Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Numerical integration (part 2)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 90.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

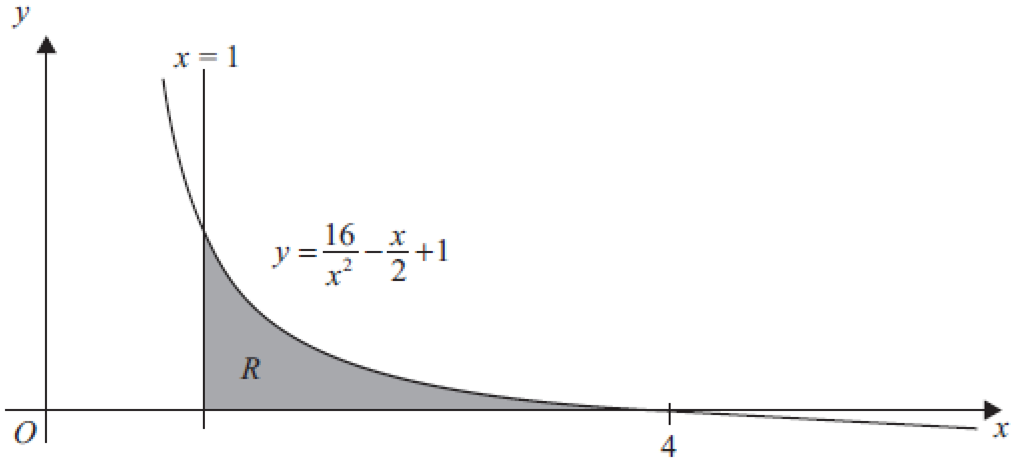

Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0.$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point (4, 0).

(*a*) Complete the table with the values of y corresponding to $x = 2$ and 2.5.

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(*b*) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) Use integration to find the exact value for the area of R.

(5)

(Total 11 marks)

2. $$y = \frac{5}{3x^2 - 2}$$

(*a*) Copy and complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

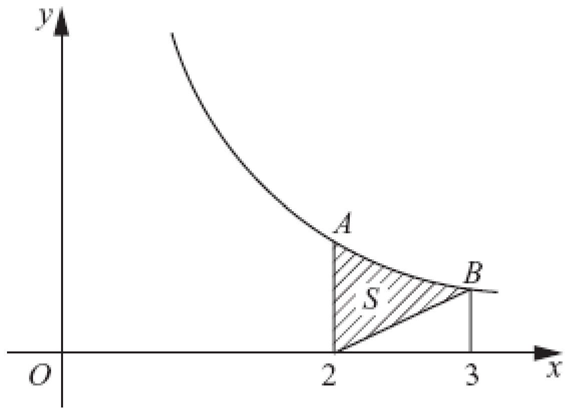

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(*c*) Use your answer to part (*b*) to find an approximate value for the area of S.

(3)

(Total 9 marks)

3. $y = \sqrt{(3^x + x)}$

(*a*) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(*b*) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x + x)}\ \mathrm{d}x$$

You must show clearly how you obtained your answer.

(4)

(Total 6 marks)

4.

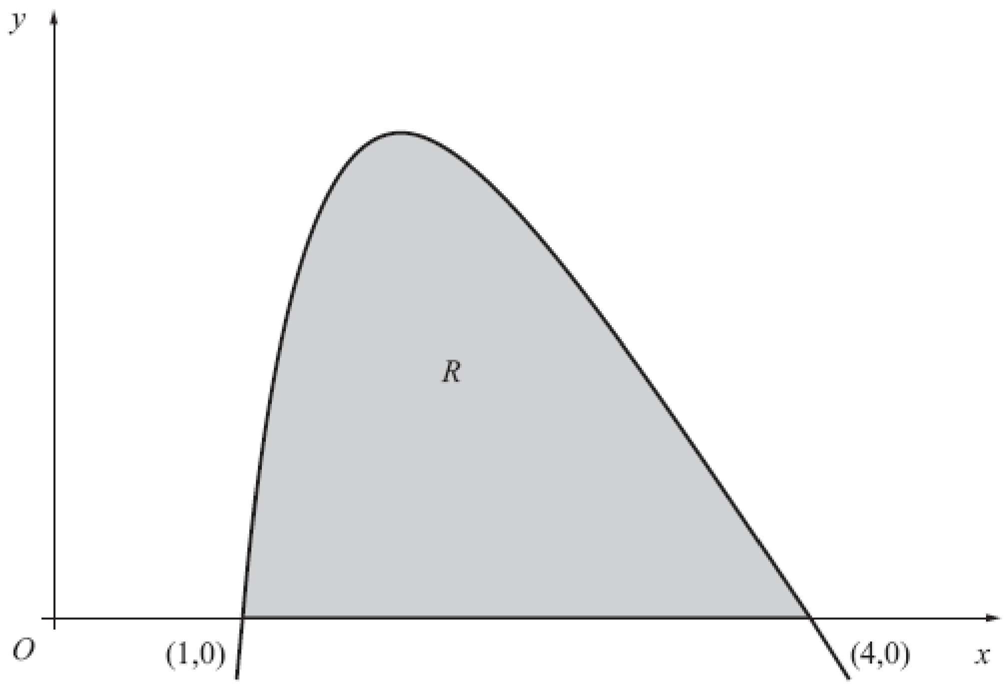

Figure 3

The finite region R, as shown in Figure 3, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0.$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(*a*) Copy and complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(*b*) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) Use integration to find the exact value for the area of R.

(6)

(Total 12 marks)

5.

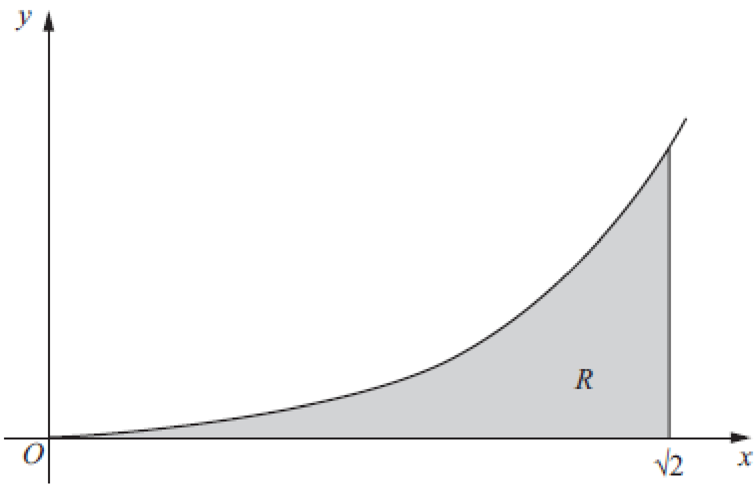

Figure 4

Figure 4 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 4, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(*a*) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(*c*) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_{2}^{4} (u-2)\ln u \ \mathrm{d}u .$$

(4)

(*d*) Hence, or otherwise, find the exact area of R.

(6)

(Total 15 marks)

Turn over

6.

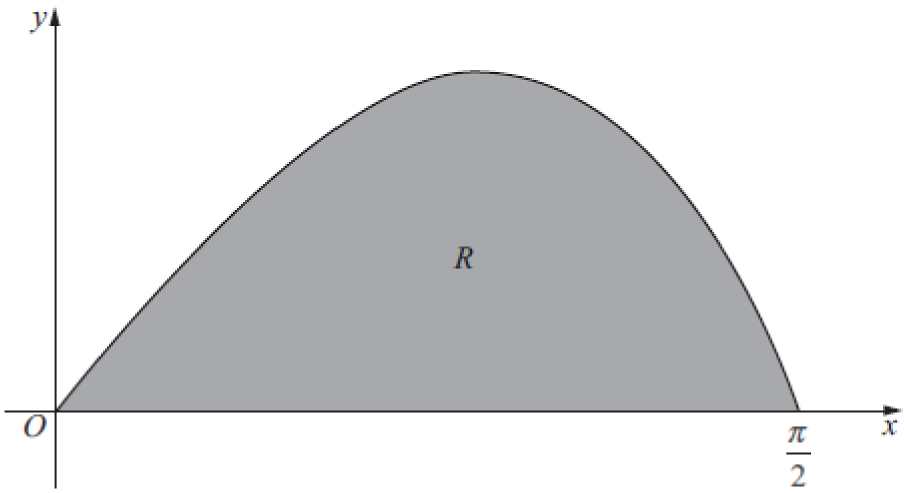

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \le x \le \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant.

(5)

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures.

(3)

(Total 12 marks)

7.

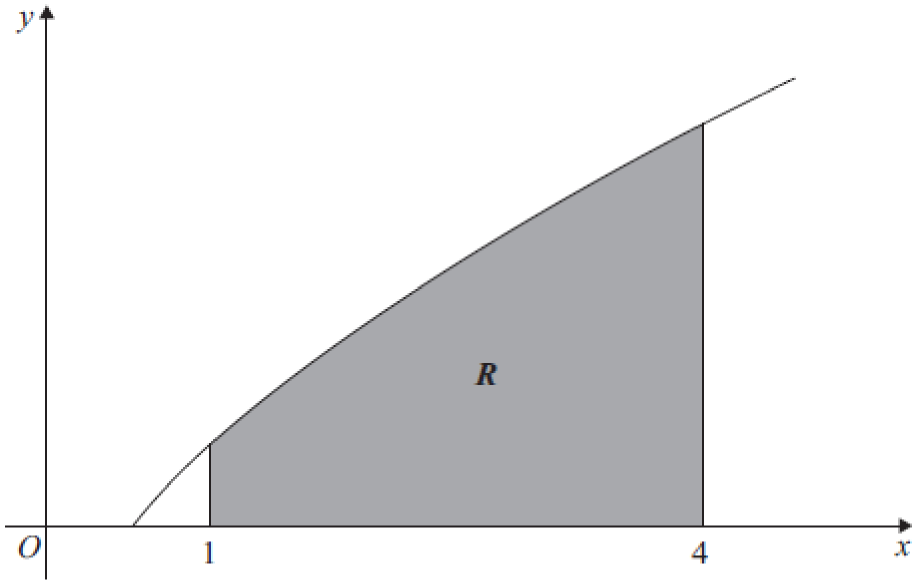

Figure 6

Figure 6 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 6, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(*a*) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*b*) Find $\int x^{\frac{1}{2}} \ln 2x \; dx$.

(4)

(*c*) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

(Total 11 marks)

8. $I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx.$

(*a*) Given that $y = \frac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(*b*) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(*c*) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

(Total 14 marks)

TOTAL FOR PAPER: 90 MARKS