GCE A level Mathematics (9AM0) – Shadow Paper (Set 1)
9MA0-02 Pure Mathematics 2

October 2020 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1(a)	$h = 0.75$	B1	1.1a
	$A \approx \frac{0.75}{2}\{0.5108 + 0.9258 + 2(0.6794 + 0.7878 + 0.8660)\}$	M1	1.1b
	= awrt 2.29	A1	1.1b
	For reference: **The integration on a calculator gives 2.29901322** **The full accuracy for *y* values gives 2.288637923** **The accuracy from the table gives 2.288625**		
		(3)	

(b)	4 × their (a)
	If (a) is correct, allow awrt 9.16 or awrt 9.15 even with no working.
	Only allow 9.16 if (a) is correct and working is shown e.g. 4 × 2.29
	If (a) is incorrect allow 4 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 9.196052879		
		(1)	
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 9.2</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as 9.196 ≈ 9.20 • Very accurate as 9.196 to 2 sf is 9.2 • 9.16 < 9.196 so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • 9.196 – 9.16 = 0.036 so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 9.20 or awrt 9.16 or 9.15 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{9.196-9.16}{9.196}\right\rvert\times100 = 0.39\%$ or $\left\lvert\frac{9.196-9.15}{9.196}\right\rvert\times100 = 0.50\%$ or $\left\lvert\frac{9.196-9.1545}{9.196}\right\rvert\times100 = 0.45\%$ or $\left\lvert\frac{9.16}{9.196}\right\rvert\times100 = 99.6\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
		(5 marks)	

Question	Scheme	Marks	AOs
2	Attempts any one of $\left(\pm\overrightarrow{PQ}=\right)\pm(\mathbf{q}-\mathbf{p}),\ \left(\pm\overrightarrow{PR}=\right)\pm(\mathbf{r}-\mathbf{p}),\ \left(\pm\overrightarrow{QR}=\right)\pm(\mathbf{r}-\mathbf{q})$ Or e.g. $\left(\pm\overrightarrow{PQ}=\right)\pm\left(\overrightarrow{OQ}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{PR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OP}\right),\ \left(\pm\overrightarrow{QR}=\right)\pm\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)$	M1	1.1b
	Attempts e.g. $\mathbf{r}-\mathbf{q}=4(\mathbf{q}-\mathbf{p})$ $\mathbf{r}-\mathbf{p}=5(\mathbf{q}-\mathbf{p})$ $\frac{4}{5}(\mathbf{q}-\mathbf{p})=\frac{1}{5}(\mathbf{r}-\mathbf{q})$ $\mathbf{q}=\mathbf{p}+\frac{1}{5}(\mathbf{r}-\mathbf{p})$ $\mathbf{q}=\mathbf{r}+\frac{4}{5}(\mathbf{p}-\mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r}-\mathbf{q}=4\mathbf{q}-4\mathbf{p}\Rightarrow 4\mathbf{p}+\mathbf{r}=5\mathbf{q}\Rightarrow \mathbf{q}=\frac{1}{5}(\mathbf{r}+4\mathbf{p})$*	A1*	2.1
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
3(a)	$2\log(5-x)=\log(5-x)^2$	B1	1.2
	$2\log(5-x)=\log(x+7)\Rightarrow\log(5-x)^2=\log(x+7)$ $(5-x)^2=(x+7)$ **or** $2\log(5-x)=\log(x+7)\Rightarrow\log(5-x)^2-\log(x+7)=0$ $\frac{(5-x)^2}{(x+7)}=1$	M1	1.1b
	$25-10x+x^2=x+7\Rightarrow x^2-11x+18=0$*	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-11x+25=0\Rightarrow(5-x)^2-x-7=0$	B1	1.2
	$\Rightarrow(5-x)^2=x+7$ $\Rightarrow\log(5-x)^2=\log(x+7)$	M1	1.1b
	$\Rightarrow 2\log(5-x)=\log(x+7)$ *Hence proved.	A1	2.1
(b)	(i) $(x=)$ 2, 9	B1	1.1b
	(ii) 9 is **not** a solution as log(5 – 9) cannot be found	B1	2.3
		(2)	
			(5 marks)

