GCE A Level Mathematics (9MA0) – Shadow Paper (Set 1)

9MA0-02 Pure Mathematics 2

June 2022 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

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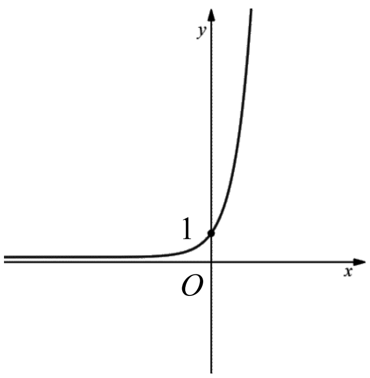
Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1	For an attempt to solve **Either** $5-4x=2+x \Rightarrow x=\dots$ **or** $4x-5=2+x \Rightarrow x=\dots$	M1	1.1b
	Either $x=\frac{3}{5}$ **or** $x=\frac{7}{3}$	A1	1.1b
	For an attempt to solve **Both** $5-4x=2+x \Rightarrow x=\dots$ **and** $4x-5=2+x \Rightarrow x=\dots$	dM1	1.1b
	For **both** $x=\frac{3}{5}$ **and** $x=\frac{7}{3}$ with no extra solutions	A1	1.1b
		(4)	
ALT	**Alternative by squaring:**		
	$(5-4x)^2=(2+x)^2 \Rightarrow 16x^2-40x+25=4+4x+x^2$	M1	1.1b
	$15x^2-44x+21=0$	A1	1.1b
	$15x^2-44x+21=0 \Rightarrow x=\dots$	dM1	1.1b
	For **both** $x=\frac{3}{5}$ **and** $x=\frac{7}{3}$ with no extra solutions	A1	1.1b
			(4 marks)

Note this question requires working to be shown not <u>just answers written down</u>. But correct equations seen followed by the correct answers can score full marks.

Question	Scheme		Marks	AOs
2(a)		Correct shape or correct intercept – see notes	B1	1.2
		Fully correct – see notes	B1	1.1b
			(2)	
(b)	$6^x=200 \Rightarrow x=\log_6 200$ or e.g. $x\log 6=\log 200 \Rightarrow x=\frac{\log 200}{\log 6}$		M1	1.1b
	$\Rightarrow (x=)$ awrt 2.957		A1	1.1b
			(2)	
				(4 marks)

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Question	Scheme	Marks	AOs
3(a)(i)	$a_1=3,\ a_2=0,\ a_3=1.5,\ a_4=3\ldots$	B1	1.1b
(ii)	3	B1	1.1b
		(2)	
(b)	$\sum_{n=1}^{85} a_n = 28\times(3+0+1.5)+3$ o.e.	M1	3.1a
	$= 129$	A1	1.1b
		(2)	
			(4 marks)
Notes:			

Question	Scheme	Marks	AOs
4	$\frac{-5(x+h)^2-(-5x^2)}{h}=\ldots$	M1	2.1
	$\frac{-5(x+h)^2-(-5x^2)}{h}=\frac{-10xh-5h^2}{h}$	A1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\lim_{h\to 0}(-10x-5h)=-10x$	A1	2.5
		(3)	
			(3 marks)
Notes:			

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Question	Scheme	Marks	AOs
5(a)	States or uses $h = 1.25$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{"1.25"}{2}\left(1.58+2.75+2\left(3.39+3.83+4.17\right)\right)$	M1	1.1b
	= awrt 16.1 or $\frac{2569}{160}$	A1	1.1b
		(3)	
(b)(i)	$\int_1^6 \log_2\left(3x\right)^8 \, dx = 8\times 16.1 =$ awrt 128.45 or e.g. $\frac{2569}{20}$	B1ft	2.2a
(ii)	$\int_1^6 \log_2 48x \, dx = \int_1^6 \log_2\left(16\times 3x\right) \, dx = \int_1^6 \left(4+\log_2 3x\right) dx$ $= \left[4x\right]_1^6 + \int_1^6 \log_2 3x \, dx = 24-4+\int_1^6 \log_2 3x \, dx...$	M1	3.1a
	Awrt 36.1 or $\frac{5769}{160}$	A1ft	1.1b
		(3)	
			(6 marks)

Question	Scheme	Marks	AOs
6(a)	$\left(f'(x)=\right) -8\sin\left(2x-3\right)-5$	M1 A1	1.1b 1.1b
	Sets $f'(x) = -8\sin\left(2x-3\right)-5 = 0 \Rightarrow x = ...$	dM1	3.1a
	$x = 3.41$ Cao	A1	3.2a
		(4)	
(b)	Explains that f(2) > 0, f(3) < 0 **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1 = 2 - \frac{4\cos\left(4-3\right)-5\times 2+15}{"-8\sin\left(2\times 2-3\right)-5"}$ (NB f(2) = 7.161… and f'(2) = –11.7317…)	M1	1.1b
	x_1 = awrt 2.61	A1	1.1b
		(2)	
			(7 marks)

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Question	Scheme	Marks	AOs
7(a)	$\sqrt{9-4x}=3(1\pm\ldots)^{\frac{1}{2}}$	B1	1.1b
	$\left(1-\text{"}\frac{4x}{9}\text{"}\right)^{\frac{1}{2}}=\ldots+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(\text{"}\frac{4x}{9}\text{"}\right)^2}{2!}$ or $+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(\text{"}\frac{4x}{9}\text{"}\right)^3}{3!}$	M1	1.1b
	$\left(1-\frac{4x}{9}\right)^{\frac{1}{2}}=1+\frac{1}{2}\times\left(-\frac{4x}{9}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{4x}{9}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{4x}{9}\right)^3}{3!}$	A1	1.1b
	$\sqrt{9-4x}=3-\frac{2x}{3}-\frac{2x^2}{27}-\frac{4x^3}{243}$	A1	1.1b
		(4)	
(b)	States that the approximation will be an **overestimate** since all terms (after the first one) in the expansion are negative (since $x>0$)	B1	3.2b
		(1)	
			(5 marks)

Question	Scheme	Marks	AOs
8	$y=\frac{4(x-2)(x-9)}{9\sqrt{x}}=\frac{4}{9}x^{\frac{3}{2}}-\frac{44}{9}x^{\frac{1}{2}}+8x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\int\left(\frac{4}{9}x^{\frac{3}{2}}-\frac{44}{9}x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)\mathrm{d}x=\frac{8}{45}x^{\frac{5}{2}}-\frac{88}{9}x^{\frac{3}{2}}+16x^{\frac{1}{2}}(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 9 and applies to their $\frac{8}{45}x^{\frac{5}{2}}-\frac{88}{9}x^{\frac{3}{2}}+16x^{\frac{1}{2}}$	M1	2.2a
	$\left(\frac{216}{5}-88+48\right)-\left(\frac{32}{45}\sqrt{2}-\frac{176}{27}\sqrt{2}+16\sqrt{2}\right)=\frac{16}{5}-\frac{1376}{135}\sqrt{2}$ Area $R=\frac{16}{5}-\frac{1376}{135}\sqrt{2}$ or $\frac{1376}{135}\sqrt{2}-\frac{16}{5}$	A1	2.1
		(6)	
			(6 marks)

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Question	Scheme	Marks	AOs
9(a)	Deduces that $A = \pm 40$ **or** $b = 90$	B1	3.4
	Deduces that $A = \pm 40$ **and** $b = 90$ Note that B0B1 is not possible	B1	3.4
	Uses $t = 0$, $h = 35 \Rightarrow \alpha = ...$ E.g. $35 = \text{"}40\text{"}\cos(\alpha) \Rightarrow \alpha = ...$	M1	3.4
	$h = \lvert \pm 40\cos(90t + 28.6)° \rvert$	A1	3.3
		(4)	
(b)	$h = \lvert \pm 40\cos(90t \pm 90)° \rvert$ (or any multiple of 90 for the value of α)	B1	3.5b
		(1)	
			(5 marks)

