Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Integration

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 119.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. $$\text{f}(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) (i) Hence find $\int \text{f}(x)\ \text{d}x$.

(ii) Find $\int_1^2 \text{f}(x)\ \text{d}x$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

(Total 10 marks)

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2. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x\ \ \text{d}x$.

(Total 6 marks)

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3. (*a*) Use integration to find

$$\int \frac{1}{x^3} \ln x \,\text{d}x.$$

(5)

(*b*) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \,\text{d}x.$$

(2)

(Total 7 marks)

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4. (*a*) Use integration by parts to find $\int x \sin 3x\ \ \text{d}x$.

(3)

(*b*) Using your answer to part (*a*), find $\int x^2 \cos 3x\ \ \text{d}x$.

(3)

(Total 6 marks)

Turn over

5.

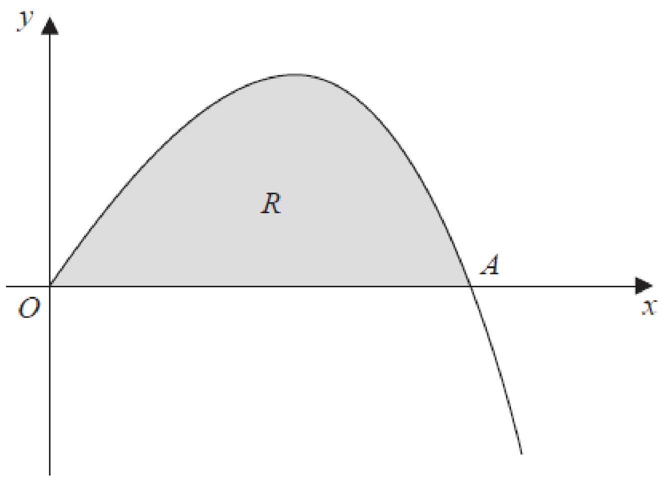

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of ln 2, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x} \, dx$.

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of ln 2.

(3)

(Total 8 marks)

Turn over

6.

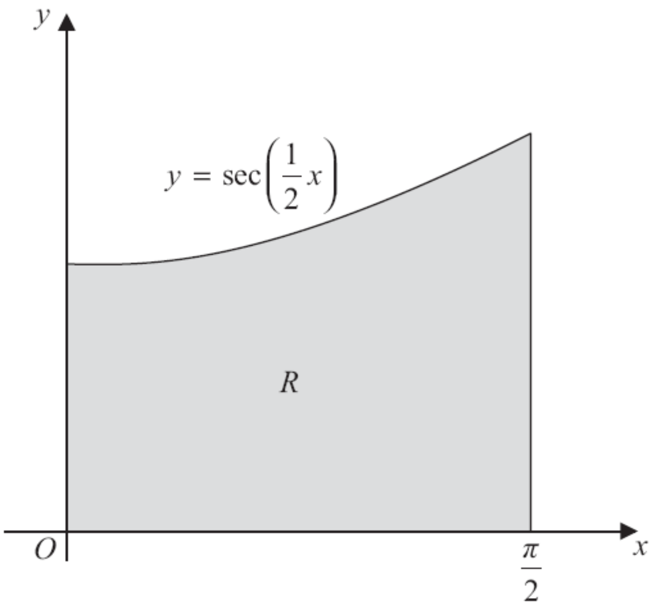

Figure 2

Figure 2 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \qquad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(*a*) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(*b*) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(*c*) Use calculus to find the exact volume of the solid formed.

(4)

(Total 8 marks)

7. (*a*) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(*b*) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$.

(3)

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\,\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$.

(6)

(Total 12 marks)

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8. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y.$$

(6)

(ii) (*a*) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta \,\mathrm{d}\theta,$$

where λ is a constant to be determined.

(5)

(*b*) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total 15 marks)

9.

