**GCE A level Mathematics (9AM0) – Shadow Paper (Set 1)**
**9MA0-01 Pure Mathematics 1**

**October 2020 Shadow Paper mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.**

**This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2020.**

| **Guidance on the use of codes within this document** |
| --- |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.<br><br>A1 – accuracy mark. This mark is generally given for a correct answer following correct working.<br><br>B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.<br><br>Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1 (a)** | $(1+12x)^{\frac{1}{2}} =$ <br> $1 + \frac{1}{2} \times 12x + \frac{\frac{1}{2}\times-\frac{1}{2}}{2!} \times (12x)^2 + \frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!} \times (12x)^3$ | M1<br>A1 | 1.1b<br>1.1b |
| | $= 1 + 6x - 18x^2 + 108x^3 + \ldots$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Substitutes $x = \frac{1}{36}$ into $(1+12x)^{\frac{1}{2}}$ to give $\frac{\sqrt{12}}{3}$ | M1 | 1.1b |
| | Explains that $x = \frac{1}{36}$ is substituted into $1 + 6x - 18x^2 + 108x^3$ and you multiply the result by 3 | A1ft | 2.4 |
| | | **(2)** | |
| | | | **(5 marks)** |

**Note:**
**(b)** Sub in and equate M0A0.
Suggesting multiplying by wrong value with no calculator M0A0.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2** | $5^{4q-2} = 7^{160} \Rightarrow (4q-2)\log 5 = 160\log 7$ | M1 | 1.1b |
| | $\Rightarrow 4q = \frac{160\log 7}{\log 5} + 2 \Rightarrow q = \ldots$ | dM1 | 2.1 |
| | $p =$ awrt 48.9 | A1 | 1.1b |
| | | **(3)** | |
| | | | **(3 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3 (a)** | $\overrightarrow{PQ} = (5\mathbf{i}-3\mathbf{j}-2\mathbf{k})-(3\mathbf{i}+2\mathbf{j}-5\mathbf{k})$ | M1 | 1.1b |
| | $= 2\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | States $\overrightarrow{OR} = 2\times\overrightarrow{PQ}$ | M1 | 1.1b |
| | Explains that as *OR* is parallel to *PQ*, so *OPQR* is a trapezium. | A1 | 2.4 |
| | | **(2)** | |
| | | | **(4 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4 (a)** | Either attempts $\dfrac{5x-9}{x-3}=9 \Rightarrow x=...$ <br> Or attempts $f^{-1}(x)$ and substitutes in $x=9$ | M1 | 3.1a |
| | $\dfrac{9}{2}$ oe | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | Attempts $ff(x)=\dfrac{5\times\left(\dfrac{5x-9}{x-3}\right)-9}{\left(\dfrac{5x-9}{x-3}\right)-3}=\dfrac{5\times(5x-9)-9(x-3)}{5x-9-3(x-3)}$ | M1, dM1 | 1.1b 1.1b |
| | $=\dfrac{8x-9}{x}=8-\dfrac{9}{x}$ | A1 | 2.1 |
| | | **(3)** | |
| | | | **(5 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5 (a)** | Uses $73=10+9d \Rightarrow d=(7)$ | M1 | 3.1b |
| | Uses $10+4\times"7"=...$ | M1 | 3.4 |
| | = 38 (km h $^{-1}$ ) | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Uses $73=10r^9 \Rightarrow r=(1.2472)$ | M1 | 3.1b |
| | Uses $10\times"1.2472^7"=...$ | M1 | 3.4 |
| | = awrt 46.9 (km h $^{-1}$ ) | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |

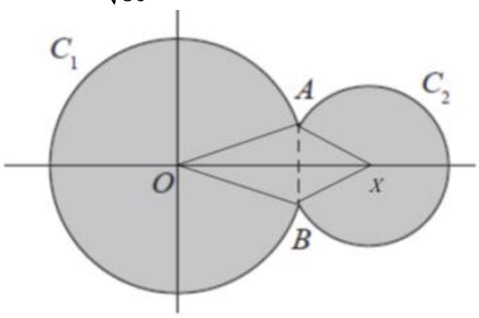
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6 (a)** | $R = \sqrt{13}$ | B1 | 1.1b |