Question	Scheme	Marks	AOs
4	${}^9C_5 a^4 (3x)^5$	M1	1.1b
	$\frac{9!}{5!4!} a^4 \times 3^5 = 489888 \Rightarrow a = ...$	dM1	2.1
	$a = 2$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
5	$60 - 3^{x+2} = 6 \times 3^x$	B1	1.1b
	$\Rightarrow 60 - 9 \times 3^x = 6 \times 3^x \Rightarrow 3^x = 4$ or e.g. $\Rightarrow \frac{60}{3^x} - \frac{3}{2} = 6 \Rightarrow 3^x = 4$	M1	1.1b
	$3^x = 4 \Rightarrow x = ...$	dM1	1.1b
	$x = \log_3 4$	A1cso	1.1b
		(4)	
	Alternative		
	$y = 6 \times 3^x \Rightarrow 3^x = \frac{y}{6} \Rightarrow y = 60 - 9 \times \frac{y}{6}$	B1	1.1b
	$9y + 6y = 60 \Rightarrow 15y = 60 \Rightarrow 15 \times 3^x = 60 \Rightarrow 3^x = 4$	M1	1.1b
	$3^x = 4 \Rightarrow x = ...$	dM1	1.1b
	$x = \log_3 4$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $60 - 3^{x+2} = 6 \times 3^x$ or equivalent e.g. $60 - 3^{x+2} - 6 \times 3^x = 0$

M1: Uses $3^{x+2} = 3^2 \times 3^x$ oe e.g. $\frac{3^{x+2}}{3^x} = 3^2$ to obtain an equation in 3^x and attempts to make 3^x the subject.

See scheme but e.g. $y = 3^x \Rightarrow 6 \times 3^x = 60 - 3^{x+2} \Rightarrow 6y = 60 - 9y \Rightarrow y = ...$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $3^x = k$ where $k > 1$

e.g. $3^x = k \Rightarrow x = \log_3 k$

or $3^x = k \Rightarrow \log 3^x = \log k \Rightarrow x \log 3 = \log k \Rightarrow x = ...$

or $3^x = k \Rightarrow \ln 3^x = \ln k \Rightarrow x \ln 3 = \ln k \Rightarrow x = ...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $3^x = 4$ so $x = 1.2618..$ but you may need to check

A1cso: $x = \log_3 4$ or $\frac{\log 4}{\log 3}$ or $\frac{\ln 4}{\ln 3}$

Ignore any attempts to find the y-coordinate

Question	Scheme	Marks	AOs
6(a)	$x^2+10x-5=(Ax+B)(x+4)+C$ or $Ax(x+4)+B(x+4)+C$ $\Rightarrow A=\ldots, B=\ldots, C=\ldots$ **Or** $x+4\overline{)x^2+10x-5}$ giving quotient $x+6$: x^2+4x; $6x-5$; $6x+24$; -29	M1	1.1b
	Two of $A=1, B=6, C=-29$	A1	1.1b
	All three of $A=1, B=6, C=-29$	A1	1.1b
		(3)	
6(b)	$\int \frac{x^2+10x-5}{x+4}\,dx = \int x+6-\frac{29}{x+4}\,dx = \ldots-29\ln(x+4)$	M1	1.1b
	$=\frac{1}{2}x^2+6x-29\ln(x+4)\quad(+c)$	A1ft	1.1b
	$\int_0^8 \frac{x^2+10x-5}{x+4}\,dx=\left[\frac{1}{2}x^2+6x-29\ln(x+4)\right]_0^8$ = (32 + 48 – 29ln12) – (0 + 0 – 29ln4) $= 32+48+29\ln\left(\frac{4}{12}\right)$ or $32+48+29\ln 3^{-1}$	M1	2.1
	= 80 – 29 ln3	A1	1.1b
		(4)	
			(7 marks)

Question	Scheme	Marks	AOs
7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{16x^2+x}{4\sqrt{x}}$ **<u>Notes:</u>** Look for $\frac{16x^2+x}{4\sqrt{x}} \to ...x^{\frac{3}{2}} + ...x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + ...$ **or** $... + Qx^{-\frac{1}{2}}$ Alternatively uses the quotient rule on $\frac{16x^2+x}{4\sqrt{x}}$. Condone slips but if rule is not quoted expect $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{4\sqrt{x}(Ax+B)-(16x^2+x)Cx^{-\frac{1}{2}}}{(4\sqrt{x})^2}(A,B,C>0)$	M1	1.1b
	E.g. $4\times\frac{3}{2}x^{\frac{1}{2}} + \frac{1}{4}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6\sqrt{x} + \frac{1}{8\sqrt{x}} - \frac{6}{x} = \frac{48x^2+x-48\sqrt{x}}{8x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$48x^2+x-48\sqrt{x}=0 \Rightarrow 48x^{\frac{3}{2}}+x^{\frac{1}{2}}-48=0$	M1	1.1b
	E.g. $48x^{\frac{3}{2}} = 48-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}} = 1-\frac{\sqrt{x}}{48} \Rightarrow x = \left(1-\frac{\sqrt{x}}{48}\right)^{\frac{2}{3}}$	A1	2.1
		(3)	
(c)	$x_2 = \sqrt[3]{\left(1-\frac{\sqrt{3}}{48}\right)^2}$	M1	1.1b
	x_2 = awrt 0.9758	A1	1.1b
	x = 0.9862	A1	2.2a
		(3)	
		(10 marks)	

Via firstly integrating

Question	Scheme	Marks	AOs
8	$f'(x)=9x^2+ax+5 \Rightarrow f(x)=3x^3+\frac{1}{2}ax^2+5x+c$	M1 A1	1.1b 1.1b
	$"c"=-3$	B1	2.2a
	$f(-3)=0 \Rightarrow 3\times(-3)^3+\frac{1}{2}a(-3)^2-5(-3)-3=0$	dM1	3.1a
	$a=\ldots\ (22)$	dM1	1.1b
	$(f(x)=)3x^3+11x^2+5x-3$ Or Equivalent e.g. $(f(x)=)(x+3)(3x^2+2x-1)$ $(f(x)=)(x+3)(3x-1)(x+1)$	A1cso	2.1
		(6)	
			(6 marks)

Via firstly using factor

Question	Scheme	Marks	AOs
8 Alt	$f(x)=(x+3)(Ax^2+Bx+C)$	M1 A1	1.1b 1.1b
	$f(x)=Ax^3+(3A+B)x^2+(3B+C)x+3C \Rightarrow C=-3$	B1	2.2a
	$f'(x)=3Ax^2+2(3A+B)x+(3B+C)$ and $f'(x)=9x^2+ax+5$ $\Rightarrow A=\ldots$	dM1	3.1a
	Full method to get *A*, *B* and *C*	dM1	1.1b
	$f(x)=(x+3)(3x^2+2x-1)$	A1cso	2.1
		(6)	
			(6 marks)

