Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 15 October 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3A**

Further Mathematics

Advanced

Paper 3A: Further Pure Mathematics 1 Shadow Set 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. Use l'Hospital's Rule to show that

$$\lim_{x\to 0}\frac{\left(e^{x(x+3)}-\cos^2 x\right)}{\sin(4x)}=\frac{3}{4}$$

(5)

(Total for Question 1 is 5 marks)

2.

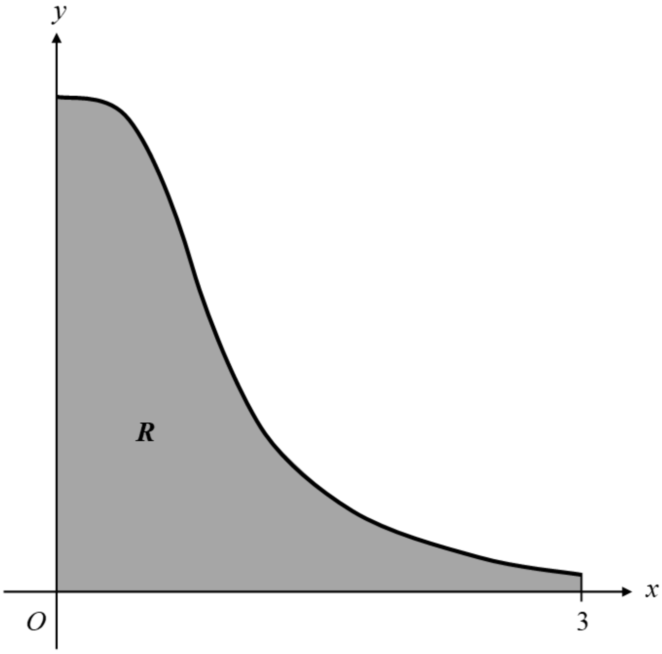

Figure 1

Figure 1 shows a sketch of the curve with equation y = f (x) where

$$f(x)=ke^{-x^2} \qquad x\in[0,3]$$

and k is a positive constant.

The shaded region R is enclosed by the curve, the x axis, and the lines $x = 0$ and $x = 3$
The area of R is 1

Use Simpson's rule with 6 intervals to obtain an estimate for the value of k

(6)

(Total for Question 2 is 6 marks)

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3. The points A, B and C, with position vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = 7\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = 8\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ respectively, lie on the plane Π

(*a*) Find $\overrightarrow{AB} \times \overrightarrow{AC}$

(3)

(*b*) Find an equation for Π in the form $\mathbf{r}.\mathbf{n} = p$

(2)

The point D has position vector $7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$

(*c*) Determine the volume of the tetrahedron $ABCD$

(4)

(Total for Question 3 is 9 marks)

4.

$$\mathrm{f}(x) = x^3\cos(2x)$$

Use Leibnitz's theorem to show that the coefficient of $\left(x - \frac{\pi}{2}\right)^7$ in the Taylor series expansion of f (x) about $\frac{\pi}{2}$ is

$$\frac{a}{b}\left(\pi^2 - c\right)$$

where a , b and c are positive integers to be determined and the fraction $\frac{a}{b}$ is given in its simplest form.

(8)

$$\left[\begin{array}{c} \textit{The Taylor series expansion of } \mathrm{f}(x) \textit{ about } x = k \textit{ is given by} \\ \mathrm{f}(x) = \mathrm{f}(k) + (x-k)\mathrm{f}'(k) + \frac{(x-k)^2}{2!}\mathrm{f}''(k) + \ldots + \frac{(x-k)^r}{r!}\mathrm{f}^{(r)}(k) + \ldots \end{array}\right]$$

(Total for Question 4 is 8 marks)

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5. The ellipse E has equation

$$\frac{x^2}{25}+\frac{y^2}{49}=1$$

The points S_1 and S_2 are the foci of E.

(a) Find the coordinates of S_1 and S_2

(3)

For any two points P_1 and P_2 that lie on E and for which the quadrilateral $P_1S_1P_2S_2$ is a parallelogram:

(b) Prove that the perimeter of $P_1S_1P_2S_2$ is 28

(4)

(Total for Question 5 is 7 marks)

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6. The speeds, in metres per second, v_A and v_B at which two particles, A and B, are moving are modelled by

$$v_A = \left|2t^2 - 15t + 18\right|$$
$$v_B = \left|23 - 6t\right|$$

respectively, where t is the time in seconds after the motion of the particles has begun.

Use algebra to find the range of time for which particle B is moving more quickly than particle A.

(8)

(Total for Question 6 is 8 marks)

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7. The points $P(36p^2, 24p)$ and $Q(36q^2, 24q)$, $p \neq q$, lie on the parabola C with equation

$$y^2 = 16x$$

The tangent to C at point P and the tangent to C at point Q meet at the point A

(*a*) Show that the coordinates of A are

$$\left(-36pq\ ,-12\left(p+q\right)\right)$$

(6)

The normal to C at P and the normal to C at Q meet at the point B.

(*b*) Show that the coordinates of B are

$$\left(4\left(9p^2+9q^2+9pq+2\right)\ ,-108pq\left(p+q\right)\right)$$

(6)

Given that the points P and Q vary such that A always lies on the line $y + 6 = 0$

(*c*) find an equation for the locus of B, giving your answer in a simplified form.

(4)

(Total for Question 7 is 16 marks)

8.

$$f(x) = \frac{2}{11 + 4\sin x - 5\cos x}$$

Using the substitution $t = \tan\left(\frac{x}{2}\right)$

(*a*) show that f (*x*) can be written in the form

$$\frac{2(1+t^2)}{(4t+1)^2 + 5}$$

(3)

(*b*) Hence solve, for $0 < x < 2\pi$, the equation

$$f(x) = \frac{5}{43}$$

giving your answers to 3 decimal places.

(5)

(*c*) Use the result of part (a) to show that

$$\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} f(x)\,dx = k\left(\arctan\left(\frac{\sqrt{5} - 4\sqrt{15}}{5}\right) - \arctan\left(\frac{\sqrt{5} + 4\sqrt{15}}{5}\right) + \pi\right)$$

where *k* is a constant to be determined.

(8)

(Total for Question 8 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS