Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

AS and A level Further Mathematics
Decision Mathematics 1

Practice Paper
Linear programming

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

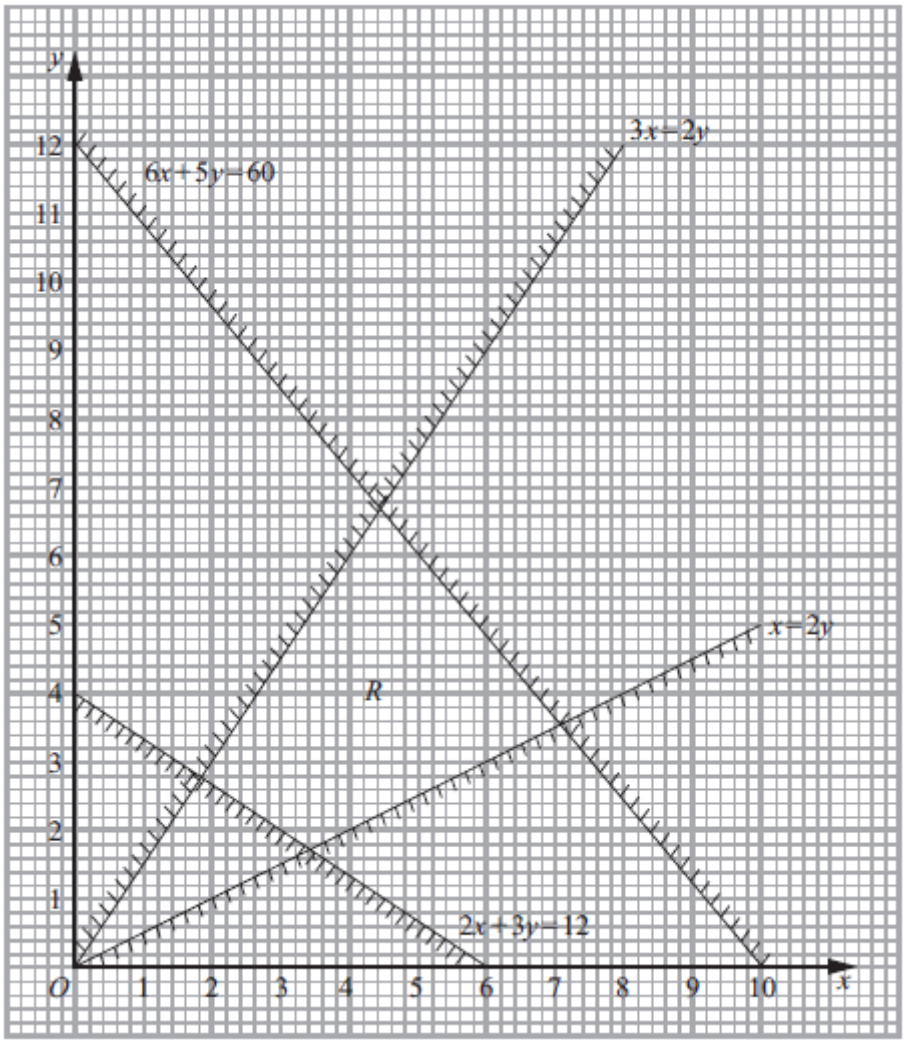

Figure 1

Figure 1 shows the constraints of a linear programming problem in x and y, where R is the feasible region.

(a) Write down the inequalities that form region R.

(2)

The objective is to maximise $3x + y$.

(b) Find the optimal values of x and y. You must make your method clear.

(4)

(c) Obtain the optimal value of the objective function.

(1)

Given that integer values of x and y are now required,

(*d*) write down the optimal values of x and y.

(1)

(Total 8 marks)

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2. A linear programming problem in x and y is described as follows.

Maximise $P = 2x + 3y$

subject to

$$x \geq 25$$

$$y \geq 25$$

$$7x + 8y \leq 840$$

$$4y \leq 5x$$

$$5y \geq 3x$$

$$x, y \geq 0$$

(*a*) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(*b*) Use the objective line method to find the optimal vertex, V, of the feasible region. You must clearly draw and label your objective line and the vertex V.

(3)

(*c*) Calculate the exact coordinates of vertex V.

(2)

Given that an integer solution is required,

(*d*) determine the optimal solution with integer coordinates. You must make your method clear.

(2)

(Total 11 marks)

3.

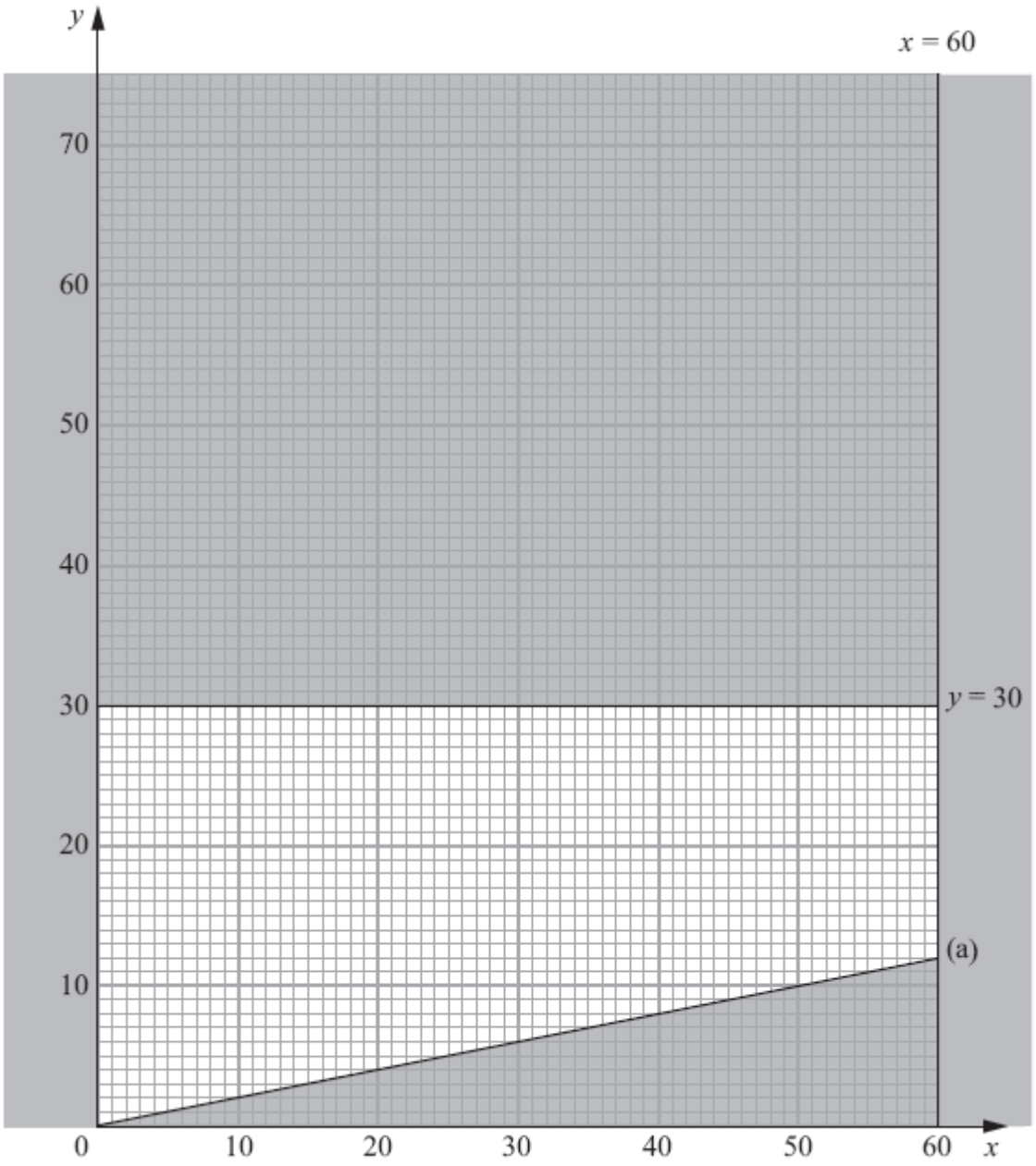

Figure 2

Lethna is producing floral arrangements for an awards ceremony.

