GCE A level Further Mathematics (9FM0) – Shadow Paper (Set 1)
9FM0-3B AL Further Statistics 1

October 2021 Shadow Paper mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2021.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question	Scheme	Marks	AOs
1(a)	$x = 4 \times 43 - 46 - 33 - 38 = 55$	B1	3.4
		(1)	
(b)	$\nu = 4 - 1 = 3$ since the only constraint is that the totals agree	B1	2.4
		(1)	
(c)	H_0: The die is unbiased	B1	2.1
	H_1: The die is biased		
	Test Statistic $= \frac{(46-43)^2}{43} + \frac{(33-43)^2}{43} + \frac{(38-43)^2}{43} + \frac{(55-43)^2}{43}$	M1	1.1b
	$= 6.4651\ldots.$	A1	1.1b
	$\chi^2_{(3,0.05)} = 7.815$	B1	1.1b
	Not in the critical region since 7.815 > "6.465…" therefore insufficient evidence to reject H_0 Inconclusive test - consistent with the **die** being unbiased.	A1	3.5a
		(5)	
			(7 marks)

Question	Scheme	Marks	AOs
2(a)	$F \sim$ Poisson (4.8)	M1	3.3
	$P(F \geq 3) = 0.85746..$ *awrt 0.857*	A1cso	1.1b
		(2)	
(b)	$A \sim$ B(7, "0.857")	M1	3.3
	$\mathrm{P}(A \leq 3) = 0.010\,068\ \ldots$	A1	1.1b
		(2)	
(c)	$P(F = 10) = 0.0147..$	B1	1.1b
	$E \sim$ B(120, "0.0147..") $\Rightarrow$ mean $= 120 \times 0.0147$ [= 1.764]	M1	3.3
	$E \sim$ Po("1.764…") $\Rightarrow \mathrm{P}(E \geq 4) = [1 - P(E \leqslant 3)]$	M1	3.4
	$=$ 0.103	A1cso	2.1
		(4)	
(d)	The number of periods is large and the probability of getting 10 faults is small	B1 (1)	2.4
(e)	H_0: $\lambda = 16.8$ H_1: $\lambda \neq 16.8$	B1	2.5
		(1)	
(f)	$X \sim \mathrm{Po}(16.8)$	B1	3.3
	$P(X \geqslant 23) = 1 - P(X \leqslant 22)$	M1	1.1b
	= 0.08687…	A1	1.1b
	0.087… > 0.025 or no evidence to reject H_0 There is insufficient evidence at the 5% level of significance that the number of **faults produced** is different on a Saturday	A1 (4)	2.2b
			(14 marks)

Question	Scheme	Marks	AOs
3	$\overline{X} \approx N(216, \ldots)$ oe	M1	3.1a
	$\overline{X} \approx N(216, 0.6048)$	A1	1.1b
	$P(\overline{X} > 218) = P\left(Z > \frac{217-216}{\sqrt{"0.6048"}}\right)$ [= awrt 1.29]	dM1	3.4
	$p = 0.099..$	A1	1.1b
		(4)	
			(4 marks)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="7">4(a)</td><td>$2\mathrm{E}(N) + 5 = 9.6$ or $\mathrm{E}(N) = 2.3$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>$a + 0.5 + 3b + 4c + 1 = 2.3$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\frac{a}{0.25+a+0.1} = \frac{6}{13}$ so $a = 0.3$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>$b = 2c$,
$b = 0.1$ and $c = 0.05$</td><td></td><td></td></tr>
<tr><td>$E(N^2) = 1 \times "0.3" + 4 \times 0.25 + 9 \times "0.1" + 16 \times "0.05" + 25 \times 0.2 [= 8]$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\mathrm{Var}(N) = "8" - "2.3"^2$</td><td>dM1</td><td>1.1b</td></tr>
<tr><td>$= 2.71$</td><td>A1*</td><td>2.1</td></tr>
<tr><td></td><td>(6)</td><td></td></tr>
<tr><td rowspan="4">(b)</td><td><table><tr><td>Winnings</td><td>0.10</td><td>0.10</td><td>0.10</td><td>0.20</td><td>0.20</td><td>0.20</td></tr><tr><td>P($N = n$)</td><td>a</td><td>0.2</td><td>0.05</td><td>0.25</td><td>b</td><td>c</td></tr></table></td><td>M1</td><td>3.3</td></tr>
<tr><td>$0.1 \times 0.1 + 0.3 \times 0.1 + 0.25 \times 0.1 + 0.1 \times 0.2 + 0.05 \times 0.2 + 0.2 \times 0.2$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>= £0.135</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td>(c)</td><td>Poisson distribution will assign substantial probability to $N > 5$</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(10 marks)</td></tr>
</table>

