Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Wednesday 7 October 2020

Afternoon (Time: 2 hours)	Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1 **Shadow Paper Set 1**

You must have: Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. (*a*) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1+12x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(*b*) Explain how you could use $x = \frac{1}{36}$ in the expansion to find an approximation for $\sqrt{12}$

There is no need to carry out the calculation.

(2)

(Total for Question 1 is 5 marks)

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2. By taking logarithms of both sides, solve the equation

$$5^{4q-2} = 7^{160}$$

giving the value of q to one decimal place.

(Total for Question 2 is 3 marks)

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3. Relative to a fixed origin O

- point P has position vector $3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$
- point Q has position vector $5\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$
- point R has position vector $4\mathbf{i} - 10\mathbf{j} + 6\mathbf{k}$

(*a*) Find $\overrightarrow{PQ}$

(2)

(*b*) Show that quadrilateral *OPQR* is a trapezium, giving reasons for your answer.

(2)

(Total for Question 3 is 4 marks)

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4. The function f is defined by $\mathrm{f}(x) = \frac{5x-9}{x-3}$, $x \in \mathbb{R}$, $x \neq 3$

(*a*) Find $\mathrm{f}^{-1}(9)$

(2)

(*b*) Show that $\mathrm{ff}(x) = a + \frac{b}{x}$, where a and b are integers to be found.

(3)

(Total for Question 4 is 5 marks)

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5. A racing bicycle has ten forward gears.

The fastest speed of the car

- in 1st gear is 10 km h^{-1}
- in 10th gear is 73 km h^{-1}

Given that the fastest speed of the racing bicycle in successive gears is modelled by an **arithmetic sequence**,

(*a*) find the fastest speed of the racing bicycle in 5th gear.

(3)

Given that the fastest speed of the racing bicycle in successive gears is modelled by a **geometric sequence**,

(*b*) find the fastest speed of the racing bicycle in 8th gear.

(3)

(Total for Question 5 is 6 marks)

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6. (*a*) Express $2\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α in radians to 4 decimal places.

(3)

The temperature, θ °C, inside a room on a given day is modelled by the equation

$$\theta = 13 + 2\sin\left(\frac{\pi t}{6} - 2\right) + 3\cos\left(\frac{\pi t}{6} - 2\right) \qquad 0 \leq t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (*a*),

(*b*) deduce the maximum temperature of the room during this day,

(1)

(*c*) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

(Total for Question 6 is 7 marks)

7.

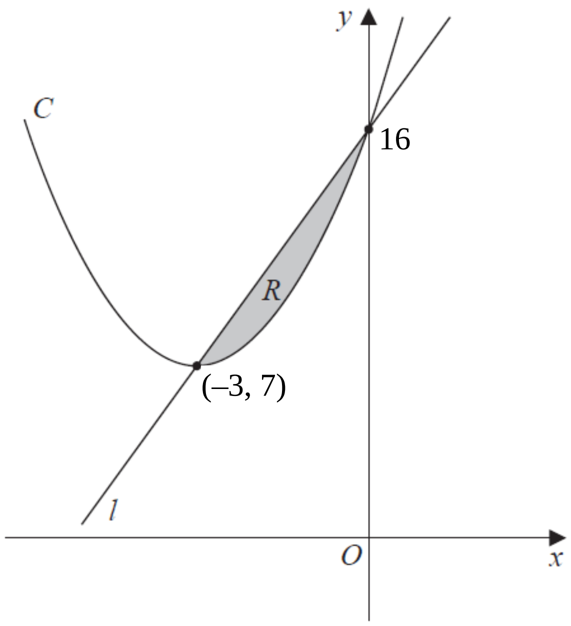

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \text{f}(x)$ and a straight line l.

The curve C meets l at the points $(-3, 7)$ and $(0, 16)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that $\text{f}(x)$ is a quadratic function in x and that $(-3, 7)$ is the minimum turning point of $y = \text{f}(x)$, use inequalities to define R.

(Total for Question 7 is 5 marks)

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8. A new computer game was released by a company.

The company monitored the total number of computer games sold, n, at time m months after the computer game was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of m.

(*You do not need to evaluate any unknown constants in your equation.*)

(Total for Question 8 is 2 marks)

9.