She will produce two types of arrangement, Celebration and Party.

Let x be the number of Celebration arrangements made.

Let y be the number of Party arrangements made.

Figure 2 shows three constraints, other than $x, y \geq 0$.

The rejected region has been shaded.

Given that two of the three constraints are $y \leq 30$ and $x \leq 60$,

(*a*) write down, as an inequality, the third constraint shown in Figure 6.

(2)

Each Celebration arrangement includes 2 white roses and 4 red roses.

Each Party arrangement includes 1 white rose and 5 red roses.

Lethna wishes to use at least 70 white roses and at least 200 red roses.

(*b*) Write down two further inequalities to represent this information.

(3)

(*c*) Add two lines and shading to Diagram 1 in the answer book to represent these two inequalities.

(2)

(*d*) Hence determine the feasible region and label it R.

(1)

The times taken to produce each Celebration arrangement and each Party arrangement are 10 minutes and 4 minutes respectively. Lethna wishes to minimise the total time taken to produce the arrangements.

(*e*) Write down the objective function, T, in terms of x and y.

(1)

(*f*) Use point testing to find the optimal number of each type of arrangement Lethna should produce, and find the total time she will take.

(4)

(Total 13 marks)

4.

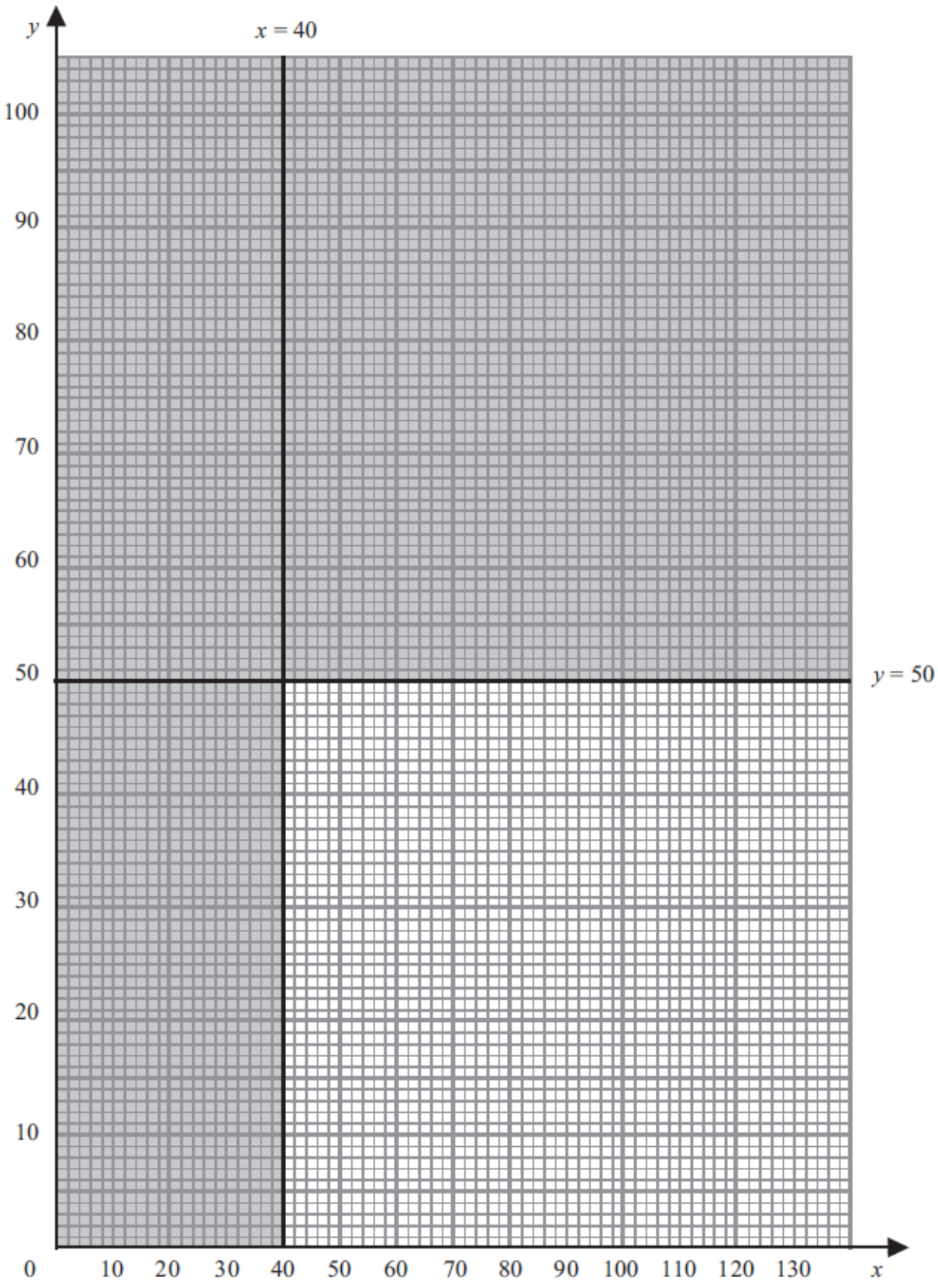
y
x = 40
100
90
80
70
60
50
y = 50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
110
120
130
x

Figure 3

Edgar has recently bought a field in which he intends to plant apple trees and plum trees.

He can use linear programming to determine the number of each type of tree he should plant.

Let x be the number of apple trees he plants and y be the number of plum trees he plants.

Two of the constraints are

$$x \geq 40$$

$$y \leq 50$$

These are shown on the graph in Figure 3, where the rejected region is shaded out.

(*a*) Use these two constraints to write down two statements that describe the number of apple trees and plum trees Edgar can plant.

(1)

Two further constraints are

$$3x + 4y \leq 360$$

$$x \leq 2y$$

(*b*) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R.

(4)

Edgar will make a profit of £60 from each apple tree and £20 from each plum tree. He wishes to maximise his profit, P.

(*c*) Write down the objective function.

(1)

(*d*) Use an objective line to determine the optimal point of the feasible region, R. You must make your method clear.

(4)

(*e*) Find Edgar's maximum profit.

(1)

(Total 11 marks)

5.

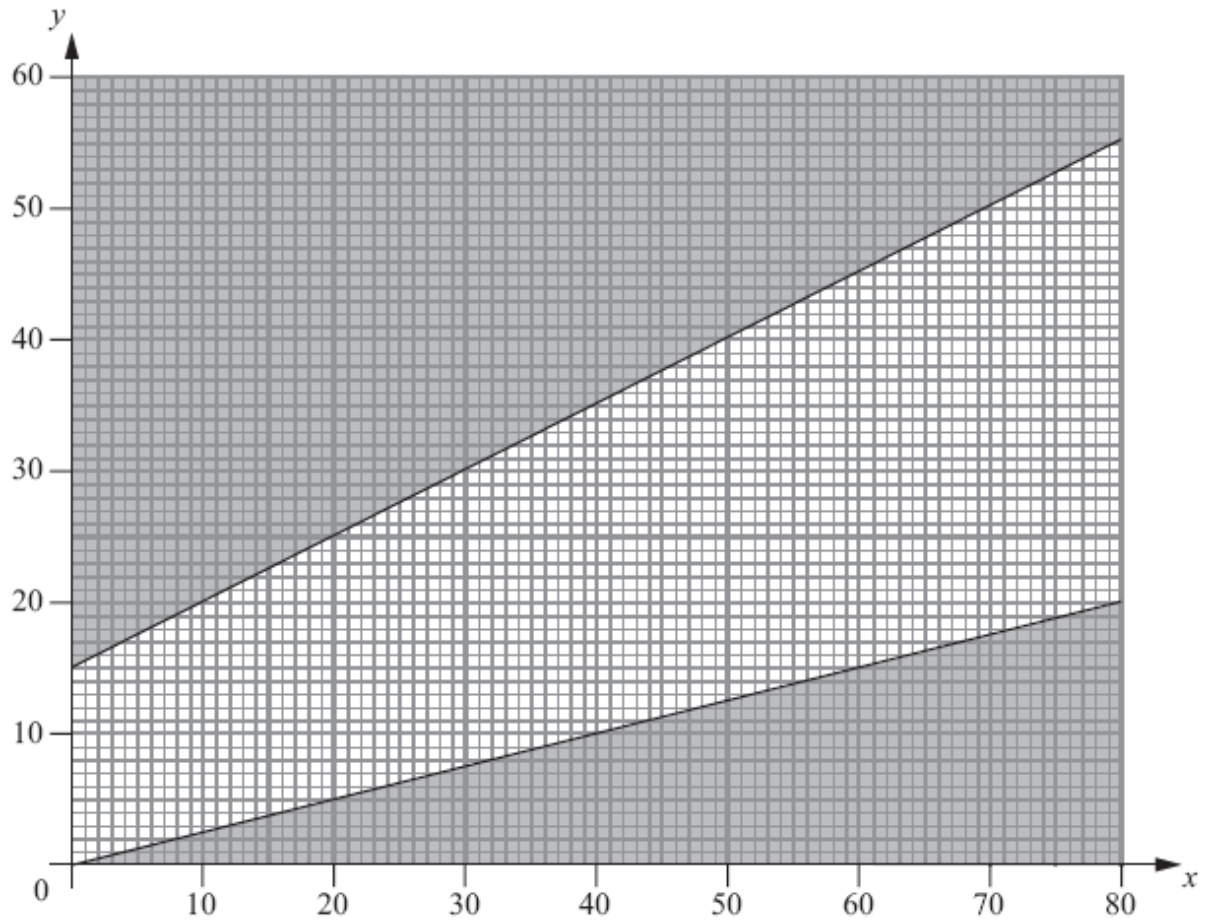

Figure 4

The graph in Figure 4 is being used to solve a linear programming problem.

Two of the constraints have been drawn on the graph and the rejected regions shaded out.

(*a*) Write down the constraints shown on the graph.

(4)

Two further constraints are

$$x + y \geq 30$$

and $\quad 5x + 8y \leq 400$

(*b*) Add two lines and shading to Graph 1 **in your answer book** to represent these constraints. Hence determine the feasible region and label it *R*.

(4)

The objective is to

$$\text{minimise } 15x + 10y.$$

(*c*) Draw a profit line on Graph 1 and use it to find the optimal solution. You must label your profit line clearly.

(3)

(Total 11 marks)

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6. A caterer can make three different sizes of salad; small, medium and large.