| | $\tan\alpha = \frac{3}{2} \Rightarrow \alpha = ...$ | M1 | 1.1b |
| | $\alpha =$ awrt 0.9828 | A1 | 1.1b |
| | | **(3)** | |
| | $\theta = 13 + \sqrt{13}\sin\left(\frac{\pi t}{6} + 0.9828 - 2\right)$ | | |
| **(b)** | $\left(13+\sqrt{13}\right)$ °C or awrt 16.61 °C | B1ft | 2.2a |
| | | **(1)** | |
| **(c)** | $\frac{\pi t}{6} + 0.9828 - 2 = \frac{\pi}{2} \Rightarrow t =$ | M1 | 3.1b |
| | $t =$ awrt 4.94 | A1 | 1.1b |
| | Either 04:57 or 4:97am or 4 hours 57 minutes after midnight. | A1 | 3.2a |
| | | **(3)** | |
| | | | **(7 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7** | Attempts equation of line<br>Eg Substitutes $(-3,7)$ into $y = mx + 16$ and finds $m$ | M1 | 1.1b |
| | Equation of $l$ is $y = 3x + 16$ | A1 | 1.1b |
| | Attempts equation of $C$<br>Eg Attempts to use the intercept $(0,16)$ within the equation<br>$y = a(x \pm 3)^2 + 7$, in order to find $a$<br>$(\Rightarrow a = 1)$ | M1 | 3.1a |
| | Equation of $C$ is $y = (x+3)^2 + 7$ or $y = x^2 + 6x + 16$ | A1 | 1.1b |
| | Region $R$ is defined by $(x+3)^2 + 7 < y < 3x + 16$ o.e.<br>*Non-strict inequalities $(\le)$ as well as strict inequalities $(<)$ are acceptable. | B1ft | 2.5 |
| | | **(5)** | |
| | | | **(5 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8** | Any equation of the correct form, involving $n$ and an exponential in $m$. So allow for example $n = e^{\pm m}$, $n = Ae^{\pm m}$, $n = Ae^{\pm km}$ condoning $n = A + Be^{\pm m}$ | M1 | 3.1b |
| | $n = Ae^{kt}$ (where $A$ and $k$ are positive constants) | A1 | 1.1b |
| | | **(2)** | |
| | | | **(2 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9(a)** | $f(x) = 9(x^2 - 6)e^{-2x}$ | | |
| | Differentiates to $e^{-2x} \times 18x + 9(x^2 - 6) \times -2e^{-2x}$ | M1<br>A1 | 1.1b<br>1.1b |
| | $f'(x) = 18e^{-2x}\{x - (x^2 - 6)\} = 18(6 + x - x^2)e^{-2x}$ | A1 | 2.1 |
| | | **(3)** | |
| **(b)** | States roots of $f'(x) = 0$ $x = -2, 3$ | B1 | 1.1b |
| | Substitutes one $x$ value to find a $y$ value | M1 | 1.1b |
| | Stationary points are $(-2, -18e^4)$ and $(3, 27e^{-6})$ | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | (i) Range $[-36e^4, \infty)$ o.e. such as $g(x) \geqslant -36e^4$ | B1ft | 2.5 |
| | (ii) For<br>• Either attempting to find $2f(0) - 2 = 18 \times -6 - 2 = (-110)$ and identifying this as the **lower bound**<br>• Or attempting to find $2 \times "27e^{-6}" - 2$ and identifying this as the **upper bound** | M1 | 3.1a |
| | Range $[-110, 54e^{-6} - 2]$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(9 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10 (a)** | $x = u^2 - 1 \Rightarrow dx = 2u du$ oe | B1 | 1.1b |
| | Full substitution $\int \frac{5dx}{(x-1)(5+2\sqrt{x+1})} = \int \frac{5\times 2u\, du}{(u^2+1-1)(5+2u)}$ | M1 | 1.1b |
| | Finds correct limits e.g. $a$ = 3, $b$ = 4 | B1 | 1.1b |
| | $= \int \frac{5\times 2\not{u}\ du}{u^{\not{2}}(5+2u)} = \int \frac{10\ du}{u(5+2u)}$ * | A1* | 2.1 |
| | | **(4)** | |
| **(b)** | $\frac{10}{u(5+2u)} = \frac{A}{u} + \frac{B}{5+2u} \Rightarrow A = ..., B = ...$ | M1 | 1.1b |