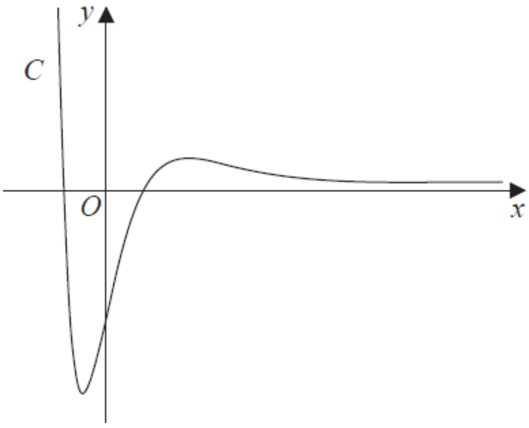

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 9(x^2 - 6)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(*a*) Show that $\mathrm{f}'(x) = 18(6 + x - x^2)\mathrm{e}^{-2x}$

(3)

(*b*) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 2 \qquad x \geq 0$$

(*c*) Find (i) the range of g,

(ii) the range of h.

(3)

(Total for Question 9 is 9 marks)

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10. (*a*) Use the substitution $x = u^2 - 1$ to show that

$$\int_8^{15} \frac{5\,\mathrm{d}x}{(x+1)\left(5+2\sqrt{x+1}\right)} = \int_a^b \frac{10\,\mathrm{d}u}{u(5+2u)}$$

where a and b are positive constants to be found.

√**(4)**

(*b*) Hence, using algebraic integration, show that

$$\int_8^{15} \frac{5\,\mathrm{d}x}{(x+1)\left(5+2\sqrt{x+1}\right)} = \ln p$$

where p is a rational constant to be found.

(6)

(Total for Question 10 is 10 marks)

11.

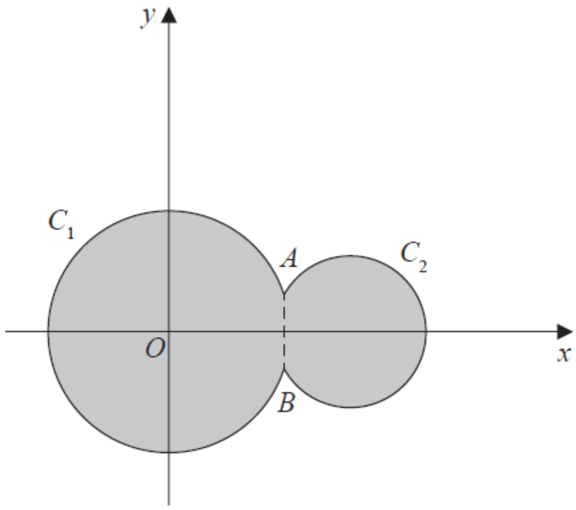

Figure 3

Circle C_1 has equation $x^2 + y^2 = 144$

Circle C_2 has equation $(x - 16)^2 + y^2 = 80$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle AOB = 1.171 radians to 4 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

(Total for Question 11 is 8 marks)

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12. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\sec\theta - \cos\theta \equiv \sin\theta\tan\theta \qquad \theta \neq (180n)^\circ \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x \leq 360°$

$$\sec x - \cos x = \sin x \tan(2x - 30°)$$

(5)

(Total for Question 12 is 8 marks)

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13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n - 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 4$

(a) show that

$$k^2 - 10k - 16 = 0$$

(3)

(b) For this sequence explain why $k \neq 8$

(1)

(c) Find the value of

$$\sum_{r=1}^{58} a_r$$

(3)

(Total for Question 13 is 7 marks)

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14. A large spherical balloon is rapidly deflating. At time t seconds the balloon has radius r cm and volume V cm^3.

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 80 cm
- after 5 seconds the radius of the balloon is 30 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

(Total for Question 14 is 10 marks)

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15. The curve C has equation

$$x^3 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-27x^2}{x^6 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[6]{\frac{81}{2}}$

(3)

(Total for Question 15 is 7 marks)

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16. Prove by contradiction that there are no positive integers a and b such that

$$4a^2 - b^2 = 49$$

(4)

TOTAL FOR PAPER IS 100 MARKS