The caterer will make a total of at least 280 salads.

The caterer wants at least 35% of the salads to be small and no more than 20% of the salads to be large.

The caterer has enough ingredients to make 400 small salads or 300 medium salads or 200 large salads.

The profit on each small, medium and large salad is 40p, 60p and 85p respectively. The caterer wants to maximise his total profit.

Let x represent the number of small salads, y represent the number of medium salads and z represent the number of large salads.

Formulate this information as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should **not** attempt to solve the problem.

(Total 8 marks)

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7. Charlie needs to buy storage containers.

There are two different types of storage container available, standard and deluxe.

Standard containers cost £20 and deluxe containers cost £65. Let x be the number of standard containers and y be the number of deluxe containers.

The maximum budget available is £520.

(*a*) Write down an inequality, in terms of x and y, to model this constraint.

(1)

Three further constraints are:

$x \geq 2$

$-x + 24y \geq 24$

$7x + 8y \leq 112$

(*b*) Add lines and shading to Diagram 1 in the answer book to represent all four constraints. Hence determine the feasible region and label it R.

(4)

The capacity of a deluxe container is 50% greater than the capacity of a standard container. Charlie wishes to maximise the total capacity.

(*c*) State an objective function, in terms of x and y.

(1)

(*d*) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex V.

(3)

(*e*) Calculate the exact coordinates of vertex V.

(2)

(*f*) Determine the number of each type of container that Charlie should buy. You must make your method clear and calculate the cost of purchasing the storage containers.

(3)

(Total 14 marks)

8.

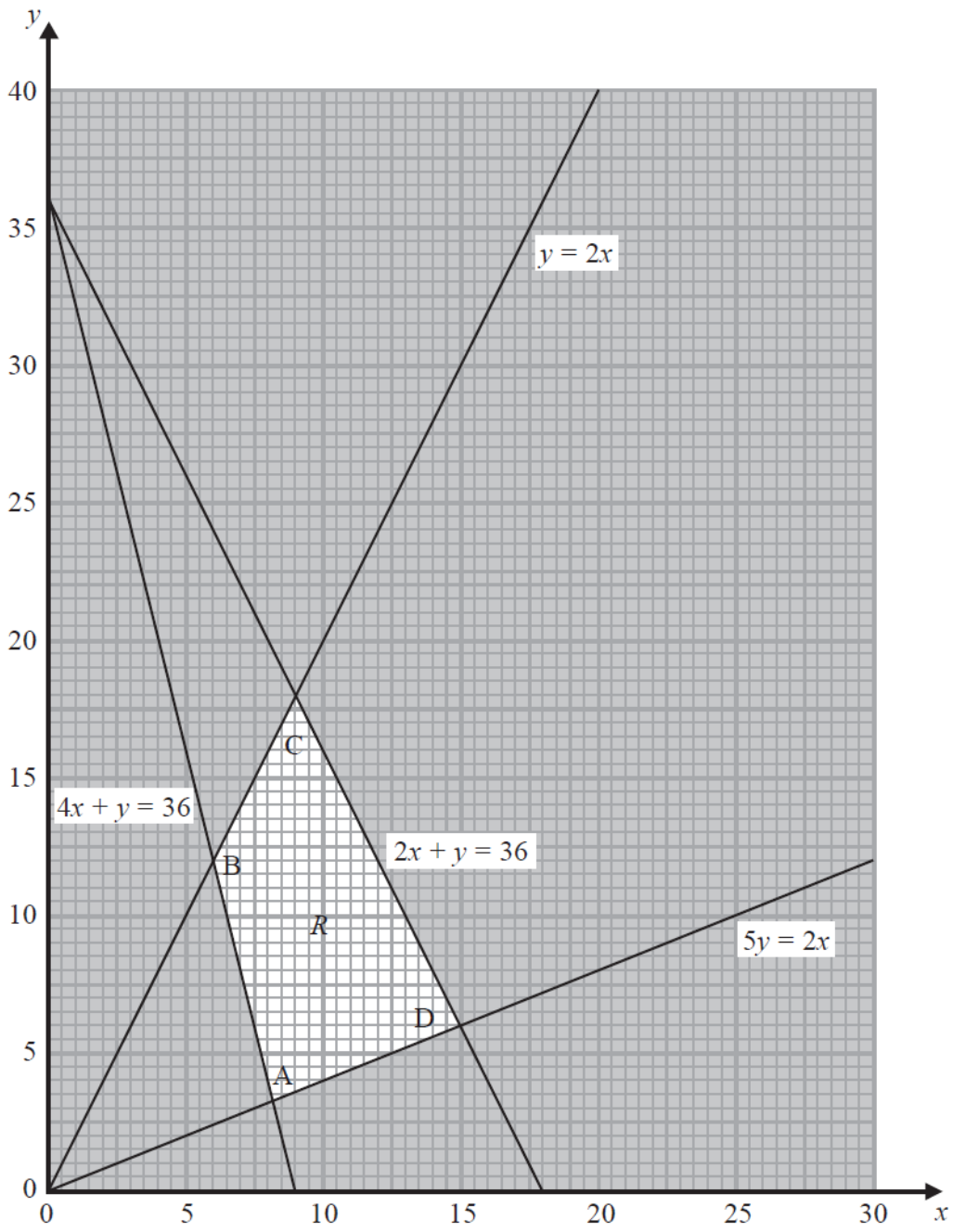

Figure 5

The graph in Figure 5 is being used to solve a linear programming problem. The four constraints have been drawn on the graph and the rejected regions have been shaded out. The four vertices of the feasible region R are labelled A, B, C and D.

(*a*) Write down the constraints represented on the graph.

(2)

The objective function, P, is given by

$$P = x + ky$$

where k is a positive constant.

The minimum value of the function P is given by the coordinates of vertex A **and** the maximum value of the function P is given by the coordinates of vertex D.

(*b*) Find the range of possible values for k. You must make your method clear.

(6)

(Total 8 marks)

9.

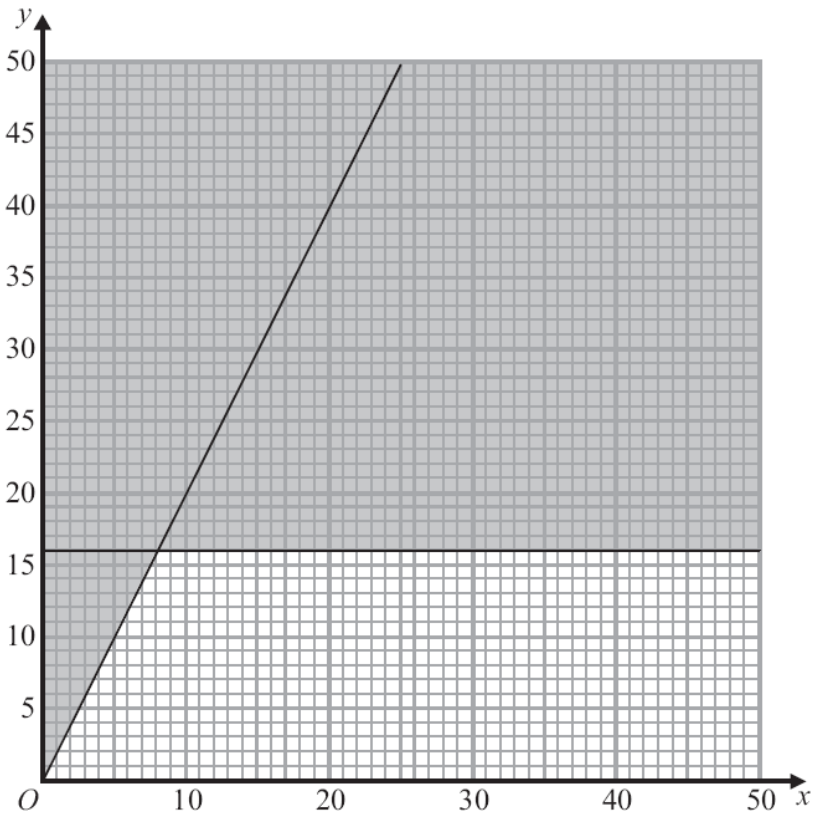

Figure 6

A company makes two types of garden bench, the 'Rustic' and the 'Contemporary'. The company wishes to maximise its profit and decides to use linear programming.

Let x be the number of 'Rustic' benches made each week and y be the number of 'Contemporary' benches made each week.

The graph in Figure 6 is being used to solve this linear programming problem.

Two of the constraints have been drawn on the graph and the rejected region shaded out.

(a) Write down the constraints shown on the graph giving your answers as inequalities in terms of x and y.

(3)

It takes 4 working hours to make one 'Rustic' bench and 3 working hours to make one 'Contemporary' bench. There are 120 working hours available in each week.

(b) Write down an inequality to represent this information.

(2)

Market research shows that 'Rustic' benches should be at most $\frac{3}{4}$ of the total benches made each week.

(*c*) Write down, and simplify, an inequality to represent this information. Your inequality must have integer coefficients.

(2)

(*d*) Add two lines and shading to Diagram 1 in your answer book to represent the inequalities of (*b*) and (*c*). Hence determine and label the feasible region, R.

(3)

The profit on each 'Rustic' bench and each 'Contemporary' bench is £45 and £30 respectively.

(*e*) Write down the objective function, P, in terms of x and y.

(1)

(*f*) Determine the coordinates of each of the vertices of the feasible region and hence use the vertex method to determine the optimal point.

(4)

(*g*) State the maximum weekly profit the company could make.

(1)

(Total 16 marks)

TOTAL FOR PAPER: 100 MARKS