| | Correct PF. $\frac{10}{u(5+2u)} = \frac{2}{u} - \frac{4}{5+2u}$ | A1 | 1.1b |
| | $\int \frac{10\,du}{u(5+2u)}$ = 2 ln $u$ – 2 ln (5 + 2$u$) (+ $c$) | dM1<br>A1ft | 3.1a<br>1.1b |
| | Uses limits $u = "4", u = "3"$ with some correct ln work leading to $k \ln b$<br>E.g. $\int_3^4 \frac{10\,du}{u(5+2u)} = (2\ln 4 - 2\ln 13) - (2\ln 3 - 2\ln 11) = 2\ln\frac{44}{39}$ | M1 | 1.1b |
| | ln $p$ = $\frac{1936}{1521}$ | A1 | 2.1 |
| | | **(6)** | |
| | | | **(10 marks)** |
| **Notes: Mark (a) and (b) together as one complete question** | | | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11 (a)** | Solves $x^2+y^2=144$ and $(x-16)^2+y^2=80$ simultaneously to find *x* or *y*<br>E.g. $(x-16)^2+144-x^2=80 \Rightarrow x=...$ | M1 | 3.1a |
| | Either $\Rightarrow -32x+400=80 \Rightarrow x=10$<br>Or $y=\ 2\sqrt{22}$ awrt $\pm 6.63$ | A1 | 1.1b |
| | Attempts to find the angle *AOB* in circle $C_1$<br>Eg Attempts $\cos\alpha=\frac{"10"}{12}$ to find $\alpha$ then $\times 2$ | M1 | 3.1a |
| | Angle *AOB* = $2\times\cos^{-1}\left(\frac{10}{12}\right)$ = 1.171 rads (4sf) | A1 | 2.1 |
| | | **(4)** | |
| **(b)** | Attempts 12 × (2$\pi$ – 1.171) = 61.35 | M1 | 1.1b |
| | Attempts to find angle *AXB* or *AXO* in circle $C_2$<br>$\cos\beta=\frac{16-"10"}{\sqrt{80}}\Rightarrow\beta=...$ (Note *AXB* =1.6709 rads, AXO=0.835…rads)<br> | M1 | 3.1a |
| | Attempts $12\times(2\pi-1.171)+\sqrt{80}\times(2\pi-2\beta)$<br>$=12\times(2\pi-1.171)+\sqrt{80}\times(2\pi-1.67)$ | dM1 | 2.1 |
| | (61.35 + 41.26)<br>= awrt 102.6 | A1 | 1.1b |
| | | **(4)** | |
| | | | **(8 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12 (a)** | States or uses $\sec\theta = \dfrac{1}{\cos\theta}$ | B1 | 1.2 |
| | $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta}$ | M1 | 2.1 |
| | $= \dfrac{\sin^2\theta}{\cos\theta} = \sin\theta \times \dfrac{\sin\theta}{\cos\theta} = \sin\theta\tan\theta$ | A1 | 2.1 |
| | | **(3)** | |
| **(b)** | $\sec x \cos x = \sin x \tan(2x - 30°)$<br>$\Rightarrow \sin x \tan x = \sin x \tan(2x - 30°)$ | | |
| | $\Rightarrow \tan x = \tan(2x - 30°) \Rightarrow x = 2x - 30°$ | M1 | 3.1a |
| | $x = 30°$ | A1 | 1.1b |
| | Also $\tan x = \tan(2x - 30°) \Rightarrow x + 180° = 2x - 30°$ | M1 | 2.1 |
| | $x = 210°$ | A1 | 1.1b |
| | Deduces $x = 180°$ | B1 | 2.2a |
| | | **(5)** | |
| | | | **(8 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **13 (a)** | Uses the sequence formula $a_{n+1} = \frac{k\left(a_n - 2\right)}{a_n}$ once with $a_1 = 4$ | M1 | 1.1b |
| | $\left(a_1 = 4\right), a_2 = \frac{k}{2}, a_3 = k-4, a_4 = \frac{k(k-6)}{k-4}$ <br> Finds four consecutive terms and sets $a_4$ equal to $a_1$ (oe) | M1 | 3.1a |
| | $\frac{k(k-6)}{k-4} = 4 \Rightarrow k^2 - 6k = 4k - 16 \Rightarrow k^2 - 10k + 16 = 0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | States that when $k = 8$, all terms are the same and concludes that the sequence does not have a period of order 3 | B1 | 2.3 |
| | | **(1)** | |
