GCE A level Further Mathematics (9FM0) – Shadow Paper (Set 1)
9FM0-4D AL Decision 2

October 2021 Shadow Paper mark scheme

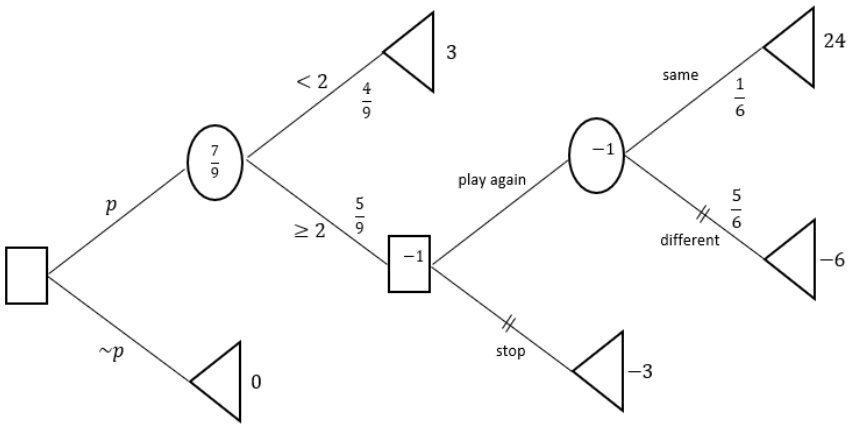
Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.

This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper which was published in December 2021.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

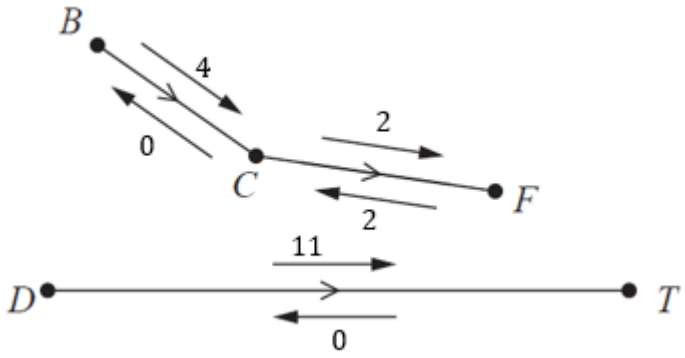
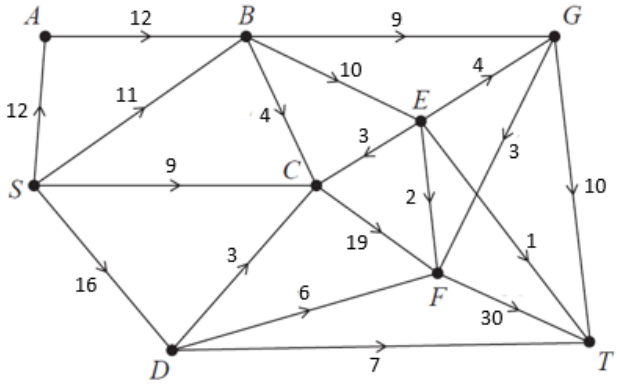
Question	Scheme	Marks	AOs
1	Let x_{ij} be 0 or 1 $\begin{cases} 1 & \text{if worker } (i) \text{ does task } (j) \\ 0 & \text{otherwise} \end{cases}$	B1	3.3
	where $i \in \{\text{P}, \text{Q}, \text{R}, \text{S}\}$ and $j \in \{1, 2, 3\}$	B1	2.5
	minimise $C = 43x_{\text{P1}} + 56x_{\text{P2}} + 51x_{\text{P3}} + 43x_{\text{Q1}} + 59x_{\text{Q2}} + '100'x_{\text{Q3}}$ $+ '100'x_{\text{R1}} + 38x_{\text{R2}} + 62x_{\text{R3}} + 61x_{\text{S1}} + 54x_{\text{S2}} + 48x_{\text{S3}}$	M1 A1	3.3 1.1b
	Subject to $\sum x_{\text{P}j} = 1, \sum x_{\text{Q}j} = 1, \sum x_{\text{R}j} = 1, \sum x_{\text{S}j} = 1$ $\sum x_{i1} = 1, \sum x_{i2} = 1, \sum x_{i3} = 1$	M1 A1	3.3 1.1b
		(6)	
			(6 marks)

