$	M1	1.1b
	$k = -1, -31$	A1	2.2a
		(5)	
		(9 marks)	
	Notes		

Question	Scheme	Marks	AOs
8(a)	$A = 2000$	B1	3.4
	$6000 = 2000e^{2k}$ or $e^{2k} = 3$	M1	1.1b
	$e^{2k} = 3 \Rightarrow 2k = ln\,3 \Rightarrow k = \ldots$	M1	2.1
	$N = 2000e^{\left(\frac{1}{2}ln3\right)t}$ or $N = 2000e^{0.549t}$	A1	3.3
		(4)	
(b)	$\frac{dN}{dt} = 2000 \times \left(\frac{1}{2}ln\,3\right)e^{\left(\frac{1}{2}ln\,3\right)t}$ or $\frac{dN}{dt} = 2000 \times 0.549e^{0.549t}$ $\left(\frac{dN}{dt}\right)_{t=7} = 2000 \times \left(\frac{1}{2}ln\,3\right)e^{7\times\frac{1}{2}ln\,3}$ or $\left(\frac{dN}{dt}\right)_{t=7} = 2000 \times 0.549e^{0.549\times7}$	M1	3.1b
	= awrt 51000 or 54000	A1	1.1b
		(2)	
(c)	$6000e^{0.2\times\left(\frac{1}{2}ln\,3\right)T} = 2000e^{\left(\frac{1}{2}ln\,3\right)T}$ or $6000e^{0.2\times''0.549''t} = 2000e^{''0.549''t}$	M1	3.4
	Correct method of getting a linear equation in T $ln3 + 0.2\frac{1}{2}ln\,3\,T = \frac{1}{2}ln\,3\,T$	M1	2.1
	$T = 2.5$	A1	1.1b
		(3)	
			(9 marks)

Notes

Question	Scheme	Marks	AOs
9(a)(i)	$36x^2 + 43x + 9$ $\equiv A(3x+2)(1-5x) + B(1-5x) + C(3x+2)^2$ $\Rightarrow B = \ldots$ or $C = \ldots$	M1	1.1b
	$B = 1$ and $C = 4$	A1	1.1b
(a)(ii)	$2A + B + 4C = 17$ $\Rightarrow 17 = 2A + 1 + 16 \Rightarrow A = \ldots$	M1	2.1
	$A = 0$*	A1*	1.1b
		(4)	

(b)(i)	$\frac{1}{(3x+2)^2} = (3x+2)^{-2} = 2^{-2}\left(1+\frac{3}{2}x\right)^{-2}$ or $(3x+2)^{-2} = 2^{-2}+\ldots$	M1	3.1a
	$\left(1+\frac{3}{2}x\right)^{-2} = 1-2\left(\frac{3}{2}x\right)+\frac{-2(-2-1)}{2!}\left(\frac{3}{2}x\right)^2+\ldots$	M1	1.1b
	$2^{-2}\left(1+\frac{3}{2}x\right)^{-2} = \frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2+\ldots$	A1	1.1b
	$\frac{1}{(1-5x)} = (1-5x)^{-1} = 1+5x+\frac{-1(-1-1)}{2!}(5x)^2+.$	M1	1.1b
	$\frac{1}{(3x+2)^2}+\frac{4}{1-5x} = \frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2+\ldots+4+20x+100x^2+\ldots$	dM1	2.1
	$= \frac{17}{4}+\frac{77}{4}x+\frac{1627}{16}x^2+\ldots$	A1	1.1b
(b)(ii)	$\lvert x\rvert < \frac{1}{5}$	B1	2.2a
		(7)	
			(11 marks)

Question	Scheme	Marks	AOs
10(a)	$\frac{\cos 2\theta-1-\sin 2\theta}{1+\cos 2\theta+\sin 2\theta} = \frac{2\cos^2\theta-1-1-2\sin\theta\cos\theta}{1-2\sin^2\theta+1+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{2\cos^2\theta-2-2\sin\theta\cos\theta}{2-2\sin^2\theta+2\sin\theta\cos\theta}$	A1	1.1b
	$= \frac{-\sin^2\theta-\sin\theta\cos\theta}{\cos^2\theta+\sin\theta\cos\theta} = \frac{-\sin\theta(\sin\theta+\cos\theta)}{\cos\theta(\cos\theta+\sin\theta)}$	dM1	2.1
	$= \frac{-\sin\theta}{\cos\theta} = -\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{\cos 6x-1+\sin 6x}{1+\cos 6x+6x} = 4\sin 3x \Rightarrow -\tan 3x = 4\sin 3x$ o.e	M1	3.1a
	$\Rightarrow -\sin 3x-4\sin 3x\cos 3x = 0$ $\Rightarrow -\sin 3x(1+4\cos 3x) = 0$ $\Rightarrow (\sin 3x = 0,)\ \cos 3x = -\frac{1}{4}$ o.e	A1	1.1b
	$x = 34.8°, 60°, 85.2°, 120°, 154.8°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)
	Notes		