| **(c)** | Deduces the repeating terms are $a_{1/4} = 4, a_{2/5} = 1, a_{3/6} = -2,$ | B1 | 2.2a |
| | $\sum_{n=1}^{58} a_k = 19 \times \left(4 + 1 + -2\right) + 4$ | M1 | 3.1a |
| | $= 61$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(7 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14 (a)** | Uses the model to state $\frac{dV}{dt} = -c$ (for positive constant $c$) | B1 | 3.1b |
| | Uses $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ with their $\frac{dV}{dt} = -c$ and $\frac{dV}{dr} = 4\pi r^2$ | M1 | 2.1 |
| | $-c = 4\pi r^2 \times \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = -\frac{c}{4\pi r^2} = -\frac{k}{r^2}$ * | A1* | 2.2a |
| | | **(3)** | |
| **(b)** | $\frac{dr}{dt} = -\frac{k}{r^2} \Rightarrow \int r^2 dr = \int -k dt$ and integrates with one side "correct" | M1 | 2.1 |
| | $\frac{r^3}{3} = -kt(+\alpha)$ | A1 | 1.1b |
| | Uses $t = 0, r = 80 \Rightarrow \alpha = ...$ $\alpha = \frac{512000}{3}$ | M1 | 1.1b |
| | Uses $t = 5, r = 30$ & $\alpha = ... \Rightarrow k = ...$ $\left(\Rightarrow k = \frac{97000}{3}\right)$ | M1 | 3.4 |
| | $r^3 = 512000 - 97000t$ or exact equivalent | A1 | 3.3 |
| | | **(5)** | |
| **(c)** | Uses the equation of their model and proceeds to a limiting value for $t$ E.g. $"512000 - 97000t" ... 0 \Rightarrow t...$ | M1 | 3.4 |
| | For times up to and including $\frac{512}{97}$ seconds (awrt 5.28 | A1ft | 3.5b |
| | | **(2)** | |
| | | | **(10 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15 (a) | $x^3 \tan y = 9 \Rightarrow 3x^2 \tan y + x^3 \sec^2 y \dfrac{dy}{dx} = 0$ | M1<br>A1 | 3.1a<br>1.1b |
| | Full method to get $\dfrac{dy}{dx}$ in terms of $x$ using $\sec^2 y = 1 + \tan^2 y = 1 + f(x)$ | M1 | 1.1b |
| | $\dfrac{dy}{dx} = \dfrac{-27x^{-4}}{\left(1+\dfrac{81}{x^6}\right)} = \dfrac{-27x^2}{x^6+81}$ | A1 | 2.1 |
| | | **(4)** | |
| (b) | $\dfrac{dy}{dx} = \dfrac{-27x^2}{x^6+81}$<br>$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-54x\left(x^6+81\right)-\left(-27x^2\right)\left(6x^5\right)}{\left(x^6+81\right)^2} = \dfrac{54x\left(2x^6-81\right)}{\left(x^6+81\right)^2}$ o.e. | M1<br>A1 | 1.1b<br>1.1b |
| | States that when $x < \sqrt[6]{\dfrac{81}{2}} \Rightarrow \dfrac{d^2y}{dx^2} < 0$<br>when $x = \sqrt[6]{\dfrac{81}{2}} \Rightarrow \dfrac{d^2y}{dx^2} = 0$<br>**AND** when $x > \sqrt[6]{\dfrac{81}{2}} \Rightarrow \dfrac{d^2y}{dx^2} > 0$<br>giving a point of inflection when $x = \sqrt[6]{\dfrac{81}{2}}$ | A1 | 2.4 |
| | | **(3)** | |
| | | | **(7 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 16 | Sets up the contradiction and factorises:<br>There are positive integers *a* and *b* such that<br>$(2a+b)(2a-b) = 49$ | M1 | 2.1 |
| | If true then $2a+b=49$, $2a-b=1$ or $2a+b=7$, $2a-b=7$<br>**Award for deducing either of the above statements** | M1 | 2.2a |
| | Solutions are $a = 12.5, b = 24$ or $a = 3.5, b = 0$<br>Award for one of these | A1 | 1.1b |
| | This is a contradiction as there are no integer solutions hence there are no positive integers *p* and *q* such that $4a^2 - b^2 = 49$ | A1 | 2.1 |
| | | **(4)** | |
| | | | **(4 marks)** |