Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Numerical integration (part 1)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 70.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

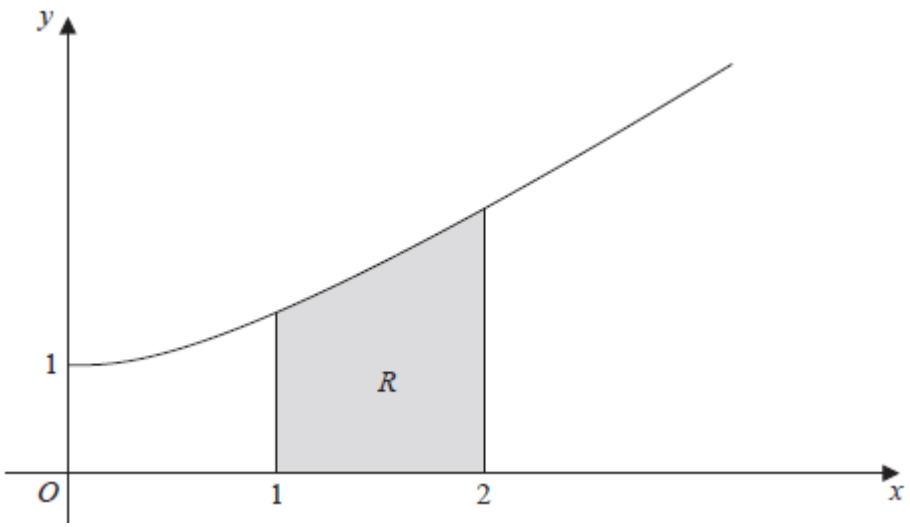

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$.

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

(*a*) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(Total 5 marks)

---

2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leq x \leq 4.$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 (8 - 2^{x-1}) \, dx$.

(3)

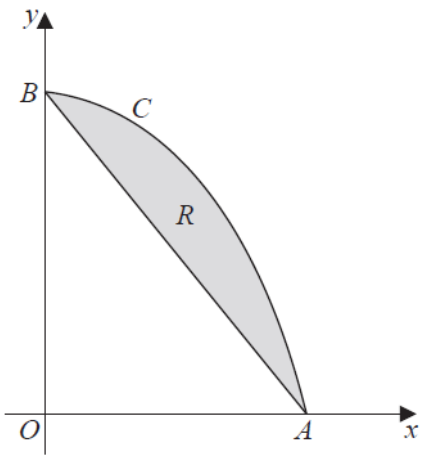

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}, \quad 0 \leq x \leq 4$.

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 2, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

(Total 6 marks)

3. (*a*) $y = 5^x + \log_2(x + 1), \quad 0 \leqslant x \leqslant 2$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821		12.502	26.585

(1)

(*b*) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x+1))\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x+1))\,\mathrm{d}x$$

giving your answer to 2 decimal places.

(1)

(Total 6 marks)

4.

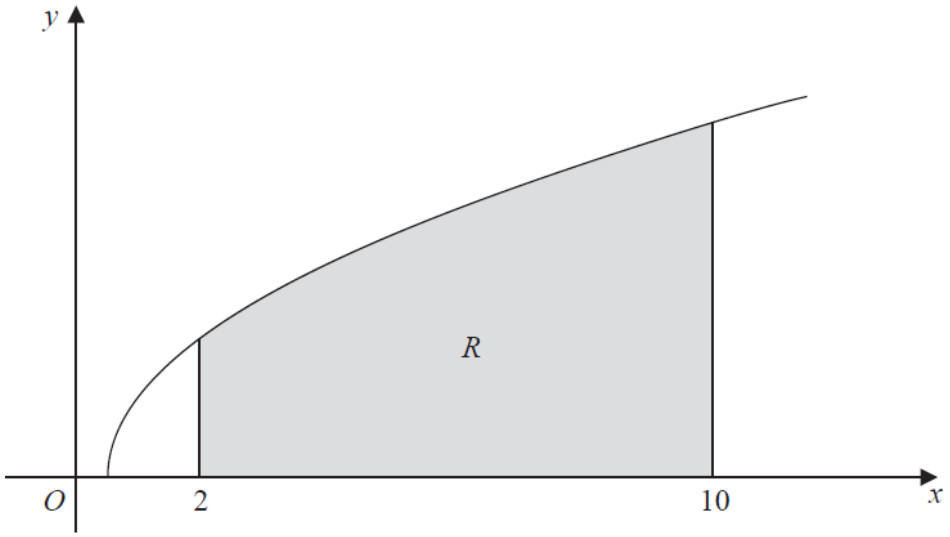

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \sqrt{(2x - 1)}, \quad x \geq 0.5$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of x and y for $y = \sqrt{(2x - 1)}$.

x	2	4	6	8	10
y	$\sqrt{3}$		$\sqrt{11}$		$\sqrt{19}$

(a) Complete the table with the values of y corresponding to $x = 4$ and $x = 8$.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(3)

(c) State whether your approximate value in part (b) is an overestimate or an underestimate for the area of R.

(1)

(Total 5 marks)

5. $y = \dfrac{5}{(x^2 + 1)}$.

(*a*) Copy and complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

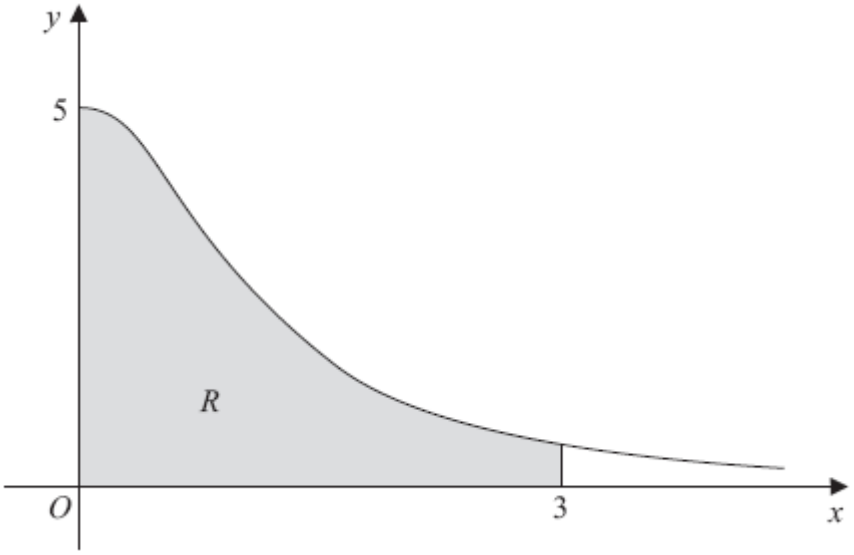

Figure 4

Figure 4 shows the region R which is bounded by the curve with equation $y = \dfrac{5}{(x^2 + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$.

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^3 4 + \frac{5}{(x^2 + 1)}\, \mathrm{d}x,$$

giving your answer to 2 decimal places.

(2)

(Total 7 marks)

6.

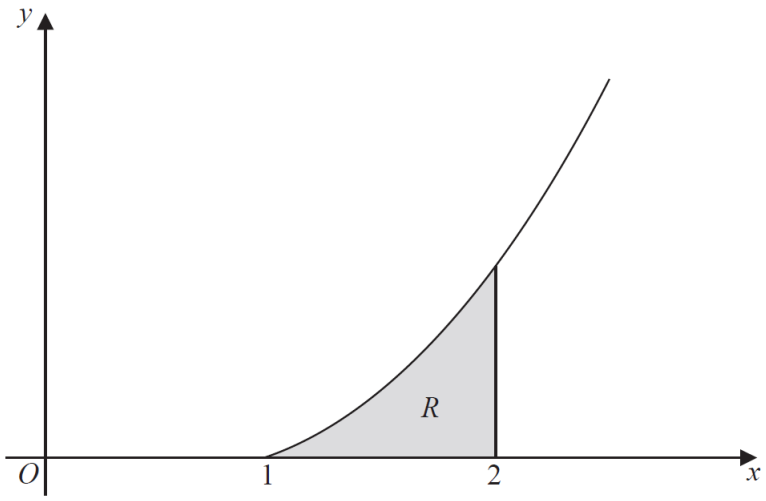

Figure 5

Figure 5 shows a sketch of part of the curve with equation $y = x^2 \ln x, \quad x \geq 1$.

The finite region R, shown shaded in Figure 5, is bounded by the curve, the x-axis and the line $x = 2$.

The table below shows corresponding values of x and y for $y = x^2 \ln x$.

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places.

(1)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(c) Use integration to find the exact value for the area of R.

(5)

(Total 9 marks)

7.

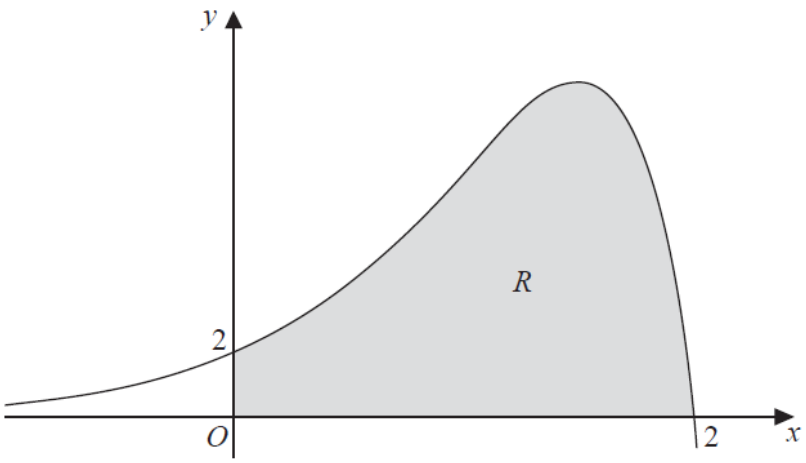

Figure 6

Figure 6 shows a sketch of part of the curve with equation

$$y = (2 - x)e^{2x}, \qquad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 6, is bounded by the curve, the x-axis and the y-axis.

The table below shows corresponding values of x and y for $y = (2 - x)e^{2x}$.

x	0	0.5	1	1.5	2
y	2	4.077	7.389	10.043	0

(a) Use the trapezium rule with all the values of y in the table, to obtain an approximation for the area of R, giving your answer to 2 decimal places.

(3)

(b) Explain how the trapezium rule can be used to give a more accurate approximation for the area of R.

(1)

(c) Use calculus, showing each step in your working, to obtain an exact value for the area of R. Give your answer in its simplest form.

(5)

(Total 9 marks)

8.

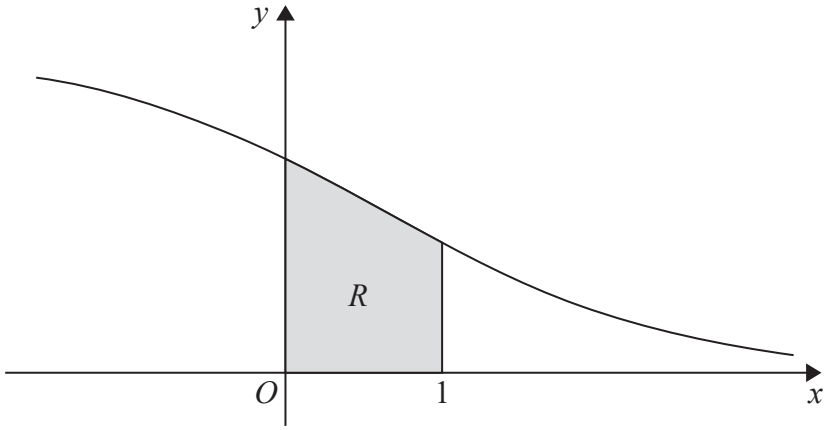

Figure 7

Figure 7 shows a sketch of part of the curve with equation $y = \dfrac{6}{(e^x + 2)}, \ x \in \mathbb{R}$

The finite region R, shown shaded in Figure 7, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \dfrac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,\mathrm{d}u$$

where a and b are constants to be determined.

(2)

(*d*) Hence use calculus to find the exact area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(Total 12 marks)

9.

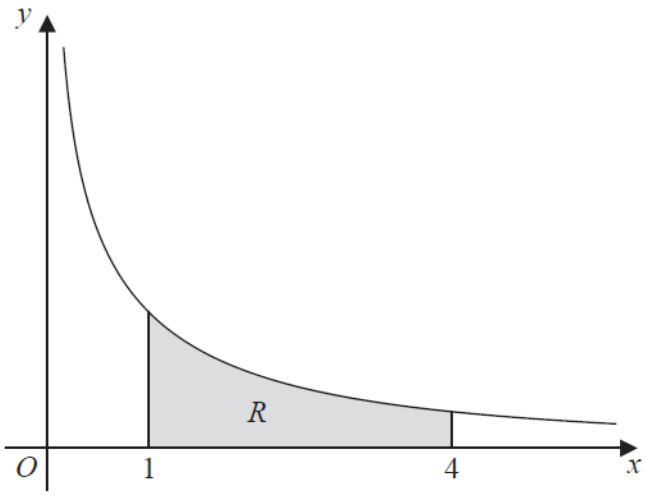

Figure 8

Figure 8 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 8, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(a) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) By reference to the curve in Figure 8, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R.

(1)

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,\mathrm{d}x$$

(6)

(Total 11 marks)

TOTAL FOR PAPER: 70 MARKS