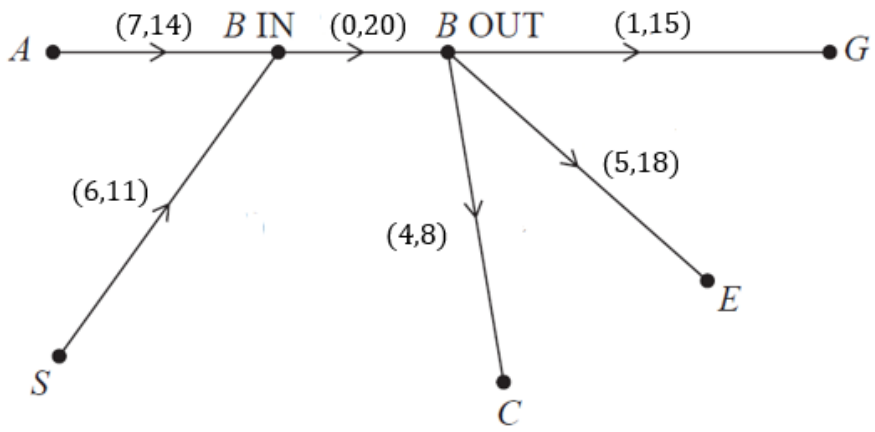
Question	Scheme	Marks	AOs
2	 EMV is £0.78 and Ralph should play the game. If he doesn't win at the first attempt he should play again since $-1 > -3$.	M1 B1 A1 M1 M1 A1 B1	3.3 1.1b 1.1b 3.4 3.4 1.1b 3.2a
		(7)	
			(7 marks)

Question	Scheme	Marks	AOs
3(a)	<table><tr><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>W</td><td>26</td><td></td><td></td><td></td></tr><tr><td>X</td><td>4</td><td>26</td><td>9</td><td></td></tr><tr><td>Y</td><td></td><td></td><td>14</td><td>2</td></tr><tr><td>Z</td><td></td><td></td><td></td><td>19</td></tr></table>	B1	1.1b
		(1)	
(b)	<table><tr><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>W</td><td>$26-\theta$</td><td></td><td></td><td>θ</td></tr><tr><td>X</td><td>$4+\theta$</td><td></td><td>$9-\theta$</td><td></td></tr><tr><td>Y</td><td></td><td></td><td>$14+\theta$</td><td>$2-\theta$</td></tr><tr><td>Z</td><td></td><td></td><td></td><td></td></tr></table> <table><tr><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>W</td><td>24</td><td></td><td></td><td>2</td></tr><tr><td>X</td><td>6</td><td>26</td><td>7</td><td></td></tr><tr><td>Y</td><td></td><td></td><td>16</td><td></td></tr><tr><td>Z</td><td></td><td></td><td></td><td>19</td></tr></table>	M1 A1	2.1 1.1b
		(2)	
(c)	<table><tr><td></td><td></td><td>25</td><td>23</td><td>27</td><td>19</td></tr><tr><td></td><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>0</td><td>W</td><td>X</td><td>4</td><td>-6</td><td>X</td></tr><tr><td>-5</td><td>X</td><td>X</td><td>X</td><td>X</td><td>7</td></tr><tr><td>-2</td><td>Y</td><td>5</td><td>0</td><td>X</td><td>3</td></tr><tr><td>1</td><td>Z</td><td>-7</td><td>2</td><td>-5</td><td>X</td></tr></table> <table><tr><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>W</td><td></td><td></td><td></td><td>$2+\theta$</td></tr><tr><td>X</td><td></td><td></td><td></td><td></td></tr><tr><td>Y</td><td></td><td></td><td></td><td></td></tr><tr><td>Z</td><td>θ</td><td></td><td></td><td>$19-\theta$</td></tr></table> <table><tr><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>W</td><td>5</td><td></td><td></td><td>21</td></tr><tr><td>X</td><td>6</td><td>26</td><td>7</td><td></td></tr><tr><td>Y</td><td></td><td></td><td>16</td><td></td></tr><tr><td>Z</td><td>19</td><td></td><td></td><td></td></tr></table> Entering cell is ZE and exiting cell is ZH	M1 A1 M1 A1	1.1b 1.1b 1.1b 2.2a
		(4)	
(d)	(£)2027	B1	1.1b
		(1)	

(e)	<table><tr><td></td><td></td><td>25</td><td>23</td><td>27</td><td>19</td></tr><tr><td></td><td></td><td>E</td><td>F</td><td>G</td><td>H</td></tr><tr><td>0</td><td>W</td><td>X</td><td>4</td><td>-6</td><td>X</td></tr><tr><td>-5</td><td>X</td><td>X</td><td>X</td><td>X</td><td>7</td></tr><tr><td>-2</td><td>Y</td><td>5</td><td>0</td><td>X</td><td>3</td></tr><tr><td>-6</td><td>Z</td><td>X</td><td>9</td><td>2</td><td>7</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> A negative II so solution is not optimal	M1 A1 A1	2.1 1.1b 2.4
		(3)	
			(11 marks)

Question	Scheme	Marks	AOs
4(a)	$x_{n+2} = 4y_{n+1} + 5 \Rightarrow x_{n+2} = 4\left(-3x_n + 2x_{n+1}\right) + 5$ Leading to $x_{n+2} - 8x_{n+1} + 12x_n = 5$ *	 B1	 2.2a
		(1)	
(b)	aux equation $m^2 - 8m + 12 = 0 \Rightarrow m = 2, m = 6$	B1	2.1
	$x_n = A(2)^n + B(6)^n$	B1	1.1b
	particular solution try $x_n = \lambda$ $\therefore \lambda - 8\lambda + 12\lambda = 5 \Rightarrow \lambda [= 1]$	M1	1.1b
	$x_n = A(2)^n + B(6)^n + 1$	A1	2.2a
	$x_1 = 1 \Rightarrow 2A + 6B = 0$	M1	1.1b
	$y_1 = a \Rightarrow x_2 = 4a + 5$	B1	3.1a
	$4A + 36B + 1 = 4a + 5$	M1	1.1b
	$A = -\frac{(a+1)}{2}, B = \frac{(a+1)}{6} \Rightarrow x_n = (a+1)(6)^{n-1} - (a+1)(2)^{n-1} + 1$ (oe)	A1	2.2a
		(8)	
(c)	As $x_6 = 38721 \Rightarrow (a+1)(6)^5 - (a+1)(2)^5 + 1 = 38721$ leading to $a = ...$	M1	3.4
	$a = 4$	A1	2.2a
		(2)	
			(11 marks)

Question	Scheme	Marks	AOs
5 (a)(i) **(ii)**	$C_1 = 14+11+11+10+12+15 = 73$ $C_2 = 15+18-3-2-3+30+15 = 70$	B1 B1	1.1b 1.1b
		(2)	
(b)	Deduces the maximum possible flow is $\leqslant 70$ litres per second	B1ft	2.2a
		(1)	
(c)	Initial flow = 32	B1	1.1b
		(1)	
(d)		M1 A1	1.1b 1.1b
		(2)	
(e)	e.g., SABGT – 5, SBGT – 2, SCFT – 2, SDFT – 4, SDT - 3	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(f)	e.g. 	B1	2.2a
		(1)	

(g)	Use of max-flow min-cut theorem Identification of cut through SA, SB, BC, CE, CF, CD and SD Value of flow = 48 Therefore it follows that the flow is maximal.	M1 A1 A1	2.1 3.1a 2.2a
		(3)	
(h)	(i)  	B1 B1	3.3 3.3
	(ii) maximum flow = 45	B1ft	2.2a
		(3)	
			(16 marks)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>Maximin</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>Action</th><th>Destination</th><th>Value</th></tr>
<tr><td rowspan="3">1</td><td>G</td><td>GT</td><td>T</td><td>17*</td></tr>
<tr><td>H</td><td>HT</td><td>T</td><td>18*</td></tr>
<tr><td>I</td><td>IT</td><td>T</td><td>20*</td></tr>
<tr><td rowspan="6">2</td><td rowspan="3">E</td><td>EG</td><td>G</td><td>Min (15,17) = 15</td></tr>
<tr><td>EH</td><td>H</td><td>Min (22,18) = 18*</td></tr>
<tr><td>EI</td><td>I</td><td>Min (16,20) = 16</td></tr>
<tr><td rowspan="3">F</td><td>FG</td><td>G</td><td>Min (19,17) = 17</td></tr>
<tr><td>FH</td><td>H</td><td>Min (14,18) = 14</td></tr>
<tr><td>FI</td><td>I</td><td>Min (27,20) = 20*</td></tr>
<tr><td rowspan="5">3</td><td>A</td><td>AF</td><td>F</td><td>Min (17,20) = 17*</td></tr>
<tr><td rowspan="2">B</td><td>BE</td><td>E</td><td>Min (24,18) = 18*</td></tr>
<tr><td>BF</td><td>F</td><td>Min (15, 20) = 15</td></tr>
<tr><td rowspan="2">C</td><td>CE</td><td>E</td><td>Min (21,18) = 18*</td></tr>
<tr><td>CF</td><td>F</td><td>Min (18, 20) = 18*</td></tr>
<tr><td>D</td><td>DE</td><td>E</td><td>Min (13,18) = 13*</td></tr>
<tr><td rowspan="4">4</td><td rowspan="4">S</td><td>SA</td><td>A</td><td>Min (23,17) = 17</td></tr>
<tr><td>SB</td><td>B</td><td>Min (24,18) = 18*</td></tr>
<tr><td>SC</td><td>C</td><td>Min (26,18) = 18*</td></tr>
<tr><td>SD</td><td>D</td><td>Min (22,13) = 13</td></tr>
</table>
</td><td>B1