Question	Scheme	Marks	AOs
9(a)	$t=0,\ \theta=20 \Rightarrow 20=A-B$ **Or** $t=8,\ \theta=42 \Rightarrow 42=A-Be^{-0.64}$	M1	3.1b
	$t=0,\ \theta=20 \Rightarrow 20=A-B$ **And** $t=8,\ \theta=42 \Rightarrow 42=A-Be^{-0.64}$ **and** $\Rightarrow A=...,\ B=...$	M1	3.1a
	At least one of: A = 66.5, B = 46.5 but allow awrt 67/awrt 47	A1 **M1 on EPEN**	1.1b
	$\theta=66.5-46.5e^{-0.08t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "66.5"(°C) (according to the model) (The model has an) upper limit of "66.5"(°C) (The model suggests that) the boiling point is "66.5"(°C)	B1ft	3.4
	Model is not appropriate as 66.5(°C) is much lower than 80(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (a)	sin 3A = sin (2A + A) = sin 2Acos A + cos 2Asin A	M1	3.1a
	$=2\sin A\cos^2 A+(1-2\sin^2 A)\sin A$	dM1	1.1b
	$=2\sin A(1-\sin^2 A)+(1-2\sin^2 A)\sin A$	ddM1	2.1
	$=3\sin A-4\sin^3 A*$	A1*	1.1b
		(4)	
(b)	$1-\sin 3x=\cos^2 x \Rightarrow \sin^2 x+3\sin x+4\sin^3 x=0$	M1	1.1b
	$\Rightarrow \sin x(4\sin^2 x+\sin x-3)=0$ $\Rightarrow \sin x(4\sin x-3)(\sin x+1)=0$ $\Rightarrow \sin x=...$	dM1	3.1a
	Two of $0^\circ, 180^\circ$, awrt $49^\circ, 131^\circ$	A1	1.1b
	All five of $0^\circ, 180^\circ$, awrt $49^\circ, 131^\circ$	A1	2.1
		(4)	
			(8 marks)

Question	Scheme	Marks	AOs
11(a)	$x=-3$ **or** $y=-7$	B1	1.1b
	$P(-3,-7)$	B1	2.2a
		(2)	
(b)	$5x+60=-4(x+3)-7 \Rightarrow x=\ldots$	M1	1.1b
	$x=-\frac{79}{9}$ or $-8\frac{7}{9}$	A1	2.1
		(2)	
(c)	$a>4$	B1	2.2a
	$y=ax \Rightarrow -7=-3a \Rightarrow a=\frac{7}{3}$	M1	3.1a
	$\left\{a : a \leqslant \frac{7}{3}\right\} \cup \{a : a>4\}$	A1	2.5
		(3)	
			(7 marks)

Question	Scheme	Marks	AOs
12(a)(i)	$y\times\frac{\mathrm{d}x}{\mathrm{d}t}=9\sin 2t\times 5\cos t$ or $9\times 2\sin t\cos t\times 5\cos t$	M1	1.2
	(Area =) $\int 9\sin 2t\times 5\cos t\,\mathrm{d}t=\int 9\times 2\sin t\cos t\times 5\cos t\,\mathrm{d}t$ or $\int 9\sin 2t\times 5\cos t\,\mathrm{d}t=\int 90\sin t\cos^2 t\,\mathrm{d}t$	dM1	1.1b
	(Area =) $\int_0^{\frac{\pi}{2}} 90\sin t\cos^2 t\,\mathrm{d}t$ *	A1*	2.1*
		(3)	
(a)(ii)	$\int 90\sin t\cos^2 t\,\mathrm{d}t=-30\cos^3 t$	M1 A1	1.1b 1.1b
	Area = $\left[-30\cos^3 t\right]_0^{\frac{\pi}{2}}=0-(-30)=30$ *	A1*	2.1
		(3)	
(b)	$9\sin 2t=5.3\Rightarrow \sin 2t=\frac{5.3}{9}$	M1	3.4
	$t=0.3148...,1.256...$	A1	1.1b
	Attempts to finds the x values at both t values	dM1	3.4
	$t=0.3148...\Rightarrow x=1.548...$ $t=1.256\Rightarrow x=4.754...$	A1	1.1b
	Width of path = awrt 3.21 metres	A1	3.2a
		(5)	
			(11 marks)

Question	Scheme	Marks	AOs
13(a)	$k = \mathrm{e}$ **or** $x \neq \mathrm{e}$	B1	2.2a
		(1)	
(b)	$$g'(x) = \frac{(\ln x - 1)\times\frac{5}{x} - (5\ln x - 6)\times\frac{1}{x}}{(\ln x - 1)^2} = \frac{1}{x(\ln x - 1)^2}$$ or $$g'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(5 - (\ln(x) - 1)^{-1}\right) = (\ln x - 1)^{-2}\times\frac{1}{x} = \frac{1}{x(\ln x - 1)^2}$$ or $$g'(x) = (\ln x - 2)^{-1}\times\frac{5}{x} - (5\ln x - 6)(\ln x - 1)^{-2}\times\frac{1}{x} = \frac{1}{x(\ln x - 1)^2}$$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 1$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $5\ln x - 6 \ldots 0$ or $\ln x - 1 \ldots 0$ or $5\ln a - 6 \ldots 0$ or $\ln a - 1 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < \mathrm{e}, \quad a > \mathrm{e}^{\frac{6}{5}}$	A1	2.2a
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2+(y-r)^2=r^2$ or $x^2+y^2-2rx-2ry+r^2=0$	B1	2.2a
	$y=8-2x,\ x^2+y^2-2rx-2ry+r^2=0$ $\Rightarrow x^2+(8-2x)^2-2rx-2r(8-2x)+r^2=0$ or $y=8-2x,\ (x-r)^2+(y-r)^2=r^2$ $\Rightarrow (x-r)^2+(8-2x-r)^2=r^2$	M1	1.1b
	$x^2+64-32x+4x^2-2rx-16r+4rx+r^2=0$ $\Rightarrow 5x^2+(2r-32)x+(r^2-16r+64)=0$ *	A1*	2.1
		(3)	
(b)	$b^2-4ac=0 \Rightarrow (2r-32)^2-4\times5\times(r^2-16r+64)=0$	M1	3.1a
	$r^2-12r+16=0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow (r-6)^2-36+16=0 \Rightarrow r=...$	dM1	1.1b
	$r=6\pm2\sqrt{5}$	A1	1.1b
		(4)	
			(7 marks)

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^n \Rightarrow S_n - rS_n = \ldots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\frac{a(1-r^{12})}{1-r} = 8\times\frac{a(1-r^6)}{1-r}$ or $8\times\frac{a(1-r^{12})}{1-r} = \frac{a(1-r^6)}{1-r}$ Equation in r^{12} and r^6 (and possibly $1-r$)	M1	3.1a
	$1-r^{12} = 8(1-r^6)$	A1	1.1b
	$r^{12} - 8r^6 + 7 = 0 \Rightarrow (r^6-7)(r^6-1) = 0 \Rightarrow r^6 = \ldots$ or e.g. $1-r^{12} = 8(1-r^6) \Rightarrow (1-r^6)(1+r^6) = 8(1-r^6) \Rightarrow r^6 = \ldots$	dM1	2.1
	$r = \sqrt[6]{7}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: 5k, 5k + 1, 5k + 2, 5k+3, 5k+4 or equivalent e.g. 5k-2, 5k – 1, 5k, 5k + 1, 5k + 2		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(5k)^2 = 25k^2 \left(= 5\times 5k^2\right)$ is a multiple of 5 $(5k+1)^2 = 25k^2 + 10k + 1 = 5\times\left(5k^2 + 2k\right) + 1$ is one more than a multiple of 5 $(5k+2)^2 = 25k^2 + 20k + 4 = 5\times\left(5k^2 + 4k\right) + 4$ is four more than a multiple of 5 $\left(\text{or } (5k-1)^2 = 25k^2 - 10k + 1 = 5\times\left(5k^2 - 2k\right) + 1\right)$ is one more than a multiple of 5 $\left(\text{or } (5k+4)^2 = 25k^2 + 40k + 16 = 5\times\left(5k^2 + 8k + 3\right) + 1\right)$ is one more than a multiple of 5	A1	1.1b
	Attempts to square in all 5 distinct cases. E.g. attempts to square 5k, 5k + 1, 5k + 2 or e.g. 5k – 1, 5k, 5k + 1	M1	2.1
	Achieves accurate results for all five cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
			(4 marks)