Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **9MA0/01**

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

June 2022, Shadow Set 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. The point $P\ (-5, -2)$ lies on the curve with equation $y = \mathrm{f}(x), \quad x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(*a*) $y = \mathrm{f}(x) - 4$

(1)

(*b*) $y = |\mathrm{f}(x)|$

(1)

(*c*) $y = 5\mathrm{f}(x + 4) - 4$

(2)

(Total for Question 1 is 4 marks)

2.

$$\mathrm{f}(x) = (x - 2)(x^3 + 8x^2 - 20x + k) - 1225 \text{ where } k \text{ is a constant}$$

Given that $(x + 3)$ is a factor of $\mathrm{f}(x)$, find the value of k.

(3)

(Total for Question 2 is 3 marks)

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3. A circle has equation

$$x^2 + y^2 + 6x - 14y = 138$$

(*a*) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

(3)

Given that P is the point on the circle that is nearest to the origin O,

(*b*) find the exact length OP

(2)

(Total for Question 3 is 5 marks)

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4. (*a*) Express $\lim\limits_{\delta x \to 0} \sum\limits_{x=2.4}^{9.6} \frac{5}{x}\,\delta x$ as an integral.

(1)

(*b*) Hence show that

$$\lim_{\delta x \to 0} \sum_{x=2.4}^{9.6} \frac{5}{x}\,\delta x = \ln p$$

where p is a constant to be found.

(2)

(Total for Question 4 is 3 marks)

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5. The mass, m kilogrammes, of a cat, t months after it is born, is modelled by the equation

$$m^2 = pt + q \qquad 0 \leq t < 170$$

where p and q are constants.

Given that

• the mass of the cat was 2.13 kg, exactly 3 months after it was born

• the mass of the cat was 3.25 kg, exactly 7 months after it was born

(a) find a complete equation for the model, giving the values of p and q to 2 significant figures.

(4)

Given that the mass of the cat was 5 kg, exactly 8 years after it was born

(b) evaluate the model, giving reasons for your answer.

(2)

(Total for Question 5 is 6 marks)

6.

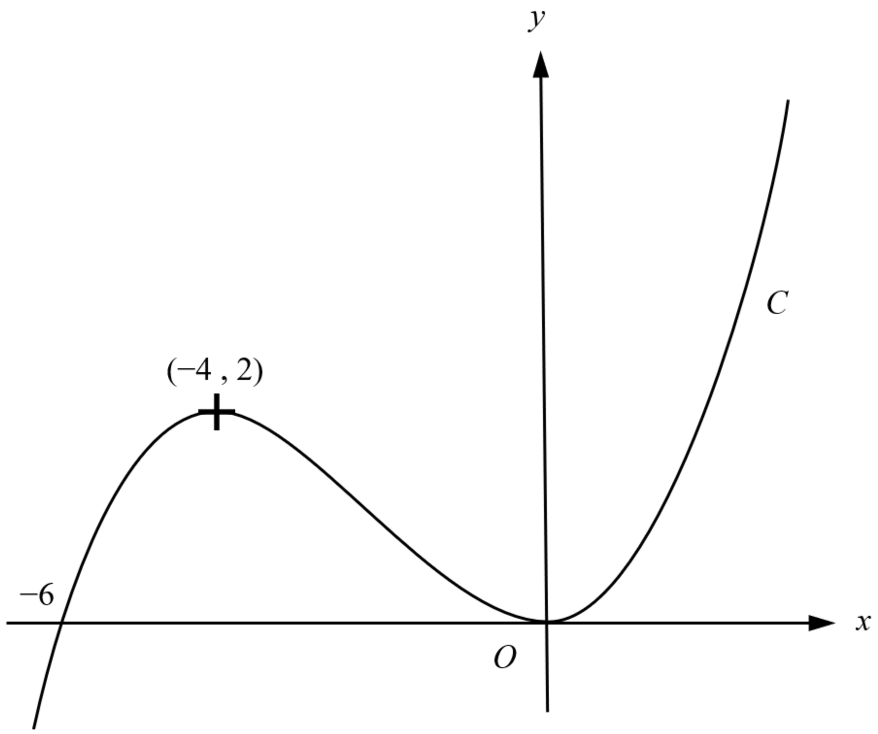

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a cubic expression in x.

The curve

• passes through (–6, 0)

• has a maximum turning point at (–4, 2)

• has a minimum turning point at the origin

(*a*) Write down the set of values of x for which

$$\mathrm{f}'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(*b*) Find the set of values of k, giving your answer in set notation.

(2)

(*c*) Find the equation of C. You may leave your answer in factorised form.

(3)

(Total for Question 6 is 6 marks)

7. (i) Given that p and q are consecutive integers such that

$$p + q \text{ is a multiple of } 3$$

use algebra to prove by contradiction that neither p nor q is a multiple of 3.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x - y)^2 < y^2 - 3x^2$

show that $y < 2x$

(2)

(Total for Question 7 is 5 marks)

8.