M1
A1
A1

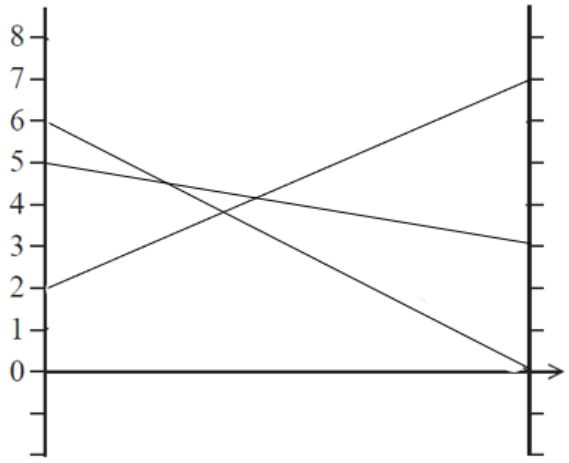
M1
A1ft
A1

M1
A1ft</td><td>3.1a

3.1a
1.1b
1.1b

1.1b
1.1b
1.1b

1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(9)</td><td></td></tr>
<tr><td>(c)</td><td>Route 1: S – B – E – H – T
Route 2: S – C – E – H – T
Route 3: S – C – F – I – T</td><td>B1ft
B1</td><td>2.2a
2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Question	Scheme	Marks	AOs
7(a)	<table><tr><td></td><td>Option X</td><td>Option Y</td><td>Option Z</td></tr><tr><td>Option Q</td><td>-4</td><td>-1</td><td>3</td></tr><tr><td>Option R</td><td>2</td><td>1</td><td>-2</td></tr></table>	M1 A1	3.1a 2.2a
		(2)	
(b)	If B plays option X, A's gains are $6p_2 = 6(1-p_1) = 6-6p_1$ If B plays option Y, A's gains are $3p_1 + 5p_2 = 3p_1 + 5(1-p_1) = -2p_1 + 5$ If B plays option Z, A's gains are $7p_1 + 2p_2 = 7p_1 + 2(1-p_1) = 5p_1 + 2$	M1 A1	3.1a 1.1b
		M1 A1	1.1b 1.1b
	$2+5p_1 = 6-6p_1 \Rightarrow p_1 = \frac{4}{11}$	A1	1.1b
	Albert should play option Q with probability $\frac{4}{11}$ and option R with probability $\frac{7}{11}$	A1ft	3.2a
		(6)	
(c)	Value of the game $= 2+5\left(\frac{4}{11}\right)-4 = -\frac{2}{11}$	B1	2.2a
		(1)	
(d)	$7q_3 = \frac{42}{11}, 6q_1 + 2q_3 = \frac{42}{11}$ or $-4q_2 + 3q_3 = -\frac{2}{11}, 2q_1 - 2q_2 = -\frac{2}{11}$	M1 A1	3.1a 1.1b
	$q_1 = \frac{5}{11}, q_2 = 0, q_3 = \frac{6}{11} \Rightarrow$ Beatrice should play option X with probability $\frac{5}{11}$, option Y, never, and option Z with probability $\frac{6}{11}$.	A1	3.2a

		(3)	
			(12 marks)