Question	Scheme	Marks	AOs
5(a)	P(at least 3 green) = $(1-0.04)^3$	M1	1.1b
	= 0.88473.. awrt 0.885	A1	1.1b
		(2)	
(b)	P(2nd yellow on 10th draw) = $9\times(0.96)^8\times(0.04)^2 = 0.0104$	M1	3.3
	= 0.00104	A1	1.1b
		(2)	
(c)	$\frac{n}{p} = 22500$ and $\frac{n(1-p)}{p^2} = 300^2$	M1 A1	3.1b 1.1b
	$225 = 1125p$ oe	M1	1.1b
	$p = 0.2$	A1	1.1b
		(4)	
(d)	H_0: $p = 0.04$ H_1: $p < 0.04$	B1	2.5
	$J \sim$ Geo(0.04)	M1	3.3
	$P(J \geq c) < 0.05 \Rightarrow (1-0.04)^{c-1} < 0.05$	M1	3.4
	$c - 1 > \frac{log\ 0.05}{log\ 0.96}$	M1	1.1b
	$c > 74.38525\ldots$ ∴ CR $J \geq 75$	A1	1.1b
		(5)	
(e)	24 is not in the Critical region	M1	1.1b
	There is no evidence to suggest that bag C contains a smaller proportion of yellow counters than bag A	A1	2.2b
		(2)	
(f)	Power of test = $P(J \geq 75 \mid p = 0.013)$	M1	2.1
	= $(1-0.013)^{74}$ oe	M1	1.1b
	= 0.3797….	A1*	1.1b
		(3)	
			(18 marks)

Question	Scheme	Marks	AOs
6(a)	$G_X(1) = 1$	M1	2.1
	$k \times 3^4 = 1 \ \therefore k = \frac{1}{81}$	A1*cso (2)	1.1b
(b)	$P(X = 2)$ is coefficient of t^2 so $G_X(t) = k(\ldots + {}^4C_2(t)^2 + \ldots)$	M1	1.1b
	$P(X = 2) = \frac{6 \times 2^2}{81} = \frac{8}{27}$	A1 (2)	1.1b
(c)	$G_W(t) = \frac{t^2}{81}(2 + t^3)^4$	M1 A1	3.1a
(d)	$G_U(t) = \frac{1}{81}(2 + t)^4 \times \frac{t(2+t)^2}{9}$	M1	3.1a
	$G_U(t) = \frac{t(2+t)^6}{729}$	A1 (2)	1.1b
(e)	$G_U'(t) = \frac{6t(2 + t)^5}{729} + \frac{(2 + t)^6}{729}$	M1	2.1
	$G_U'(1) = 3$	A1ft	1.1b
	$G_U''(t) = \frac{30t(2 + t)^4}{729} + \frac{6(2 + t)^5}{729} + \frac{6(2 + t)^5}{729}$	M1	2.1
	$G_U''(1) = \frac{22}{3}$ $\text{Var}(U) = \frac{4}{3}$	A1	1.1b
	$\text{Var}(U) = "\frac{891}{80}" + "\frac{9}{2}" - \left("\frac{9}{2}"\right)^2$	M1	2.1
	$= \frac{3501}{4}$	A1 (6)	1.1b

Question	Scheme	Marks	AOs
7(a)	Size of the test = 0.01	B1	1.2
		(1)	
(b)(i)	Let CR be $\bar{L} < k$		
	$\frac{k-22}{\frac{0.15}{\sqrt{n}}} = -2.3263$	M1	3.4
	$k = 22 - \frac{0.348945}{\sqrt{n}}$	A1	1.1b
	$\frac{"22 - \frac{0.348945}{\sqrt{n}}" - 21.8}{\frac{0.15}{\sqrt{n}}} > 1.6449$	M1d A1ft	3.4 1.1b
	$\frac{0.59568}{\sqrt{n}} < 0.2 \quad \sqrt{n} > 2.9784$ oe	M1d	1.1b
	$n = 9$	A1cso	2.1
		(6)	
(ii)	The probability of a Type II error would decrease.	B1	2.2a
		(1)	
			(8 marks)