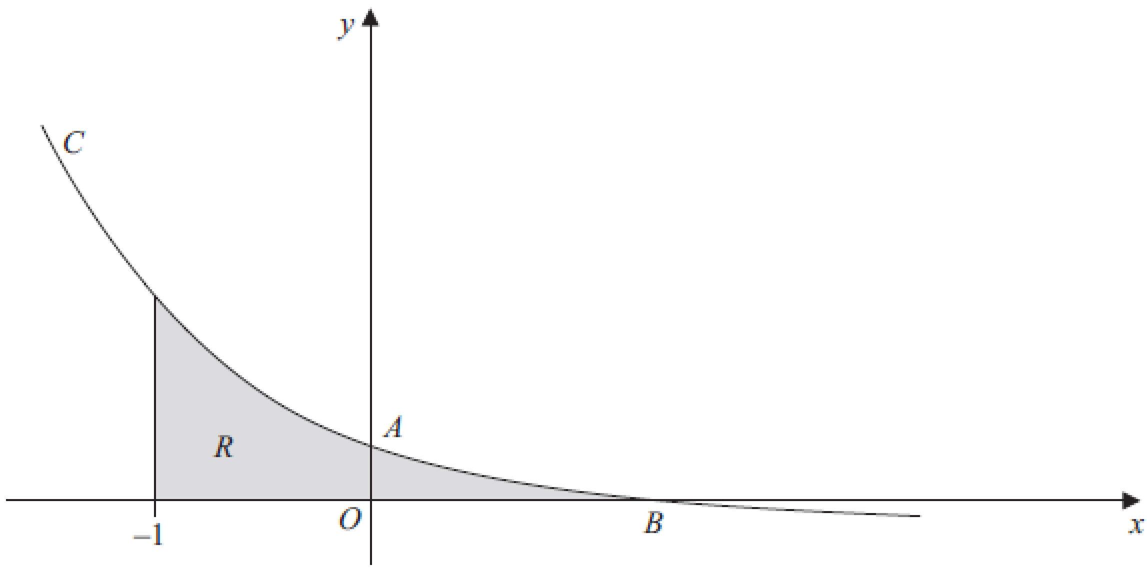

Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1.$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x-coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 3, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

(Total 15 marks)

10.

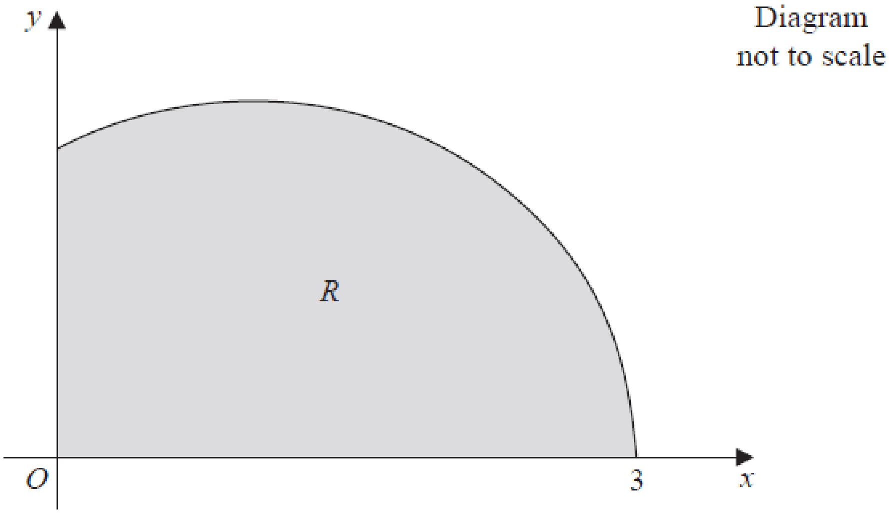

Figure 4

Figure 4 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$.

The finite region R, shown shaded in Figure 4, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

(Total 8 marks)

Turn over

11.

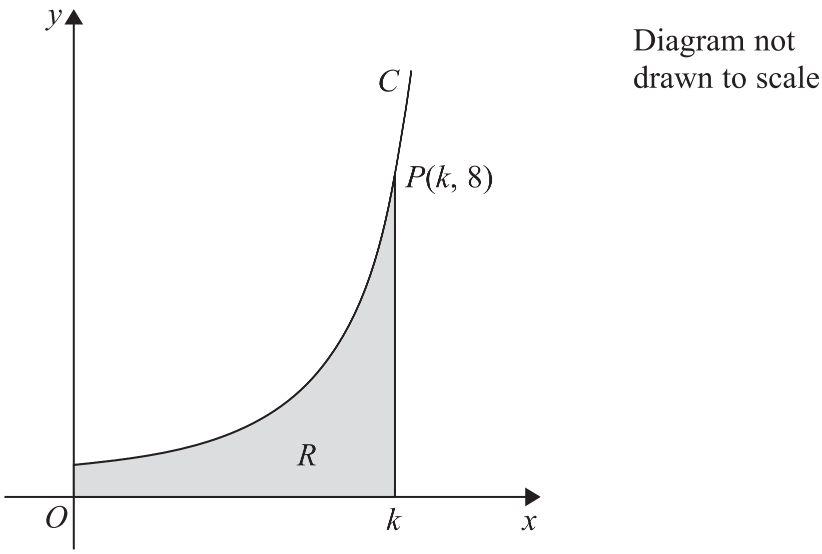

Figure 5

Figure 5 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 5, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda\int_{\alpha}^{\beta}(\theta\sec^2\theta + \tan\theta\sec^2\theta)\,\mathrm{d}\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

(Total 12 marks)

12. *a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(*c*) Hence show that the population cannot exceed 5000.

(1)

(Total 12 marks)

TOTAL FOR PAPER: 119 MARKS