Question	Scheme	Marks	AOs
11(a)	$h = 0.25$	B1	1.1b
	$A \approx \frac{1}{2} \times \frac{1}{4}\{3.77654 + 5.7709 + 2(4.2807 + 4.7867 + 5.2834)\}$	M1	1.1b
	$= 4.78$	A1	1.1b
		(3)	
(b)	$\int 3(ln\,x)^{2} - 2\,dx = x(3(ln\,x)^2 - 2) - \int 6x \times \frac{2\,ln\,x}{x}\,dx$	M1 A1	3.1a 1.1b
	$= (3(ln\,x)^2 - 2)x - 6\int ln\,x\,dx$	dM1	2.1
	$\int_4^5 3(ln\,x)^2 - 2\,dx = [3x(ln\,x)^2 - 6x\,ln\,x + 4x]_4^5$ $= 15(ln\,5)^2 - 30\,ln\,5 + 20 - 48(ln\,2)^2 + 48ln2 - 16$	ddM1	2.1
	$= 15(ln\,5)^2 - 48(ln\,2)^2 - 30\,ln\,5 + 48\,ln\,2 + 4$	A1	1.1b
		(5)	
			(8 marks)
	Notes		

Question	Scheme	Marks	AOs
12(a)	$H = ax^2 + bx + c$ and $x = 0, H = 2 \Rightarrow H = ax^2 + bx + 2$	M1	3.3
	$H = ax^2 + bx + 2$ and $x = 160, H = 34 \Rightarrow 34 = 25600a + 160b + 2$ **or** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 100 \Rightarrow 200a + b = 0$	M1 A1	3.1b 1.1b
	$H = ax^2 + bx + 2$ and $x = 160, H = 34 \Rightarrow 34 = 25600a + 160b + 2$ **and** $\frac{dH}{dx} = 2ax + b = 0$ when $x = 100 \Rightarrow 200a + b = 0$ $\Rightarrow a = ..., b = ...$	dM1	3.1b
	$H = -\frac{1}{200}x^2 + x + 2$ o.e.	A1	1.1b
		(5)	
(b)(i)	$x = 100 \Rightarrow H\left(= -\frac{1}{200}(100)^2 + 100 + 2\right) = 52m$	B1	3.4
(b)(ii)	$H = 0 \Rightarrow -\frac{1}{200}x^2 + x + 2 = 0 \Rightarrow x = ...$	M1	3.4
	$x = (-1.98039...,)201.980...$ $\Rightarrow x = 202(m)$	A1	3.2a
		(3)	
(c)	Examples must focus on why the model may not be appropriate or give values/situations where the model would break down: E.g. • The ground is unlikely to be horizontal • The ball is not a particle so has dimensions/size • The ball is unlikely to travel in a vertical plane (as it will spin) • *H* is not likely to be a quadratic function in *x*	B1	3.5b
		(1)	
			(9 marks)
	Notes		

Question	Scheme	Marks	AOs
13	$9(x-2)^2 + y^2 = 9\left(\frac{t^2+9}{t^2+3} - \frac{2t^2+6}{t^2+3}\right)^2 + \left(\frac{6\sqrt{3}t}{t^2+3}\right)^2$	M1	3.1a
	$= 9\left(\frac{3-t^2}{t^2+3}\right)^2 + \frac{108t^2}{(t^2+3)^2}$	dM1	1.1b
	$\frac{9(t^4+6t^2+9)}{(t^2+3)^2} = \frac{9(t^2+3)^2}{(t^2+3)^2} = 9*$	A1*	2.1
		(3)	

Question	Scheme	Marks	AOs

14	$y=\dfrac{x-16}{4+\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{4+\sqrt{x}-(x-16)\frac{1}{2}x^{-\frac{1}{2}}}{\left(4+\sqrt{x}\right)^2}$	M1 A1	2.1 1.1b
	$=\dfrac{4+\sqrt{x}-(x-16)\frac{1}{2}x^{-\frac{1}{2}}}{\left(4+\sqrt{x}\right)^2}=\dfrac{4+\sqrt{x}-\frac{1}{2}\sqrt{x}+8x^{-\frac{1}{2}}}{\left(4+\sqrt{x}\right)^2}=\dfrac{4\sqrt{x}+\frac{1}{2}x+8}{\sqrt{x}\left(4+\sqrt{x}\right)^2}$	M1	1.1b
	$=\dfrac{x+8\sqrt{x}+16}{2\sqrt{x}\left(4+\sqrt{x}\right)^2}=\dfrac{\left(4+\sqrt{x}\right)^2}{2\sqrt{x}\left(4+\sqrt{x}\right)^2}=\dfrac{1}{2\sqrt{x}}$	A1	2.1
		(4)	
			(4 marks)
	Notes		

Question	Scheme	Marks	AOs
15(i)	$n=1, 2^5=32, 5^1=5, (32>5)$ $n=2, 3^5=243, 5^2=25, (243>25)$ $n=3, 4^5=1024, 5^3=125, (1024>125)$ $n=4, 5^5=3125, 5^4=625, (3125>625)$	M1	2.1
	So if n Ñ $4, n\in \mathbb{N}$ then $(n+1)^5>5^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^5+7=(2p\pm1)^5+7=...$	M1	2.1
	$=32p^5+80p^4+80p^3+40p^2+10p+8$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $32p^5+80p^4+80p^3+40p^2+10p+8$ being even • acceptable statement such as "this is a contradiction so if m^5+7 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)
	Notes		