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Question	Scheme	Marks	AOs
10(a)	Attempts to solve $\frac{5}{3}=\frac{9x+8}{3x+5}\Rightarrow x=...$ Or substitutes $x=\frac{5}{3}$ into $\frac{8-5x}{3x-9}$ $\frac{8-5x}{3x-9}$	M1	3.1a
	$f^{-1}\left(\frac{5}{3}\right)=\frac{1}{12}$	A1	1.1b
		(2)	
(b)	$\left(\frac{9x+8}{3x+5}=\right)3\pm\frac{...}{3x-5}$	M1	1.1b
	$\left(\frac{9x+8}{3x+5}=\right)3-\frac{7}{3x+5}$	A1	2.1
		(2)	
(c)	$0\le g^{-1}(x)\le 3$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0)=\frac{9\times0+8}{3\times0+5}$ **or** $f(3)=\frac{9\times3+8}{3\times3+5}$	M1	3.1a
	Attempts both boundaries $f(0)=\frac{9\times0+8}{3\times0+5}$ **and** $f(3)=\frac{9\times3+8}{3\times3+5}$	dM1	1.1b
	Range $\frac{8}{5}\le fg^{-1}(x)\le\frac{5}{2}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x)=\sqrt{9-x}\Rightarrow fg^{-1}(x)=\frac{9\sqrt{9-x}+8}{3\sqrt{9-x}+5}$ $fg^{-1}(0)=\frac{9\sqrt{9-0}+8}{3\sqrt{9-0}+5}$ **or** $fg^{-1}(0)=\frac{9\sqrt{9-0}+8}{3\sqrt{9-0}+5}$	M1	3.1a
	$fg^{-1}(0)=\frac{9\sqrt{9-0}+8}{3\sqrt{9-0}+5}$ **and** $fg^{-1}(0)=\frac{9\sqrt{9-0}+8}{3\sqrt{9-0}+5}$	dM1	1.1b
	Range $\frac{8}{5}\le fg^{-1}(x)\le\frac{5}{2}$	A1	2.1
		(3)	
			(8 marks)

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Question	Scheme	Marks	AOs
11	$n(n^2+7)$		
	Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n(n^2+7)$	M1	3.1a
	Achieves $2k(4k^2+7)$ (for $n=2k$) and states "even" **Or** achieves: $(2k+1)(4k^2+4k+8)=2(2k+1)(2k^2+2k+4)$ (for $n=2k+1$) and states "even" Or e.g. achieves $(2k-1)(4k^2-4k+8)=2(2k-1)(2k^2-2k+4)$ (for $n=2k-1$) and states "even"	A1	2.2a
	Attempts even **and** odd numbers Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n(n^2+7)$	dM1	2.1
	Achieves $2k(4k^2+7)$ (for $n=2k$) and states "even" **Or** achieves: $(2k\pm1)(4k^2\pm4k+8)=2(2k\pm1)(2k^2\pm2k+4)$ (for $n=2k\pm1$) and states "even" Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
			(4 marks)

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Question	Scheme	Marks	AOs
12(a)	$\mathrm{f}(x) = -\frac{\mathrm{e}^{5x}}{3x^2 + k} \Rightarrow \mathrm{f}'(x) = \frac{-5\mathrm{e}^{5x}(3x^2 + k) + 6x\mathrm{e}^{5x}}{(3x^2 + k)^2}$ or $\mathrm{f}(x) = -\mathrm{e}^{5x}(3x^2 + k)^{-1} \Rightarrow \mathrm{f}'(x) = -5\mathrm{e}^{5x}(3x^2 + k)^{-1} + 6x\mathrm{e}^{5x}(3x^2 + k)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x) = \frac{-\mathrm{e}^{5x}(15x^2 - 6x + 5k)}{(3x^2 + k)^2}$	A1	2.1
		(3)	
(b)	If $y = \mathrm{f}(x)$ has no stationary point then $15x^2 - 6x + 5k = 0$ has no root	B1	2.2a
	Applies $b^2 - 4ac < 0$ with $a = 15$, $b = -6$, $c = 5k$	M1	2.1
	$k > \frac{3}{25}$	A1	1.1b
		(3)	
			(6 marks)

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Question	Scheme	Marks	AOs
13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{PQ} = \pm(-\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $\pm\overrightarrow{PR} = \pm(-6\mathbf{i} - 18\mathbf{j} + (a-4)\mathbf{k})$ $\pm\overrightarrow{QR} = \pm(-5\mathbf{i} - 15\mathbf{j} + (a-6)\mathbf{k})$	M1	3.1a
	Uses two of the three vectors in such a way as to find the value of a. e.g. $a - 6 = 5 \times 2$	dM1	2.1
	$a = 16$	A1	1.1b
		(3)	
	(a) Alternative:		
	$\mathbf{r}_{PQ} = 3\mathbf{i} + 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	M1	3.1a
	$3\mathbf{i} + 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} - 18\mathbf{j} + a\mathbf{k} \Rightarrow \mu = 6$ $3\mathbf{i} + 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} - 18\mathbf{j} + a\mathbf{k} \Rightarrow a = 4 + 12$	dM1	2.1
	$a = 16$	A1	1.1b
(b)	Deduces that $\overrightarrow{OS} = \mu\overrightarrow{OP} = 3\mu\mathbf{i} + 4\mu\mathbf{k}$ $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ $\overrightarrow{RS} = (3\mu + 3)\mathbf{i} + 18\mathbf{j} + (4\mu - "16")\mathbf{k}$	M1	3.1a
	Correct attempt at μ using the fact that $\overrightarrow{RS}$ is parallel to $\overrightarrow{OQ}$ $\overrightarrow{RS} = (3\mu + 3)\mathbf{i} + 18\mathbf{j} + (4\mu - "16")\mathbf{k}$ $\overrightarrow{OQ} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ $3\mu + 3 = -12 \Rightarrow \mu = \ldots$ OR $4\mu - 16 = -36 \Rightarrow \mu = \ldots$	dM1	1.1b
	$\lvert\overrightarrow{OS}\rvert = 5\sqrt{3^2 + 4^2} = 25$	A1	1.1b
		(3)	
			(6 marks)

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Question	Scheme	Marks	AOs
14(a)	$\frac{11}{(3x-2)(x+3)} \equiv \frac{A}{3x-2} + \frac{B}{x+3} \Rightarrow A = ..., B = ...$	M1	1.1b
	Either $A = 3$ or $B = -1$	A1	1.1b
	$\frac{11}{(3x-2)(x+3)} \equiv \frac{3}{3x-2} - \frac{1}{x+3}$	A1	1.1b
		(3)	
(b)	$\int \frac{1}{M}\ \mathrm{d}M = \int \frac{11}{(3t-2)(t+3)}\ \mathrm{d}t$	B1	1.1a
	$\int \frac{3}{3t-2} - \frac{1}{t+3}\ \mathrm{d}t$ =...ln $(3t-2)$ - ...ln $(t+3)$ $(+ c)$	M1	3.1a
	$\ln M = \ln(3t-2) - \ln(t+3)\ (+c)$	A1ft	1.1b
	Substitutes $t = 2.5$, $M = 5.5 \Rightarrow c = (\ln 5.5)$	M1	3.4
	$\ln M = \ln(3t-2) - \ln(t+3) + \ln 5.5$ $M = \frac{11(3t-2)}{2(t+3)}$ *	A1*	2.1
		(5)	
(c)	(i) 40 seconds	B1	3.2a
	(ii) 16.5 mg	B1	3.4
		(2)	
			(10 marks)

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Question	Scheme	Marks	AOs
15(a)	Uses the common ratio $\frac{3+2\cos\theta}{4\sin\theta}=\frac{8\cot\theta}{3+2\cos\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\cot\theta\times\sin\theta=\cos\theta$ $(3+2\cos\theta)^2=4\times 8\cos\theta$	dM1	1.1b
	Proceeds to given answer: $9+12\cos\theta+4\cos^2\theta=32\cos\theta$ $\Rightarrow 4\cos^2\theta-20\cos\theta+9=0$	A1*	2.1
		(3)	
(b)	$4\cos^2\theta-20\cos\theta+9=0\Rightarrow\cos\theta=\frac{1}{2}\quad\left(,\frac{9}{2}\right)$	M1	1.1b
	$\theta=\frac{5\pi}{3}$	A1	1.2
		(2)	
(c)	Attempts a value for either a or r e.g. $a=4\sin\left(\frac{5\pi}{3}\right)=4\times\left(-\frac{\sqrt{3}}{2}\right)$ **or** $r=\frac{3+2\cos\theta}{4\sin\theta}=\frac{3+2\times\frac{1}{2}}{4\times\left(-\frac{\sqrt{3}}{2}\right)}$	M1	3.1a
	$a=-2\sqrt{3}$ **and** $r=-\frac{2\sqrt{3}}{3}$ oe	A1	1.1b
	Uses $S_\infty=\frac{a}{1-r}=\frac{-2\sqrt{3}}{1+\frac{2\sqrt{3}}{3}}$	dM1	2.1
	Rationalises denominator $S_\infty=\frac{-2\sqrt{3}}{1+\frac{2\sqrt{3}}{3}}=\frac{-6\sqrt{3}}{3+2\sqrt{3}}\times\frac{3-2\sqrt{3}}{3-2\sqrt{3}}$	ddM1	1.1b
	$6\left(\sqrt{3}-2\right)$	A1	2.1
		(5)	
			(10 marks)

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Question	Scheme	Marks	AOs
16(a)	Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{-6\operatorname{cosec}^2 t\cot t}{-3\operatorname{cosec}^2 t}=2\cot t$	M1 A1	1.1b 1.1b
	At $t=\dfrac{3\pi}{4}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-2$, $x=-4$, $y=8$	M1	2.1
	Attempts equation of normal $y-8=\dfrac{1}{2}(x+4)$	M1	1.1b
	$y=\dfrac{1}{2}x+10$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\operatorname{cosec}^2 t=1+\cot^2 t\Rightarrow\dfrac{y-2}{3}=1+\left(\dfrac{x+1}{3}\right)^2$	M1	3.1a
	$\Rightarrow y-2=3+\dfrac{(x+1)^2}{3}\Rightarrow y=\dfrac{1}{3}(x+1)^2+5$ *	A1*	2.1
		(2)	
(c)	**Attempts the lower limit for *k*:** $\dfrac{1}{3}(x+1)^2+5=\dfrac{1}{2}x+k\Rightarrow 2x^2+x+(32-6k)=0$ $b^2-4ac=12-4\times 2(32-6k)\Rightarrow k=...$	M1	2.1
	$k=\dfrac{85}{16}$	A1	1.1b
	Attempts the upper limit for *k*: $(x,y)_{t=\frac{\pi}{4}}$: $t=\dfrac{\pi}{4}\Rightarrow x=3\cot\dfrac{\pi}{4}-1=2$, $y=3\operatorname{cosec}^2\dfrac{\pi}{4}+2=8$ (2, 8), $y=\dfrac{1}{2}x+k\Rightarrow 8=1+k\Rightarrow k=...$	M1	2.1
	$(k=)\ 7$	A1	1.1b
	$\dfrac{85}{16}<k\le 7$	A1	2.2a
		(5)	
			(12 marks)

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