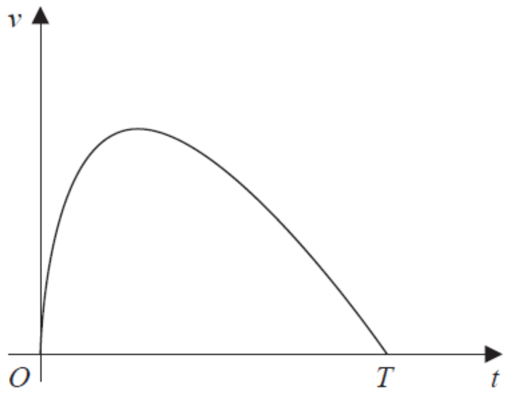

Figure 2

A train leaves Tokyo station and travels to Osaka station.

Figure 2 shows a graph of the speed of the train, v km h^{-1}, as it travels between the two stations.

The train takes T seconds to travel between the two stations.

The speed of the train is modelled by the equation

$$v = (1500 - 600t)\ln(t + 1) \qquad 0 \leq t \leq T$$

where t hours is the time after the train leaves Tokyo station.

According to the model,

(a) find the value of T

(1)

(b) show that the maximum speed of the train occurs when

$$t = \frac{7}{2 + 2\ln(t+1)} - 1$$

(4)

Using the iteration formula

$$t_{n+1} = \frac{7}{2 + 2\ln(t_n + 1)} - 1$$

with $t_1 = 1.05$

(c) (i) find the value of t_3 to 3 decimal places,

(ii) find, by repeated iteration, the time taken for the car to reach maximum speed.

(3)

(Total for Question 8 is 8 marks)

9.

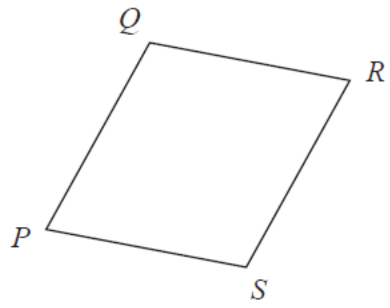

Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 5\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = \mathbf{j} - 7\mathbf{k}$

(*a*) show that parallelogram *PQRS* is a rhombus.

(2)

(*b*) Find the exact area of the rhombus *PQRS*.

(4)

(Total for Question 9 is 6 marks)

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10. An ecologist is investigating the number of lions and wildebeest in a national park.

The number of lions, measured in hundreds, L, is modelled by the equation

$$L = 1 + 2e^{0.03t}$$

where t is the number of years from the start of the investigation.

According to the model,

(*a*) find the number of lions at the start of the study,

(1)

(*b*) show that, exactly 5 years after the start of the investigation, the number of lions was increasing at a **rate** of approximately 7 per year.

(3)

The number of wildebeest, measured in thousands, W, is modelled by the equation

$$W = 10 + 4e^{-0.03t}$$

where t is the number of years from the start of the investigation.

When $t = T$, according to the models, there are 10 times as many wildebeest as lions.

(*c*) Find the value of T to one decimal place.

(4)

(Total for Question 10 is 8 marks)

11.

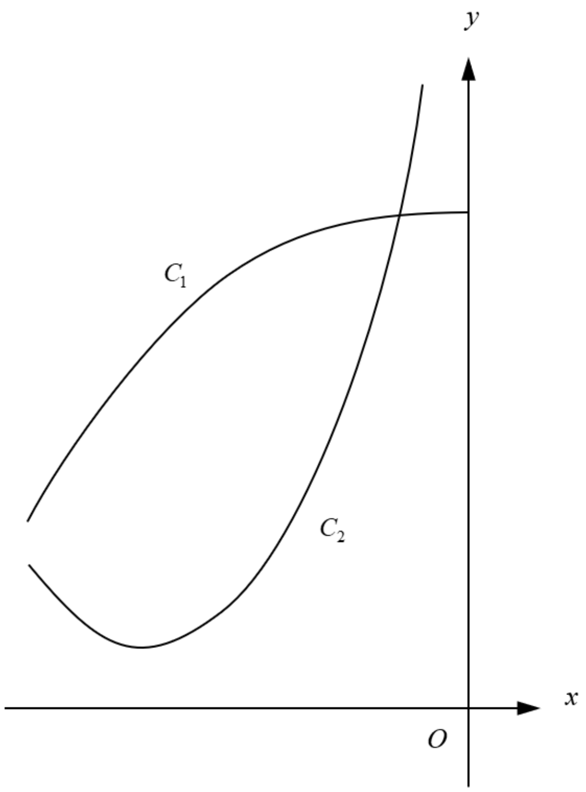

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3x^3 + 60 \qquad x < 0$$

and part of the curve C_2 with equation

$$y = 29x^2 + 100x + 90 \quad x < 0$$

(*a*) Verify that the curves intersect at $x = -\frac{1}{3}$

(2)

The curves intersect again at the point P C_2

(*b*) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

(Total for Question 11 is 7 marks)

12. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_{e}^{e^2} x^2 \ln x \, dx = \frac{1}{9}e^3\left(Ae^3 + B\right)$$

where A and B are rational constants to be found.

(5)

(Total for Question 12 is 5 marks)

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13. (i) In a geometric series, the first term is a and the common ratio is r

Show that

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

(3)

(ii) Jamal's mum asks him to buy groceries every week. She gives him some money to spend and a list of items and Jamal is allowed to keep any money which is left over. Jamal decides to try and use this money to save up to buy some new headphones that cost £36. He saves £6.60 in week 1, £6.00 in week 2, £5.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that Jamal takes n weeks to save exactly £36

(*a*) show that

$$n^2 - 23n + 120 = 0$$

(2)

(*b*) Solve the equation

$$n^2 - 23n + 160 = 0$$

(1)

(*c*) Hence state the number of weeks Jamal takes to save enough money to buy the headphones, giving a brief reason for your answer.

(1)

(Total for Question 13 is 7 marks)

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14. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Given that

$$5 \sin (x + 30°) = 3 \cos (x - 30°)$$

show that

$$\tan x = \frac{3\sqrt{3} - 5}{5\sqrt{3} - 3}$$

(4)

(*b*) Hence or otherwise solve, for $0 \leq \theta < 180°$

$$5 \sin (2\theta + 60°) = 3 \cos 2\theta$$

giving your answers to the nearest integer.

(4)

(Total for Question 14 is 8 marks)

15.

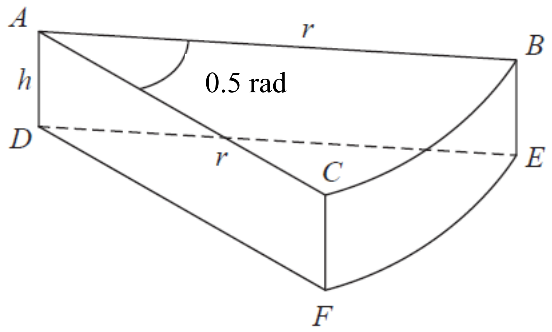

Figure 5

Harminder models a slice of the cheesecake she has made.

Figure 5 shows the outline of the cheesecake.

The slice is modelled so that

- face *ABC* is a sector of a circle with radius *r* cm and centre *A*
- angle *BAC* = 0.5 radians
- faces *ABC* and *DEF* are congruent
- edges *AD*, *CF* and *BE* are perpendicular to faces *ABC* and *DEF*
- edges *AD*, *CF* and *BE* have length *h* cm

Given that the volume of the slice of cheesecake is 125 cm^3

(*a*) show that the surface area of the slice, *S* cm^2, is given by

$$S = \frac{r^2}{2} + \frac{1250}{r}$$

making your method clear.

(4)

Using algebraic differentiation,

(*b*) find the value of *r* for which *S* has a stationary point.

(4)

(*c*) Prove, by further differentiation, that this value of *r* gives the minimum surface area of the slice.

(2)

(Total for Question 15 is 10 marks)

16.

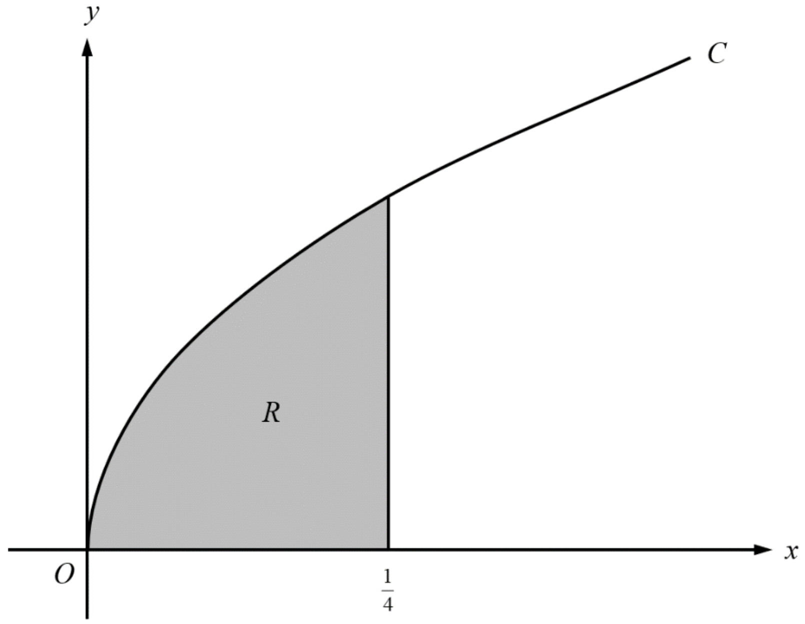

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = \cos^2 t \qquad y = 3\cos 2t + 2\cos t + 3 \qquad \frac{\pi}{4} \leq t \leq \frac{\pi}{2}$$

The region R, shown shaded in Figure 6, is bounded by C, the x-axis and the line with equation $x = \frac{1}{4}$

(*a*) Show that the area of R is given by

$$\int_{\frac{\pi}{2}}^{a} \left(-\frac{3}{2}\sin 4t - 4\cos^2 t \sin t - 3\sin 2t \right) \mathrm{d}t$$

where a is a constant to be found.

(5)

(*b*) Hence, using algebraic integration, find the exact area of R.

(4)

(Total for Question 16 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS